A Duality Web in 2 + 1 Dimensions and the Unity of Physics

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IAS

Based on:

NS and E. Witten, arXiv:1602.04251;

NS, T. Senthil, C. Wang, and E. Witten, arXiv:1606.01989;

P.-S. Hsin, NS, arXiv:1607.07457

Recent related papers:

A. Karch and D. Tong, arXiv:1606.01893;

J. Murugan and H. Nastase, arXiv:1606.01912

Three (almost) independent lines of development – the unity of physics

- The condensed matter, $3d$ quantum field theory route
- The supersymmetric route
- The AdS/CFT, large N route

The condensed matter, $3d$ QFT route

- Statistical transmutation: a massive particle coupled to a dynamical (statistical, emergent) gauge field with a Chern-Simons term can change its spin and statistics [Wilczek; Polyakov; Jain].
  - Many applications (FQHE, composite fermions, flux attachment, …)

- This does not mean that a second-quantized theory of massless interacting bosons coupled to a gauge field with a Chern-Simons term is dual to a theory of fermions (or the other way around).

The condensed matter, $3d$ QFT route

Particle/vortex duality [Peskin; Dasgupta and Halperin]

$$|D_B\Phi|^2 - |\Phi|^4 \quad \leftrightarrow \quad |D_b\hat{\Phi}|^2 - |\hat{\Phi}|^4 + \frac{1}{2\pi}Bdb$$

- LHS is XY, $O(2)$ Wilson-Fisher.
- B is a background field coupled to a global $U(1)_B$ symmetry.
- RHS is a gauged version of this theory. b is a dynamical field.
- IR duality – two different theories flowing to the same IR fixed point.
- $\Phi \leftrightarrow \mathcal{M}_{\mathrm{b}}$ is a monopole operator of b (charged under $U(1)_B$).
- $|\Phi|^2 \leftrightarrow -|\hat{\Phi}|^2$. Upon deformation: unbroken $U(1)_B$ phase is Higgs phase in the RHS; broken $U(1)_B$ phase massless b.

The condensed matter, $3d$ QFT route

Boson/fermion duality [Chen, Fisher, Wu; Barkeshli, McGreevy]

$$|D_B\Phi|^2 - |\Phi|^4 \quad \leftrightarrow \quad i\bar{\Psi}\not{D}_a\Psi + \frac{1}{2\pi}Bda$$

LHS Wilson-Fisher fixed point (B is a background gauge field)

RHS QED with gauge field a with a single fermion, a.k.a $U(1)_{1/2}$

- Arguments involve elementary fields with fractional charges and fractional level Chern-Simons terms.
- LHS is T-reversal invariant, while RHS seems like it is not.
- LHS does not need a spin structure, while RHS does. Violating gravitational ‘t Hooft matching conditions?
- The IR behavior of the RHS is debated.

The condensed matter, $3d$ QFT route

Fermion/fermion duality [Son; Wang, Senthil; Metlitski, Vishwanath]

$$i\bar{\Psi}\not{D}_A\Psi \quad \leftrightarrow \quad i\bar{\chi}\not{D}_a\chi + \frac{1}{4\pi}Ada$$

- Motivated by
  - physics of the lowest Landau level at half-filling [Halperin, Lee, Read]
  - T-Pfaffian state of topological insulators [Chen, Fidkowski, Vishwanath].
- Improperly quantized Chern-Simons term
- LHS is T-reversal invariant (with anomaly) and RHS seems like it is not. Its IR behavior is debated.

The supersymmetric route

- Many dualities of $4d\ \mathcal{N}=1$ theories (IR dualities) [NS; …]
- They motivated many dualities in $3d$
  - $\mathcal{N}=2$ [Aharony, Hanany, Intriligator, NS, Strassler; Aharony; Giveon, Kutasov; … Benini, Closset, Cremonesi; Intriligator, NS; Aharony, Razamat, NS, Willett; Park, Park; …]
  - $\mathcal{N}=4\ 3d$ mirror symmetry [Intriligator, NS; …]
- These use particle/vortex duality
- Later derived by compactification of $4d\ \mathcal{N}=1$ dualities on a circle and then flow with relevant operators [Aharony, Razamat, NS, Willett].
  - More checks
  - Leads to many new dualities

The supersymmetric route

- Many checks using supersymmetry and localization
- Related to string duality
- Connected to level/rank duality of $3d$ topological quantum field theory and $2d$ RCFT (rigorous [...; Hsin, NS]).

- Can flow from them to non-supersymmetric theories [Jain, Minwalla, Yokoyama; Gur-Ari, Yacoby; ...].
  - This motivates non-supersymmetric dualities.
  - But the flow might not be smooth.

The AdS/CFT, large N route

- Same $4d$ Vasiliev theory is dual to two different $3d$ field theories [Vasiliev ; Sezgin, Sundell; Klebanov, Polyakov; Giombi, Yin; Aharony, Gur-Ari, Yacoby; Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin].
  - Scalars coupled to a Chern-Simons gauge theory
  - Fermions coupled to a Chern-Simons gauge theory
- Hence, a purely field theoretic duality between them
- Many explicit checks of this duality at large N [Maldacena, Zhiboedov; Aharony, Giombi, Gur-Ari, Maldacena, Yacoby; Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama; Minwalla, Yokoyama; Yokoyama; Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama; Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama; …]

Synthesis [Aharony]

3 [Aharony] + 1 [Hsin, NS] conjectures

N_f scalars at $|\Phi|^4$ point coupled to $\quad$ N_f fermions coupled to

- $SU(N)_k \quad \leftrightarrow \quad U(k)_{-N+\frac{N_f}{2},-N+\frac{N_f}{2}}$
- $U(N)_{k,k} \quad \leftrightarrow \quad SU(k)_{-N+\frac{N_f}{2},-N+\frac{N_f}{2}}$
- $U(N)_{k,k+N} \quad \leftrightarrow \quad U(k)_{-N+\frac{N_f}{2},-N-k+\frac{N_f}{2}}$
- $U(N)_{k,k-N} \quad \leftrightarrow \quad U(k)_{-N+\frac{N_f}{2},-N+k+\frac{N_f}{2}}$

Fits the large N picture ($N,k \to \infty$ with finite N/k)

Fits the supersymmetric picture

Related to level/rank duality

Baryon and monopole operators match [Radicevic]

Puzzles

- Is it true for all N, k, N_f?
- How can a theory of bosons, which does not need a spin structure be dual to a theory of fermions, which needs it?
- What is the relation to the dualities in the condensed matter literature (with puzzles about quantization of coefficients, T-reversal invariance, etc.)?
- What is the precise statement of the dualities (including the coupling to background gauge fields and their Chern-Simons counterterms)?
- Are the assumptions independent? Can we assume some of these dualities and derive others?
- Are there other such dualities?

Examine $N = k = N_f = 1$

Assume: a free fermion coupled to a background A is dual to a gauged Wilson-Fisher fixed point with Chern-Simons interaction for the dynamical field b

$$i\bar{\Psi} \not{D}_A \Psi \quad \leftrightarrow \quad |D_b \Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb$$

- Same symmetries
  - $U(1)_A$
  - T-reversal invariance (with anomaly) of the RHS follows from particle/vortex duality. T-reversal is a quantum symmetry there. (More below.)
- Conversely, assuming this duality we derive the known particle/vortex duality.

$$i\bar{\Psi}\not{D}_A\Psi \quad \leftrightarrow \quad |D_b\Phi|^2 - |\Phi|^4 + \frac{1}{4\pi}bdb + \frac{1}{2\pi}Adb$$

Mapping of operators

- $\Psi \ \leftrightarrow \ \Phi^+\mathcal{M}_b$ with $\mathcal{M}_b$ a monopole operator (charged under $U(1)_A$)
- Mass term $\overline{\Psi}\,\Psi \ \leftrightarrow \ |\Phi|^2$
- Mass deformation leads to two phases depending on the sign…

$$i\bar{\Psi}\not{D}_A\Psi \quad \leftrightarrow \quad |D_b\Phi|^2 - |\Phi|^4 + \frac{1}{4\pi}bdb + \frac{1}{2\pi}Adb$$

- Mass deformation depends on the sign:
  - Φ is massive with spin $\frac{1}{2}$. It is charged under $U(1)_b$ and $U(1)_A$. It is mapped to the massive Ψ. (Statistical transmutation of massive particles.)
  - $U(1)_b$ is Higgsed. Vortex of spin $-\frac{1}{2}$ is charged under $U(1)_A$. It is mapped to the massive Ψ.
- T changes the sign of the fermion mass ($=$ boson mass square) and maps Φ particles to vortices.

$$i\bar{\Psi} \not{D}_A \Psi \quad \leftrightarrow \quad |D_b\Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb$$

- Here both sides of the duality need a spin structure (spinors in the LHS and odd Chern-Simons level in the RHS)
  - But if A is a spin$_c$ connection,

$$\int \frac{dA}{2\pi} = \frac{1}{2} \int w_2 \mathrm{mod}\, \boldsymbol{Z}\,,$$

  there is no need for a spin structure on either side.

Derive many other dualities

Starting with

$$i\bar{\Psi} \not{D}_A \Psi \quad \leftrightarrow \quad |D_b\Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb$$

we can derive other dualities by changing the two sides:

- Add a Chern-Simons counterterm for the classical field A
- Gauge it by turning A into a dynamical field a and adding a new classical field.
- Use other dualities.
- Repeat.

Another boson/fermion duality

For example, derive:

$$|D_B\Phi|^2 - |\Phi|^4 \quad \leftrightarrow \quad i\bar{\Psi}\not{D}_a\Psi + \frac{1}{2\pi}Bda - \frac{1}{4\pi}BdB$$

LHS Wilson-Fisher fixed point

RHS QED with a single fermion, a.k.a $U(1)_{1/2}$

- Derived from the other duality
- Neither side needs a spin structure when a is a spin$_c$ connection
- Need a Chern-Simons counterterm for B
- Can map the operators and check the phases
- RHS is T-reversal invariant (quantum symmetry)

A fermion/fermion duality

Derive: $i\bar{\Psi}\not{D}_A\Psi \quad \leftrightarrow$

$$i\bar{\chi}\not{D}_a\chi - \frac{2}{4\pi}bdb + \frac{1}{2\pi}adb + \frac{1}{2\pi}Adb - \frac{1}{4\pi}AdA$$

LHS free fermion

RHS QED with a single fermion, coupled to $U(1)_{-2}$ of b.

- If we incorrectly integrate out b, we find the previously mentioned version with improperly quantized Chern-Simons terms.
- No need for a spin structure when a and A are spin_c connections.
- Can map the operators and check the phases
- T-reversal invariance (with anomaly) is manifest in LHS. It acts non-trivially in the RHS (quantum symmetry).

More

- ✓ Many more dualities and relations between them
- ✓ Add gravitational Chern-Simons counterterms (more checks)
- ✓ Relation to $4d$ S-duality in half-space with these $3d$ theories on the boundary (Witten's S and T operations on $3d$ field theories)
- ✓ Generalization to arbitrary N and k
  - Using a precise version of level/rank duality
  - Problem with large N_f
  - Leads to many more dualities
- ❑ Much more can be done