Fluid/gravity duality and hydrodynamics at its limits

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M. Heller, RJ, P. Witaszczyk, 1302.0697 [PRL 110, 211602 (2013)]

Outline

What is hydrodynamics? (and higher-order hydrodynamics?)

Some questions

Methods: fluid/gravity duality and boost-invariant flow

The Borel plane and gradient expansion

Conclusions

Introduction

What is hydrodynamics?

- Universal description of the long wavelength degrees of freedom
- Applies equally well at macroscopic and microscopic scales
- Current most relevant example: quark-gluon-plasma produced at RHIC/LHC

Long wavelength description $\equiv$ gradient expansion $\equiv$ expansion in the number of derivatives

Question: What is the nature of such an expansion?

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3. Arrange all possible terms by the number of derivatives of u^μ
4. Coefficients of these terms $\equiv$ transport coefficients characteristic of the underlying microscopic theory
5. Generalized Navier-Stokes equation is just $\partial_\mu T^{\mu\nu} = 0$

$\mathcal{N} = 4$ SYM hydrodynamics:

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

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What is the nature of the gradient expansion?

- Suppose we include terms with more and more derivatives (dissipation)
- Is the resulting series asymptotic (zero radius of convergence)?
- What physics is (quantitatively) responsible for the lack of convergence?

Analogy: perturbative expansion and instanton effects...

Question 2

If the hydrodynamic series is only asymptotic, is it Borel summable?

- What are the singularities on the Borel plane and what is their physical interpretation?

These questions are very interesting but also quite theoretical...

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- In our previous work [Heller, RJ, Witaszczyk] we considered the evolution of a spacetime dual to a plasma system evolving from some nonequilibrium initial conditions and its transition to hydrodynamics

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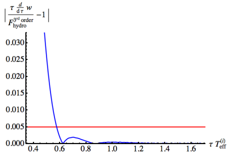

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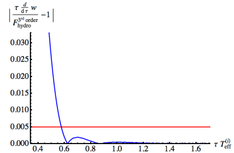

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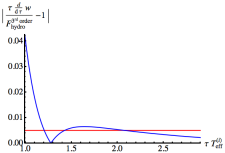

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$\mathcal{N} = 4$ SYM provides for us (through the AdS/CFT correspondence) the **only** physical system for which one can systematically compute high orders of the gradient expansion and examine the above questions

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- Approach pioneered by Bhattacharya, Hubeny, Minwalla, Rangamani
- One starts with a boosted planar black hole representing a plasma system moving with uniform velocity u^μ and with temperature T
- One promotes u^μ and T to slowly varying functions – one has to correct the metric iteratively in an expansion in gradients
- At each order one looks for a (regular) solution of

$$\text{(Linear differential operator)}[g_{\mu\nu}^{(n)}] = RHS[\{g_{\mu\nu}^{(j)}\}_{0\leq j\leq n-1}]$$

- Rather complicated to perform the expansion analytically:
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- Very simple to solve numerically (with very high precision!)
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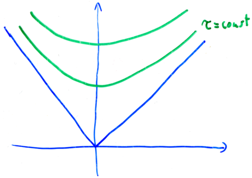
Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



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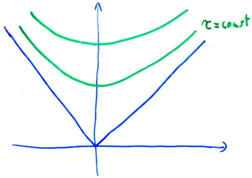

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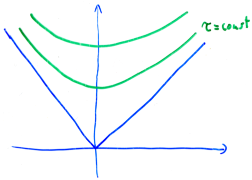
Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T_{\mu}^{\mu} = 0$ and $\partial_{\mu} T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The assumptions of symmetry fix uniquely the flow velocity
- Gradient expansion coincides with an expansion in

$$\frac{1}{\tau^{\frac{2}{3}}}$$

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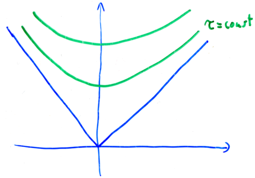

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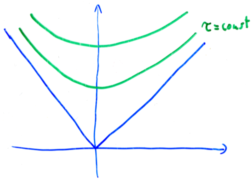

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Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}} 3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1 + 2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3 + 2\pi^2 + 24\log 2 - 24\log^2 2}{324 \cdot 2^{\frac{1}{2}} 3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \ldots$$

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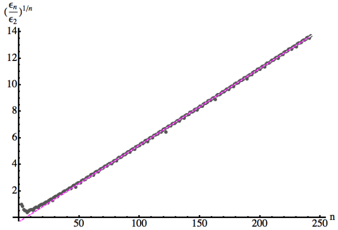

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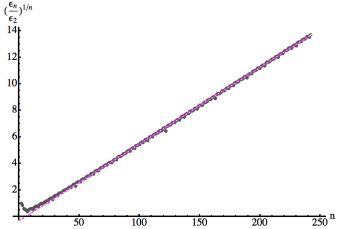

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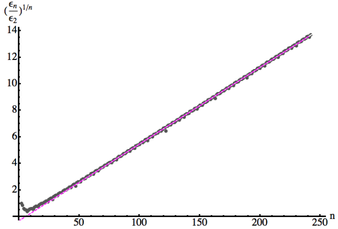

Zero radius of convergence
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Borel transform

- Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

- If there are no singularities on the real axis, a Borel resummation of the asymptotic series can be obtained from the integral

$$\varepsilon_{resum}(u) = \int_0^\infty e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$

- $\tilde{\varepsilon}(u)$ has only a finite radius of convergence. In order to locate singularities in the Borel plane, we perform a symmetric Pade approximation...
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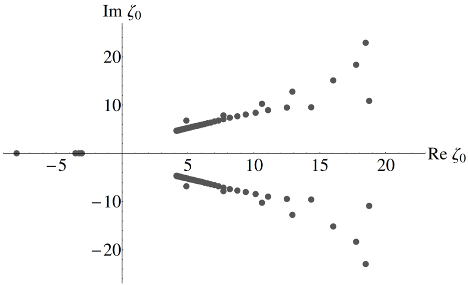
Singularities in the Borel plane

- The poles on the negative real axis are spurious
- The zeroes of the denominator of the Pade approximant coincide with the zeroes of the numerator up to 10^{-100} accuracy

Singularities in the Borel plane



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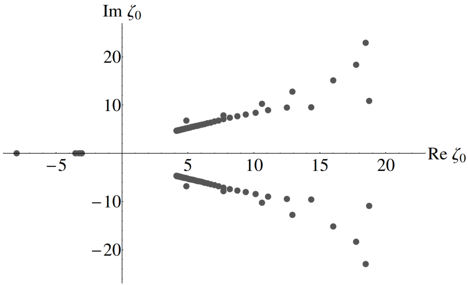

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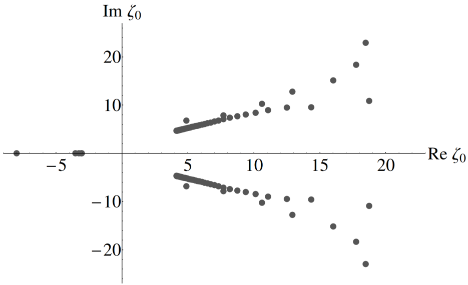

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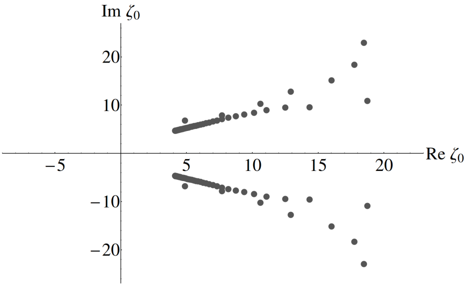

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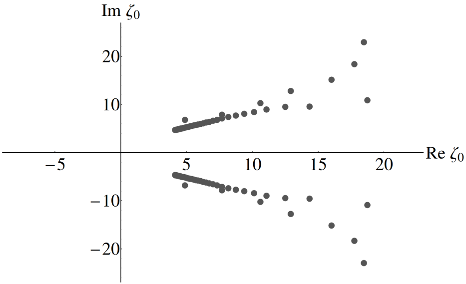

- Branch cuts on the Borel plane
- Branch points set the radius of convergence of the Borel transform
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Question:

What is the physical interpretation of the branch cut singularities?

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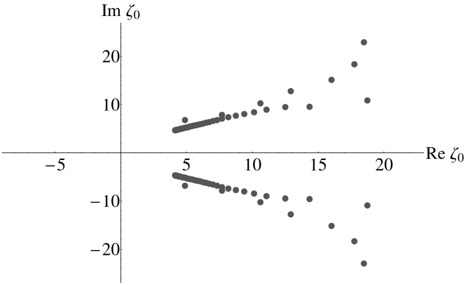

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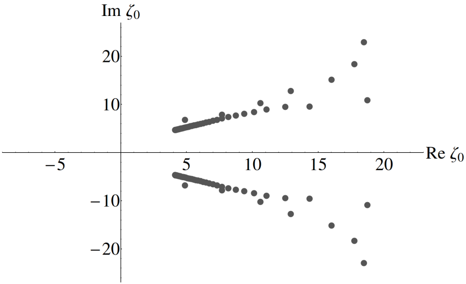

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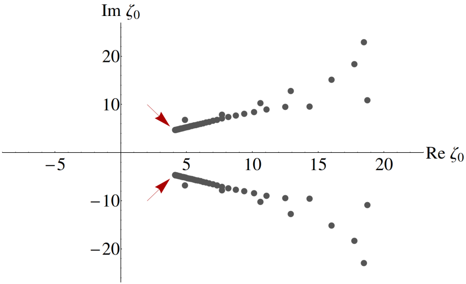

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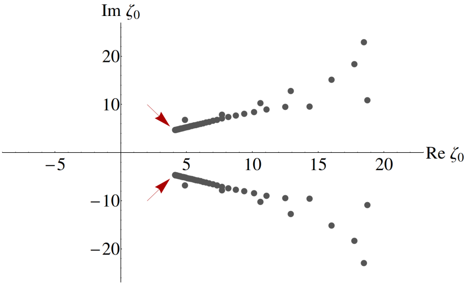

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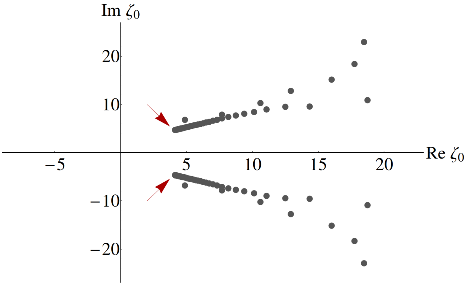

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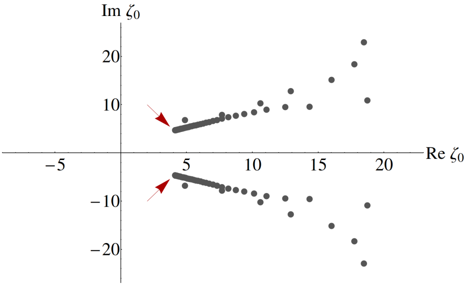

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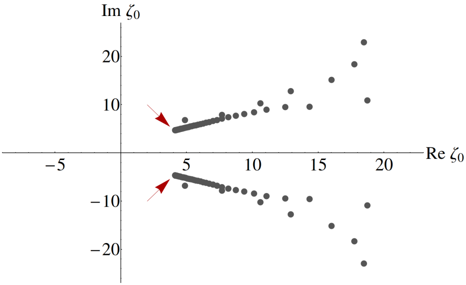

- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta} \tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right) d\zeta$$

- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\, i$) will contribute as

$$e^{-(4.12065+4.67895\, i)\,\tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode!
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i \underbrace{(3.1195 - 2.7467\, i)}_{planar\ BH\ QNM} \int \underbrace{T(\tau)}_{1/\tau^{\frac{1}{3}}} d\tau = \underbrace{-i\frac{3}{2}(3.1195 - 2.7467\, i)}_{-4.12005-4.67925\, i} \tau^{\frac{2}{3}}$$

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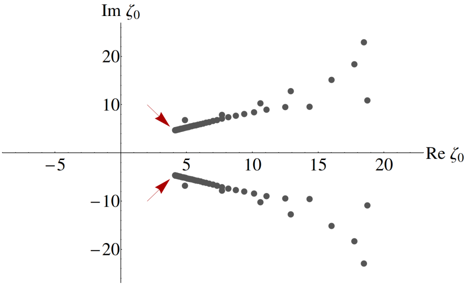

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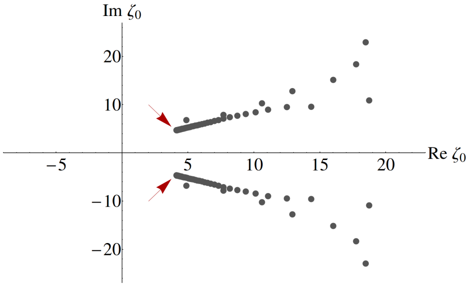

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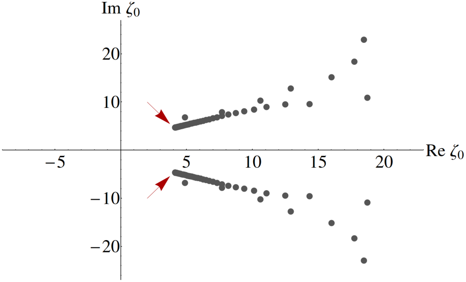

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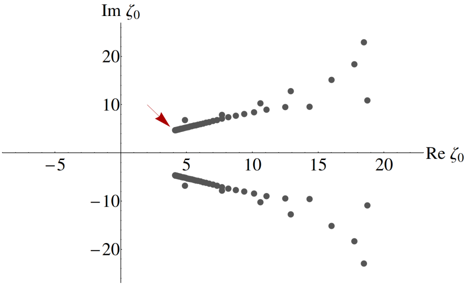

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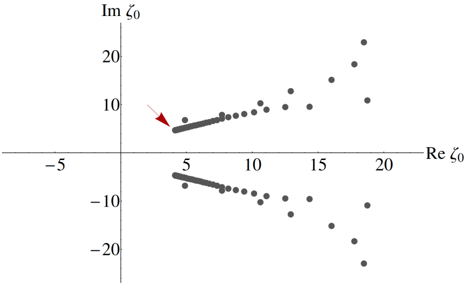

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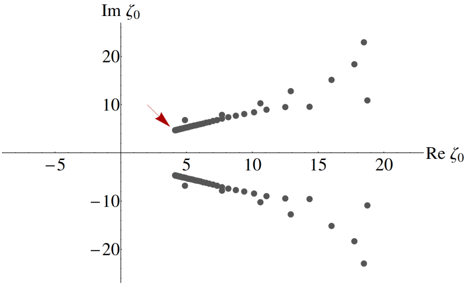

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Singularities in the Borel plane

What is the interpretation of the whole branch cut?

- Deform the contour of the inverse Borel transform to encircle the cut and extract the large τ behaviour
- We obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i} \cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The late time geometry is an evolving black hole deformed by viscous corrections (first gradient terms in the fluid/gravity duality)
- This modification of the geometry generates a nontrivial power-law modification of the quasi-normal mode which is

$$\tau^{-1.5422+0.5199\,i} \cdot e^{-i\,\frac{3}{2}(3.1195-2.7467\,i)\tau^{\frac{2}{3}}}$$

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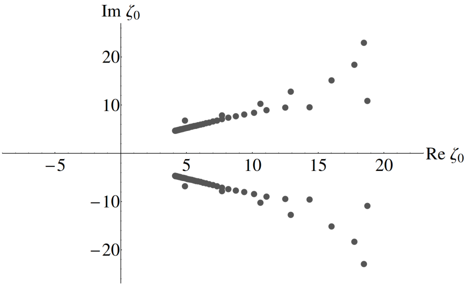

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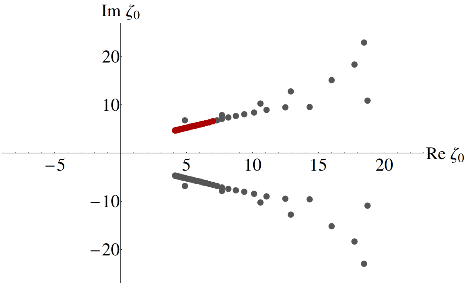

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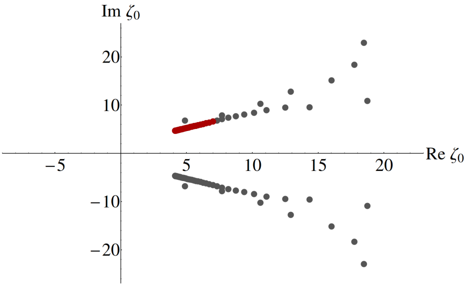

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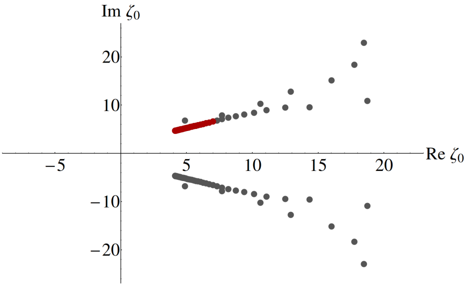

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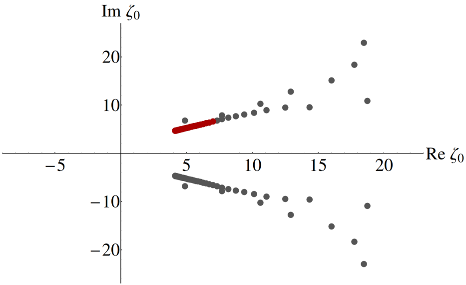

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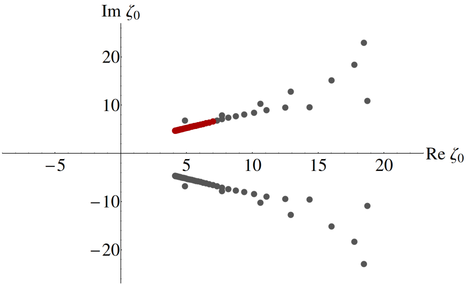

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- The hydrodynamic expansion has zero radius of convergence
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- Analogy with perturbative expansion in QFT and instanton effects
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