Some aspects of the Sachdev-Ye-Kitaev model

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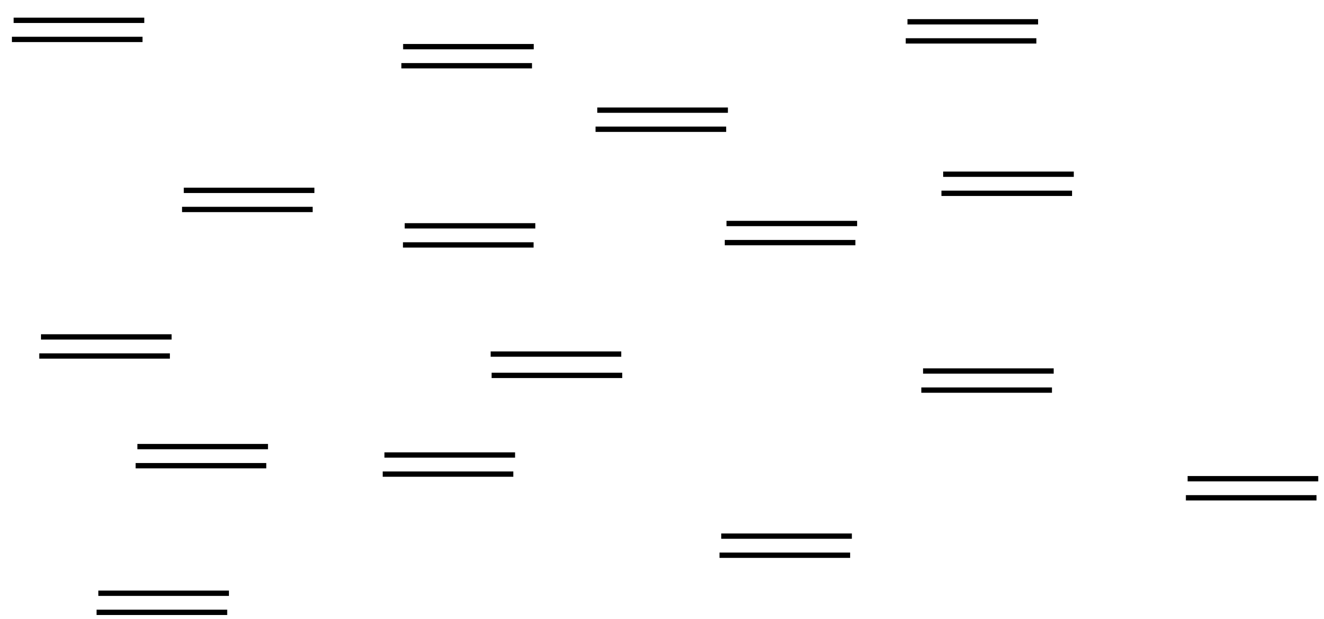
Strings 2016

SYK model

Sachdev,Ye '93, Kitaev '15

- Quantum mechanics of $N \gg 1$ fermions.



- Interaction between any four sites

- Gaussian-random coupling J_{ijkl}

SYK

$$H = \sum_{1 \leq i < j < k < l \leq N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

Majorana fermions $\{\chi_i, \chi_j\} = \delta_{ij}$

Quenched disorder

$$P(J_{ijkl}) \sim \exp\left(-12 N^3 J_{ijkl}^2 / J^2\right)$$

Three properties

- Solvable.

  Can compute correlation functions at large N

- Emergent conformal invariance

  In IR (strong coupling)
  At level of 2-pt function; broken by 4-pt function

- Maximally chaotic

  At strong coupling, has same Lyapunov exponent as a black hole, saturating the Maldacena Shenker Stanford bound

Holographic?

Sachdev '10, Kitaev '15

Outline

1. Review of 2-pt function

2. 4-pt function

3. Variants of SYK

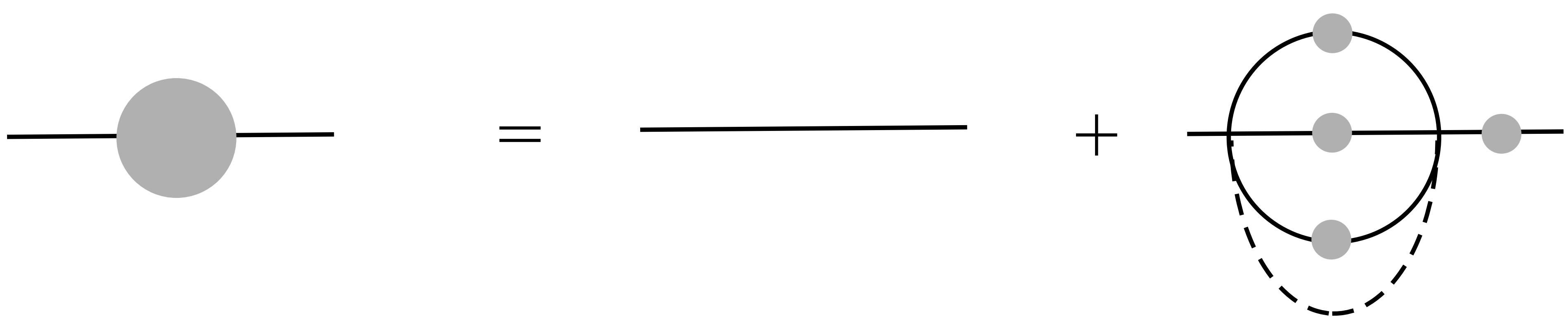
2-pt function

2-pt function

Sachdev Ye '93; Georges, Parcollet, Sachdev '01; Kitaev '15

$$L = \sum_i \frac{1}{2}\chi_i \frac{d}{d\tau}\chi_i - \sum_{i,j,k,l} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

- SYK solvable as a result of having a small & well-organized set of Feynman diagrams: nested sunsets.



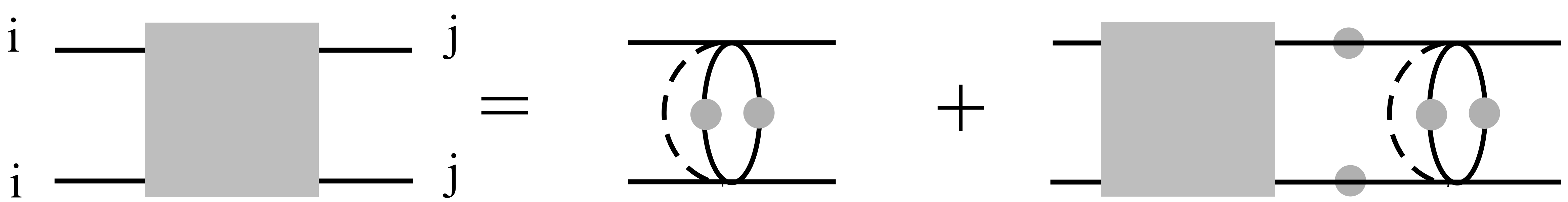
$$G(\tau) \equiv \langle T\chi_i(\tau)\chi_i(0)\rangle = \begin{cases} \frac{1}{2}\mathrm{sgn}(\tau)\ , & |J\tau| \ll 1 \\ \frac{1}{\sqrt{4\pi}}\frac{\mathrm{sgn}(\tau)}{|J\tau|^{1/2}}\ , & |J\tau| \gg 1 \end{cases}$$

4-pt function

4-pt function

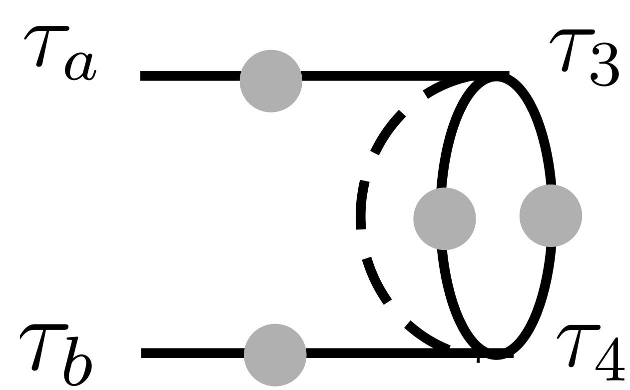
Kitaev ’15; Polchinski, V.R. ‘16; Maldacena, Stanford ‘16

- Only ladder diagrams



4-pt function: 3 steps

1) Find eigenvalues $g(\nu)$ and eigenvectors $v_{\nu\omega}(\tau_a, \tau_b)$ of kernel



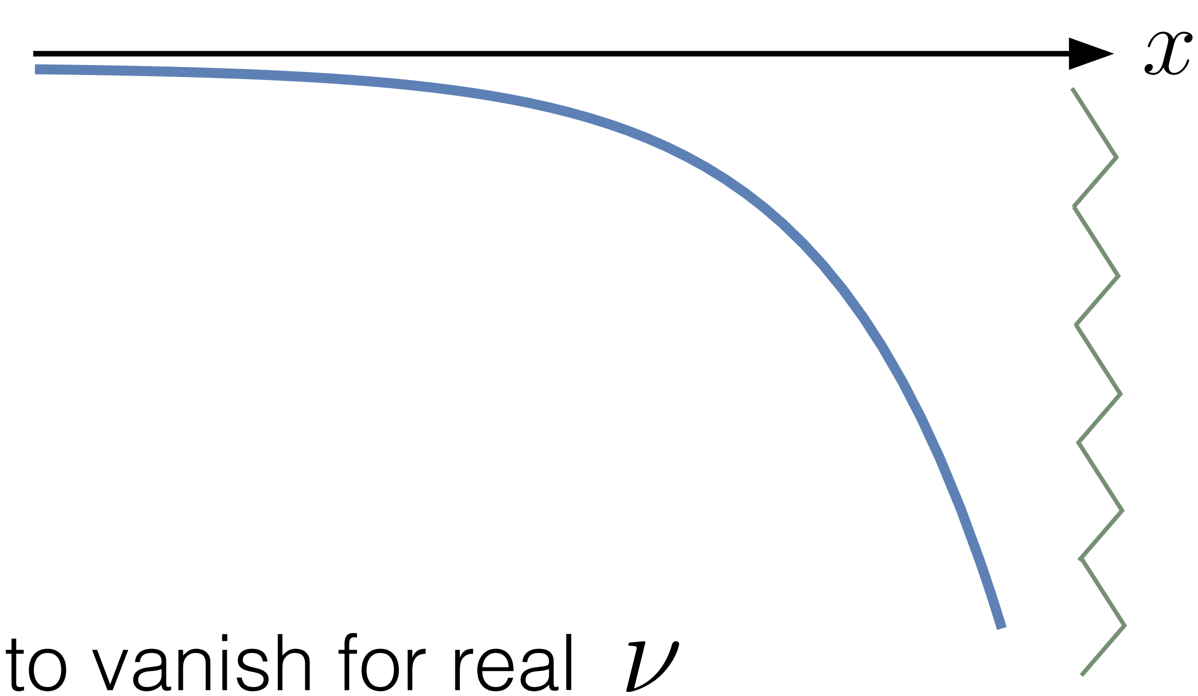
$$v_{\nu\omega}(\tau_a, \tau_b) \sim \frac{\text{sgn}(\tau_{ab})}{|\tau_{ab}|} e^{-i\omega(\tau_a+\tau_b)/2} \Big(J_\nu(|\omega\tau_{ab}|/2) + \xi_\nu J_{-\nu}(|\omega\tau_{ab}|/2) \Big)$$

$$g(\nu) = -\frac{3}{2\nu} \tan \frac{\nu\pi}{2} \qquad \xi_\nu = \frac{\tan \nu\pi/2 + 1}{\tan \nu\pi/2 - 1}$$

2) Take a complete set of eigenvectors

View Bessel eqn. as a Schrodinger eqn.

$$-\frac{d^2 J_\nu(x)}{dx^2} - e^{2x} J_\nu(x) = -\nu^2 J_\nu(x) \qquad\qquad x = \log|\omega t_{ab}/2|$$



Need ξ_ν to vanish for real ν

$$\nu = \begin{cases} ir\ , & r > 0 \qquad \text{scattering} \\ \frac{3}{2} + 2n, & n \geq 0 \qquad \text{bound} \end{cases}$$

3) Expanding 1PI four-point function in terms of eigenvectors, from Schwinger-Dyson eqn. get

$$\Gamma = \frac{1}{N} \sum_{\substack{\nu = 3/2 + 2n \\ ir}} \int d\omega \frac{v^*_{\nu\omega}(\tau_1, \tau_2) v_{\nu\omega}(\tau_3, \tau_4)}{1 - g(\nu)}$$

$$g(\nu) = -\frac{3}{2\nu} \tan \frac{\nu\pi}{2}$$

For a nice form, in terms of only the spectrum (ν for which $g(\nu) = 1$), see talk by D. Stanford

Conformal symmetry breaking

- Divergence due to $\nu = 3/2$

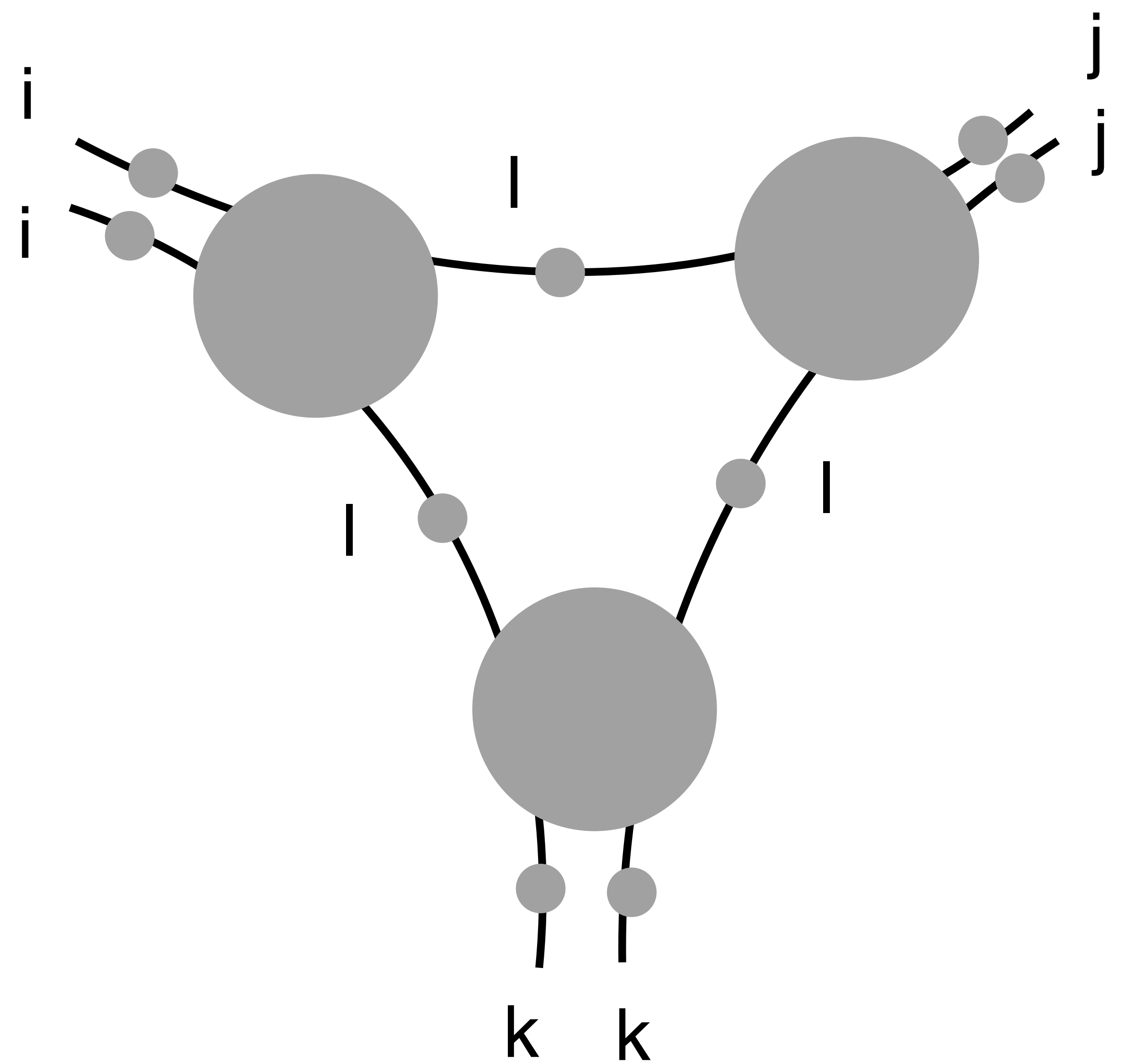
- Result of IR limit ($|J\tau| \gg 1$). Eliminate by including $\frac{1}{|J\tau|}$ corrections to IR two-point function appearing in kernel.

- Analogous to breaking that occurs in AdS_2 as studied by Almheiri, Polchinski '14

- Detailed story

Kitaev '15; Maldacena, Stanford '16; Jensen, '16
Maldacena, Stanford, Yang '16;
Engelsoy, Mertens, Verlinde '16;
Almheiri, Kang '16; Jevicki, Suzuki, Yoon, '16;
Bagrets, Altland, Kamenev, '16

6-pt function



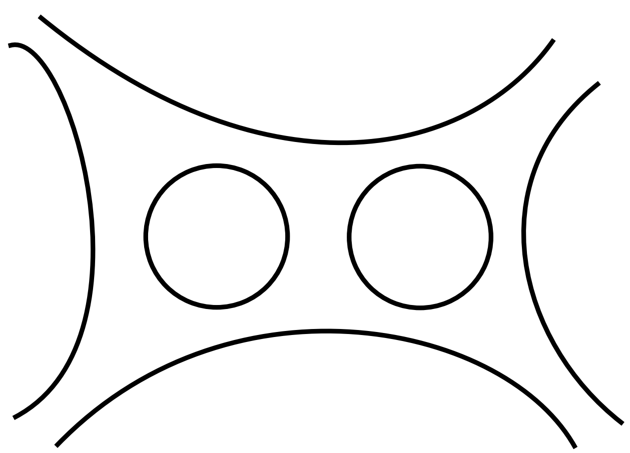
Jevicki, Suzuki, Yoon, '16
D.Gross, V.R., in progress

Variants of SYK

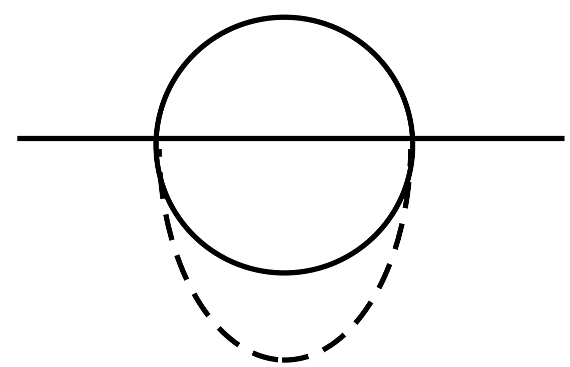
SYK: an new class

Hard

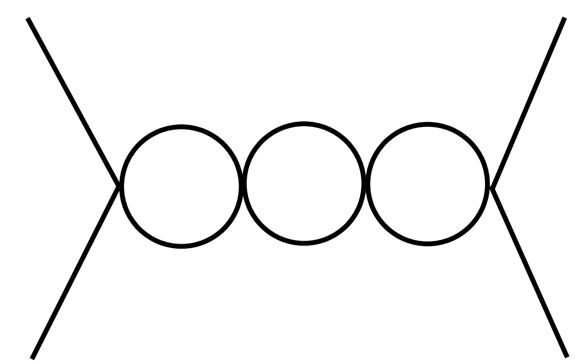
Easy



Matrix model
planar diagrams



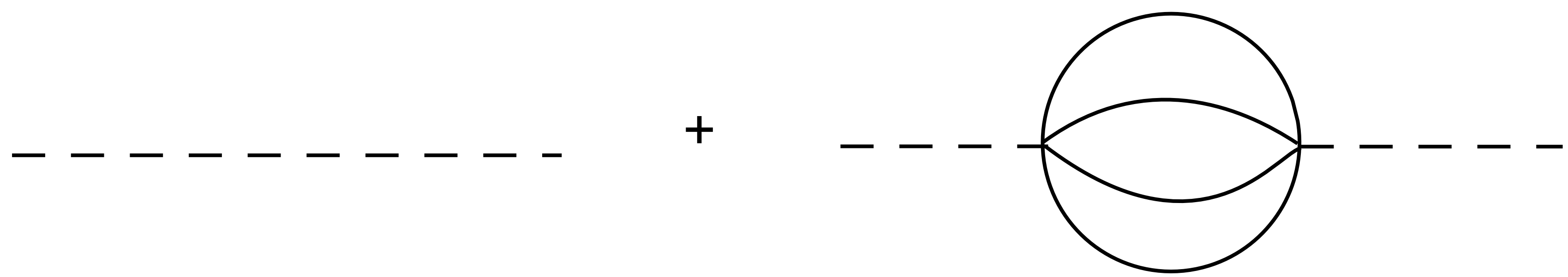
SYK
sunset diagrams



vector model
bubble diagrams

Disorder

- J_{ijkl} is Gaussian random
- Instead, let J_{ijkl} be a nearly static quantum field (e.g. momentum of a harmonic oscillator, with nearly zero frequency)

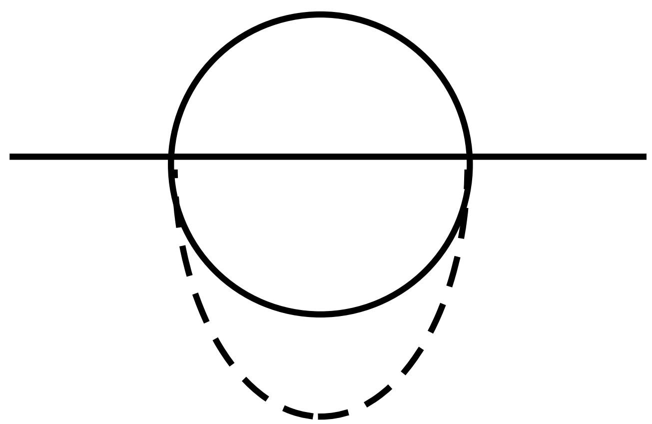


- Quantum corrections are $1/N^3$ suppressed
- So gives same correlation functions

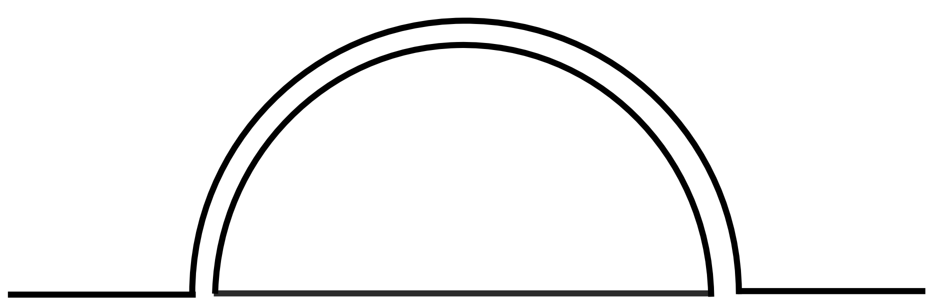
Michel, Polchinski, V.R., Suh '16

Sunsets vs Rainbows

- SYK sums sunset diagrams



- Rainbow diagrams are easier

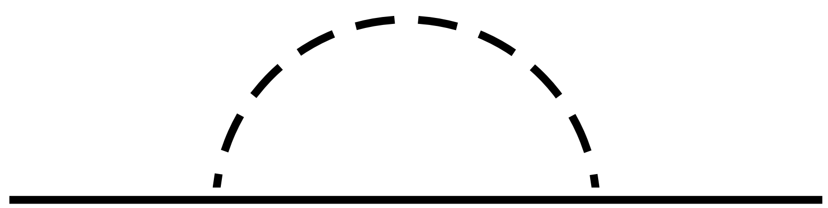


e.g. ’t Hooft model 2d QCD

Random mass fermion

- Fermion with random mass is in SYK family

$$H = \sum_{ij} J_{ij}\, c_i^\dagger c_j$$



- J_{ij} is two index, so this should be thought of as a rainbow diagram

Random mass fermion: solution

- At finite N, two-point function given by matrix integral

$$G(\omega) = -\frac{1}{N}\frac{1}{Z}\int \prod_{i\leq j} dJ_{ij}\,\mathrm{tr}\left(\frac{1}{i\omega + J}\right)\,\exp\left(-\mathrm{tr}(J^2)/2\bar{J}^2\right)$$

- Solve by method orthogonal polynomials

$$G(\omega) = \frac{i}{\omega}\frac{1}{N}\int_0^\infty ds\, e^{-s} e^{-\frac{s^2\bar{J}^2}{2\omega^2}}\, L^1_{N-1}\left(\frac{s^2\bar{J}^2}{\omega^2}\right)$$

- Can expand in powers of 1/N

- Higher-point correlators similarly given in terms of associated Laguerre polynomials

D.Gross, V.R., in progress

No chaos for some rainbow models

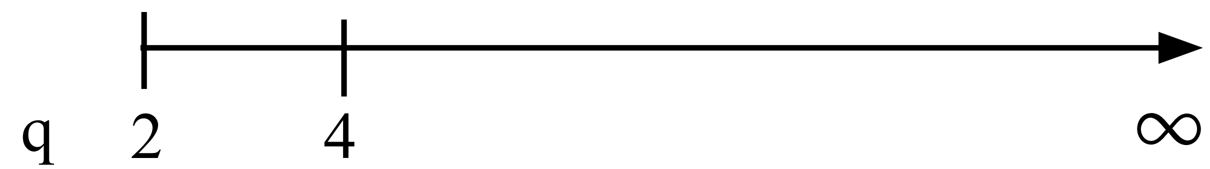
- Fermion with random mass is not chaotic
- IP/IOP models: harmonic oscillator in adjoint representation of U(N) coupled to oscillator in fundamental of U(N). Has (elaborate) rainbow diagrams. Iizuka, Okuda, Polchinski ‘08
- IOP model is not chaotic Michel, Polchinski, V.R., Suh ‘16
- Other rainbow models?

Space of SYK models

- SYK

$$H = \sum_{1 \le i_1 < \ldots < i_q \le N} J_{i_1 i_2 \ldots i_q} \chi_{i_1} \chi_{i_2} \cdots \chi_{i_q}$$

Kitaev ‘15



Maldacena, Stanford ‘16

- Bosons

spin glass? Sachdev, Fu ‘16

- SUSY

Anninos, Anous, Denef ‘16

- SY

$$H = \frac{1}{\sqrt{M}} \sum_{i,j=1}^{N} \sum_{\mu,\nu=1}^{M} J_{ij} \, S^{\mu}_{i\nu} S^{\nu}_{j\mu}$$

Sachdev, Ye ‘93

$$S^{\mu}_{i\,\nu} = \sum_{\alpha=1}^{n} c^{\mu\dagger}_{i\,\alpha} c^{\alpha}_{i\,\nu} \qquad \sum_{\mu=1}^{M} c^{\dagger}_{i\,\alpha\mu} c^{\beta\mu}_{i} = m \, \delta^{\beta}_{\alpha}$$

Summary

- SYK is a thermalizing, chaotic system
- Remarkably, it is solvable at large N.
- Nearly conformal in IR, leading to simplification.
- Two-point function given by sum of sunset diagrams
- Four-point function given by sum of ladder diagrams
- Diagrammatic structure - nested sunsets - is new
- More models with this structure?
- Dual of SYK?