STRING THEORY IN THE TWENTIETH CENTURY

John H. Schwarz

Strings 2016 – August 1, 2016

ABSTRACT

String theory has been described as 21st century science, which was discovered in the 20th century. Most of you are too young to have experienced what happened. Therefore, I think it makes sense to summarize some of the highlights in this opening lecture.

Since I only have 25 minutes, this cannot be a comprehensive history. Important omitted topics include 2d CFT, string field theory, topological string theory, string phenomenology, and contributions to pure mathematics. Even so, I probably have too many slides.

1960 – 68: The analytic S matrix

The goal was to construct the S matrix that describes hadronic scattering amplitudes by assuming

- Unitarity and analyticity of the S matrix

- Analyticity in angular momentum and Regge Pole Theory

- The bootstrap conjecture, which developed into Duality (e.g., between s-channel and t-channel resonances)

The dual resonance model

In 1968 Veneziano found an explicit realization of duality and Regge behavior in the narrow resonance approximation:

$$A(s,t) = g^2 \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))},$$

$$\alpha(s) = \alpha(0) + \alpha' s.$$

The motivation was phenomenological. Incredibly, this turned out to be a tree amplitude in a string theory!

Soon thereafter Virasoro proposed, as an alternative,

$$T = \frac{g^2\,\Gamma(-\frac{\alpha(s)}{2})\Gamma(-\frac{\alpha(t)}{2})\Gamma(-\frac{\alpha(u)}{2})}{\Gamma(-\frac{\alpha(t)+\alpha(u)}{2})\Gamma(-\frac{\alpha(s)+\alpha(u)}{2})\Gamma(-\frac{\alpha(s)+\alpha(t)}{2})},$$

which has similar virtues.

The N-particle generalization of the Veneziano formula (due to multiple contributors) is

$$A_N = g^{N-2}\int \mu(y)\prod_i dy_i \prod_{i<j}(y_i - y_j)^{\alpha' k_i\cdot k_j}.$$

For a suitable measure $\mu(y)$ this has cyclic symmetry in the N external lines. $T = \sum C_N A_N$. The coefficients C_N are Chan–Paton factors.

Shapiro's generalization of the Virasoro formula

$$T_N = g^{N-2} \int \mu(z) \prod_i d^2 z_i \prod_{i<j} |z_i - z_j|^{\alpha' k_i \cdot k_j}$$

has total symmetry in the N external lines.

Fubini (1928 - 2005) and Veneziano showed that these formulas have a consistent factorization on a spectrum of single-particle states described by an infinite number of oscillators

$$\{a_m^\mu\} \qquad \mu = 0, 1, \ldots, d-1 \qquad m = 1, 2, \ldots$$

There is one set of such oscillators in the Veneziano case and two sets in the Shapiro–Virasoro case.

Nambu, Nielsen, and Susskind independently interpreted the spectrum and amplitudes as the theory of a relativistic string: open strings in the first case and closed strings in the second case. Having found the spectrum and tree amplitudes, it became possible to study radiative corrections (loop amplitudes) starting with Kikkawa (1935 - 2013), Sakita (1930 - 2002), and Virasoro.

For lack of time, I am being sketchy about who did what. However, I would like to emphasize the contributions of two people who passed away within the past year: Stanley Mandelstam and Yoichiro Nambu.

Stanley Mandelstam (1928 - 2016)

Stanley joined Geoffrey Chew's "nuclear democracy" program in Berkeley in 1963. Its goal was to derive the hadronic S matrix from analyticity, unitarity, and the bootstrap conjecture. I was a graduate student there in the heyday of this activity (1962 - 66). Geoff and Stanley influenced my scientific development profoundly.

Stanley introduced the variables s, t, u and derived the "Mandelstam representation", which is a double dispersion relation in the stu plane.

Stanley made important contributions to Regge-pole theory and proved the existence of branch points in the angular momentum plane. He proved the finiteness of $\mathcal{N} = 4$ super YM theory, as well as the equivalence of the Sine-Gordon model and the massive Thirring model.

Among his many contributions to string theory, Stanley obtained evidence for the UV finiteness of multiloop string amplitudes, derived the fermion-fermion amplitude in the RNS model, and contributed to the construction of light-cone gauge string field theory.

Yoichiro Nambu (1921 - 2015)

After his formative years in Japan, where he worked with Tomonaga and others, Nambu came to the US in 1952. He spent a couple of years at the IAS, after which he joined Fermi's powerhouse group (Chew, Goldberger, Low, Lee, Yang, etc.) at the University of Chicago. He remained there for the rest of his career.

Nambu contributed to a wide range of topics in physics. He received the Nobel Prize in Physics in 2008 for the concept of spontaneous symmetry breaking, which he developed while studying superconductivity.

Nambu was an important early contributor to string theory in 1969 - 70. He was one of the first (perhaps the first) to recognize that the Veneziano model is actually the quantum theory of a string. He introduced the elegant Nambu–Goto action for the string. It is the area of the string world-sheet:

$$S = T \int \sqrt{- \det G_{\alpha\beta}}\, d^2\sigma,$$

where T is the string tension, the induced metric is

$$G_{\alpha\beta} = g_{\mu\nu}(x)\partial_\alpha x^\mu \partial_\beta x^\nu ,$$

and $g_{\mu\nu}(x)$ is the target-space metric.

Critical string theory

In 1970 Gross, Neveu, Scherk (1946 - 80) and I, as well as Frye and Susskind, studied one-loop amplitudes. Unexpected branch points that violate unitarity appeared in the "nonplanar" open-string loop amplitude.

In 1971 Claud Lovelace (1934 - 2012) observed that these singularities become poles if $\alpha(0) = 1$ and $d = 26$. For these choices the theory is perturbatively consistent (though there are tachyons). The new poles are interpreted as closed-string modes. This was the discovery of open-string/closed-string duality.

This discovery forced us to take $d > 4$ seriously for the first time. Since we knew about Kaluza and Klein, it should have motivated us to think about gravity, but that took a few more years.

The critical string theory has an infinite set of gauge invariances that eliminate unphysical ghosts. These are encoded in the Virasoro algebra. Two different proofs of the no-ghost theorem were devised by Brower and by Goddard and Thorn in 1972. In this case, only transverse excitations of the string give independent physical degrees of freedom, as was explained by Goddard, Goldstone, Rebbi, and Thorn in 1973.

The RNS model

In January 1971 Pierre Ramond introduced a string theory analog of the Dirac equation. His proposal was that just as the string's momentum p^μ is the zero mode of a string density $P^\mu(\sigma)$, the Dirac matrices γ^μ should be the zero modes of densities $\Gamma^\mu(\sigma)$.

Then he defined

$$F_n = \frac{1}{2\pi}\int_0^{2\pi} e^{-in\sigma}\Gamma \cdot P d\sigma \qquad n \in \mathbb{Z}.$$

In particular, $F_0 = \gamma \cdot p +$ oscillator terms.

Ramond then proposed the wave equation

$$(F_0 + M)|\psi\rangle = 0,$$

which deserves to be called the Dirac–Ramond Equation.

He also observed that the Virasoro algebra of the bosonic string theory generalizes to a super-Virasoro algebra with odd elements F_n and even elements L_n. This was one of the first superalgebras in the literature. Golfand and Likhtman's 4d super-Poincaré algebra appeared at about the same time (though its appearance was delayed by the Soviet authorities).

A couple of months later André Neveu and I introduced a second interacting bosonic string theory, which we called the dual pion model. It has a similar structure to Ramond's theory, but the periodic density $\Gamma^\mu(\sigma)$ is replaced by an antiperiodic one $H^\mu(\sigma)$. Then the modes

$$G_r = \frac{1}{2\pi}\int_0^{2\pi} e^{-ir\sigma} H \cdot P d\sigma \qquad r \in \mathbb{Z} + 1/2$$

are the odd elements of a similar super-Virasoro algebra.

Together with Thorn, we showed that these bosons and Ramond's fermions combine into a unified interacting theory.

Later in 1971 Gervais and Sakita observed that the RNS string world-sheet theory

$$S = T \int d\sigma d\tau \left(\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \gamma^\alpha \partial_\alpha \psi_\mu\right)$$

has two-dimensional supersymmetry.

In 1976 this global supersymmetry and the super-Virasoro constraints were shown to result from gauge fixing an action with local 2d supersymmetry (Brink, Di Vecchia, Howe; Deser, Zumino). This formulation was used by Polyakov in his 1981 study of string theory path integrals.

The decline of string theory

String theory, as a theory of hadrons, had problems: an unrealistic dimension of space (25 or 9) and the presence of massless particles. Moreover, the success of QCD made the effort to formulate a theory of hadrons based on strings less pressing.

Exciting developments in 1973 - 74 included the completion and acceptance of the standard model as well as grand unification. Understandably, quantum field theory regained respectability and string theory rapidly fell out of favor.

Gravity and unification

In 1974 Yoneya interpreted the massless spin 2 state (*i.e.*, symmetric tensor) in the closed-string spectrum as a graviton. Invoking a theorem of Weinberg, it was easy to show that string theory agrees with general relativity at low energies.

Also, the massless spin 1 states in the open-string spectrum could be interpreted as gauge fields. This had been shown in 1971 by Neveu and Scherk.

In 1974 Scherk and I proposed to interpret string theory as a quantum theory of gravity, unified with the other forces, rather than as a theory of hadrons. At this time, I knew what I would be doing for the rest of my career.

This proposal had several obvious advantages:

- Gravity was required by the theory
- String theory has no UV divergences
- Extra dimensions could be a good thing
- Unification of gravity with forces described by Yang–Mills theories was automatic

Supersymmetry and supergravity

Wess (1934 - 2007) and Zumino (1923 - 2014), motivated by the Gervais–Sakita result, pioneered supersymmetric field theory in 1973 - 74. Subsequent work that is relevant to string theory includes

- $\mathcal{N} = 1$, $d = 4$ supergravity (1976): Ferrara, Freedman, Van Nieuwenhuizen; Deser, Zumino

- $\mathcal{N} = 1$, $d = 10$ and $\mathcal{N} = 4$, $d = 4$ super Yang–Mills theory (1977): Brink, Scherk, JHS

- $\mathcal{N}=1$, $d=11$ supergravity (1978): Cremmer, Julia, Scherk

- $\mathcal{N}=8$, $d=4$ supergravity (1978): Cremmer, Julia

- The $AdS_4 \times S^7$ solution of $d=11$ supergravity (1980): Freund, Rubin

- $SO(8)$ gauged $\mathcal{N}=8$ supergravity (1982): de Wit, Nicolai

In 1977 Gliozzi, Scherk, and Olive (1937 - 2012) proposed a projection of the RNS string spectrum (both for bosons and fermions) – the GSO Projection – that removes roughly half of the states including the tachyon. They showed that after the projection the number of bosons and fermions is equal at every mass level.

This was compelling evidence for 10d spacetime supersymmetry of the GSO-projected theory. But it was not a proof. Also, it was not clear what theories were possible.

Superstrings

In 1979 Michael Green and I began a collaboration with the goal of understanding why the GSO-projected RNS string theory has spacetime supersymmetry. When possible, we also collaborated with Lars Brink. Our collaboration achieved the following:

- Formulated (and named) the type I, type IIA, and type IIB superstring theories

- Developed a light-cone gauge formalism for the GSO projected theory and used it to prove spacetime supersymmetry of the spectrum and interactions

- Used this formalism to compute various tree and one-loop amplitudes and elucidate their properties

- Formulated superstring world-sheet theories with manifest 10d spacetime supersymmetry (and local kappa symmetry)

Anomalies

In 1983 Alvarez-Gaumé and Witten derived the formulas for all gauge and gravitational anomalies in any dimension and used them to exhibit miraculous cancelations in type IIB superstring theory.

The type IIB theory seemed unrealistic at the time, since it does not contain any YM gauge fields in flat 10d spacetime. Nowadays nonperturbative vacua of type IIB superstring theory are considered promising candidates for realistic particle physics phenomenology. These studies go by the name of F-theory.

Classically, Type I superstring theory can have an $SO(n)$ or $Sp(n)$ gauge group. In every case a hexagon diagram appears to give a nonzero gauge anomaly, so it seemed unlikely that type I superstring theory could give a consistent parity-violating quantum theory.

In 1984 (at the Aspen Center for Physics) Green and I discovered that all gauge and gravitational anomalies can cancel for a theory with $\mathcal{N} = 1$ supersymmetry in 10d Minkowski spacetime only if the YM gauge group is

$$SO(32) \qquad \text{or} \qquad E_8 \times E_8.$$

Within a few months Gross, Harvey, Martinec, and Rohm introduced the heterotic string for $Spin(32)/\mathbb{Z}_2$ and $E_8 \times E_8$. Shortly after that Candelas, Horowitz, Strominger, and Witten introduced Calabi–Yau compactification. Applied to the $E_8 \times E_8$ heterotic string theory, they showed that it could give 4d effective theories with many realistic features.

After a decade in the shadows, superstring theory – with the goal of unification – suddenly became a mainstream activity.

T duality and mirror symmetry

The equivalence of circles of radius R and α'/R appeared in a 1984 paper of Kikkawa and Yamasaki. This was followed by Narain's 1986 identification of the moduli space of toroidally compactified heterotic strings

$$\mathcal{M} = \frac{O(16+n,n)}{O(16+n) \times O(n) \times O(16+n,n;\mathbb{Z})}.$$

T duality also relates the Type IIA and IIB theories.

In 1996 Strominger, Yau, and Zaslow showed that mirror pairs of Calabi–Yau spaces have T^3 fibrations, and mirror symmetry is T duality applied to the fibers.

S duality

Nonperturbative strong coupling/weak coupling duality was proposed for gauge theory in 1977 by Montonen and Olive. In 1979 Osborn observed that the proposal was most plausible for $\mathcal{N} = 4$ super YM.

In 1990 Font, Ibanez, Lüst, and Quevedo argued that T^6 compactification of the heterotic string results in a nonperturbative $SL(2, \mathbb{Z})$ duality group in addition to Narain's perturbative $SO(22, 6; \mathbb{Z})$ duality group. They introduced the name S duality.

In 1994 Sen produced nontrivial evidence to support the Montonen–Olive conjecture for $\mathcal{N} = 4$ gauge theories and string theories. Following that, Vafa and Witten provided much additional evidence.

In 1994 Hull and Townsend identified an $SL(2, \mathbb{Z})$ duality group for the type IIB superstring theory in $\mathbb{R}^{9,1}$. They also generalized this to $E_{n+1}(\mathbb{Z})$ “U dualities” for the type IIB theory in $T^n \times \mathbb{R}^{9-n,1}$.

The second superstring revolution 1994-96

- Seiberg–Witten theory and Seiberg duality

- The web of dualities relating the various superstring theories and M-theory. This was mostly due to Witten.

- D-branes: Polchinski

- Black-hole entropy: Strominger and Vafa

- F-theory: Vafa

- Matrix Theory: Banks, Fischler, Shenker, Susskind

AdS/CFT

Maldacena 1997

Witten 1998

Gubser, Klebanov, and Polyakov 1998

Understanding AdS/CFT has kept us busy ever since its discovery. It really is 21st century science!

Conclusion

The history of string theory is one of unification: of particles and forces, of math and physics, of people and cultures. I am both proud and honored to have been part of this adventure.

I find it it sobering to suppose that the final theory that we are dreaming of has been (and will be) discovered many times in other parts of the Universe, not to mention the multiverse.

THE END