

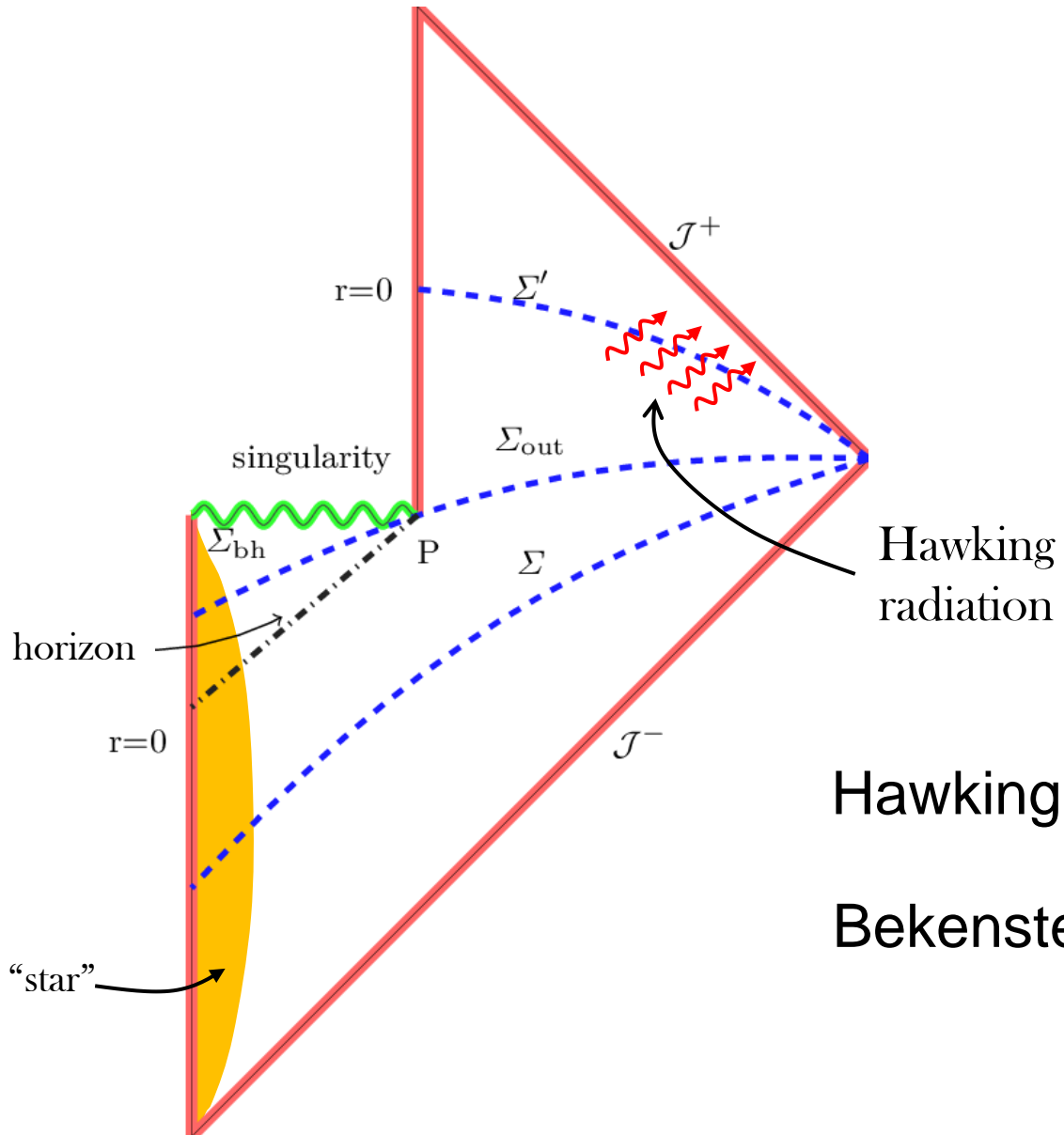


**BLACK HOLE  
INFORMATION PARADOX  
DISCUSSION**

**STRINGS 2021**

(June 21 – July 2, 2021)

# Black hole information paradox:

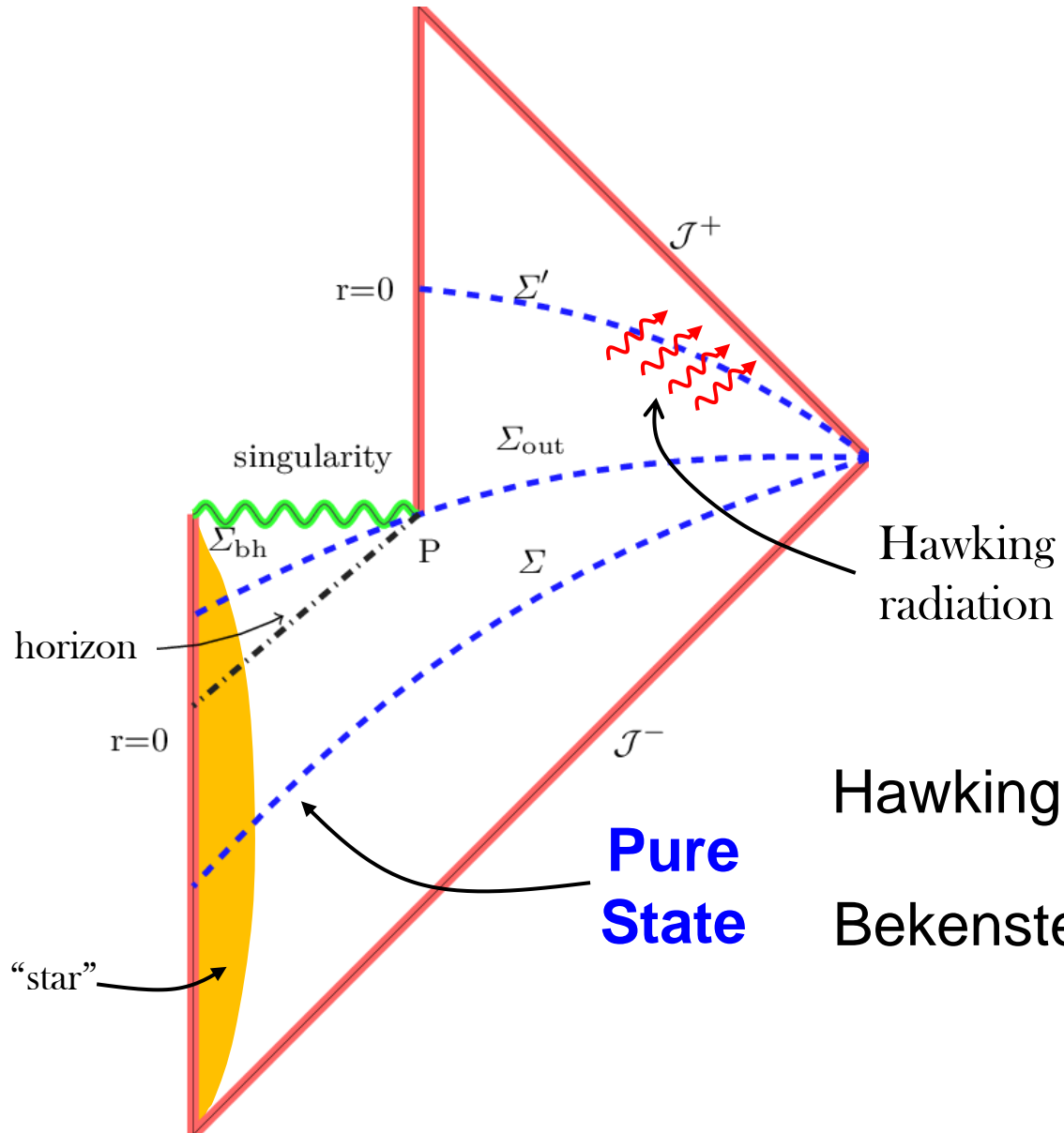


Hawking temperature:  $T = \frac{\hbar\kappa}{2\pi}$

Bekenstein-Hawking entropy:

$$S_{BH} = \frac{A_{horiz}}{4G_N\hbar}$$

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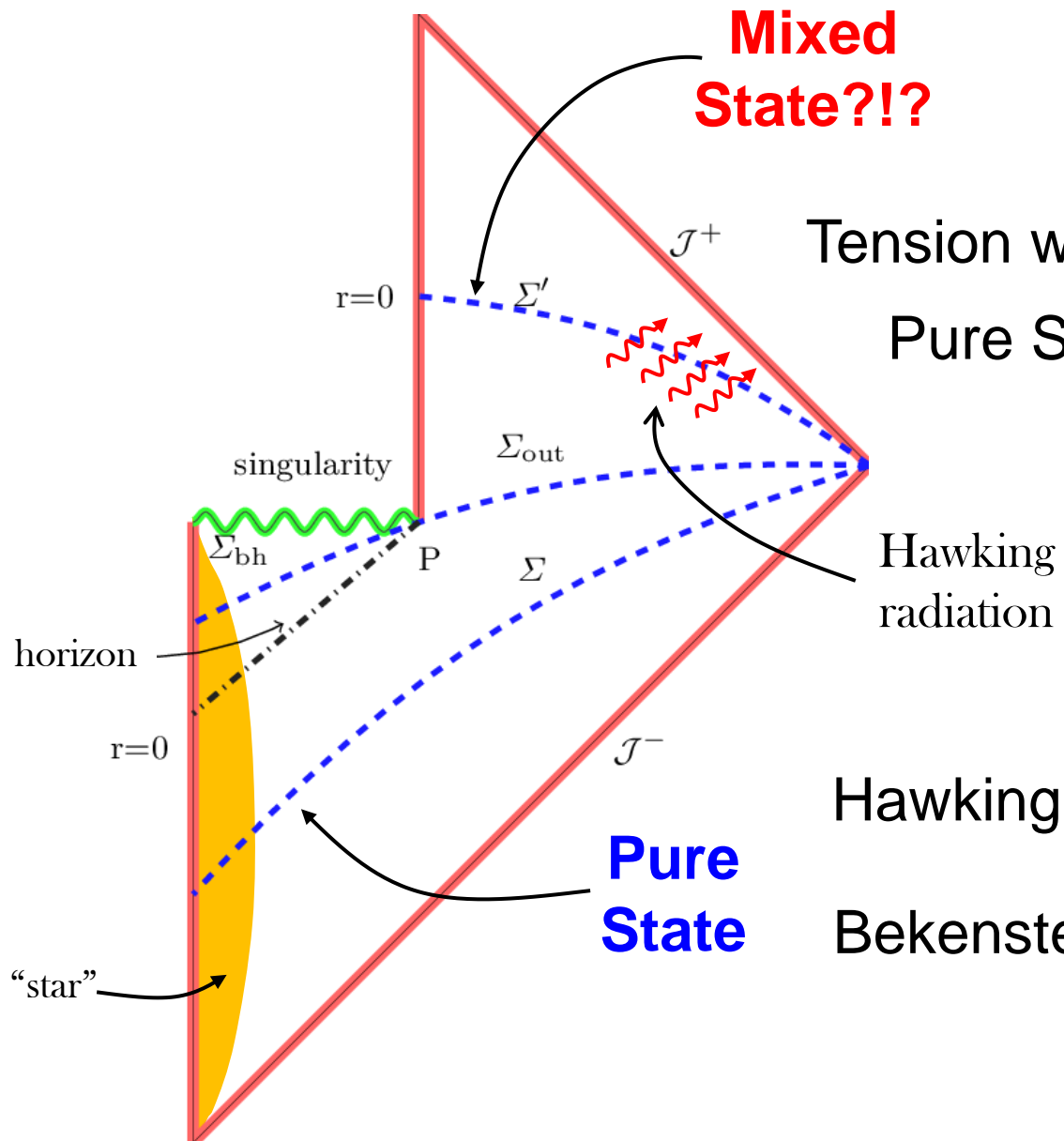


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# Black hole information paradox:



Tension with Unitary Time Evolution:

Pure State  $\rightarrow$  Mixed State

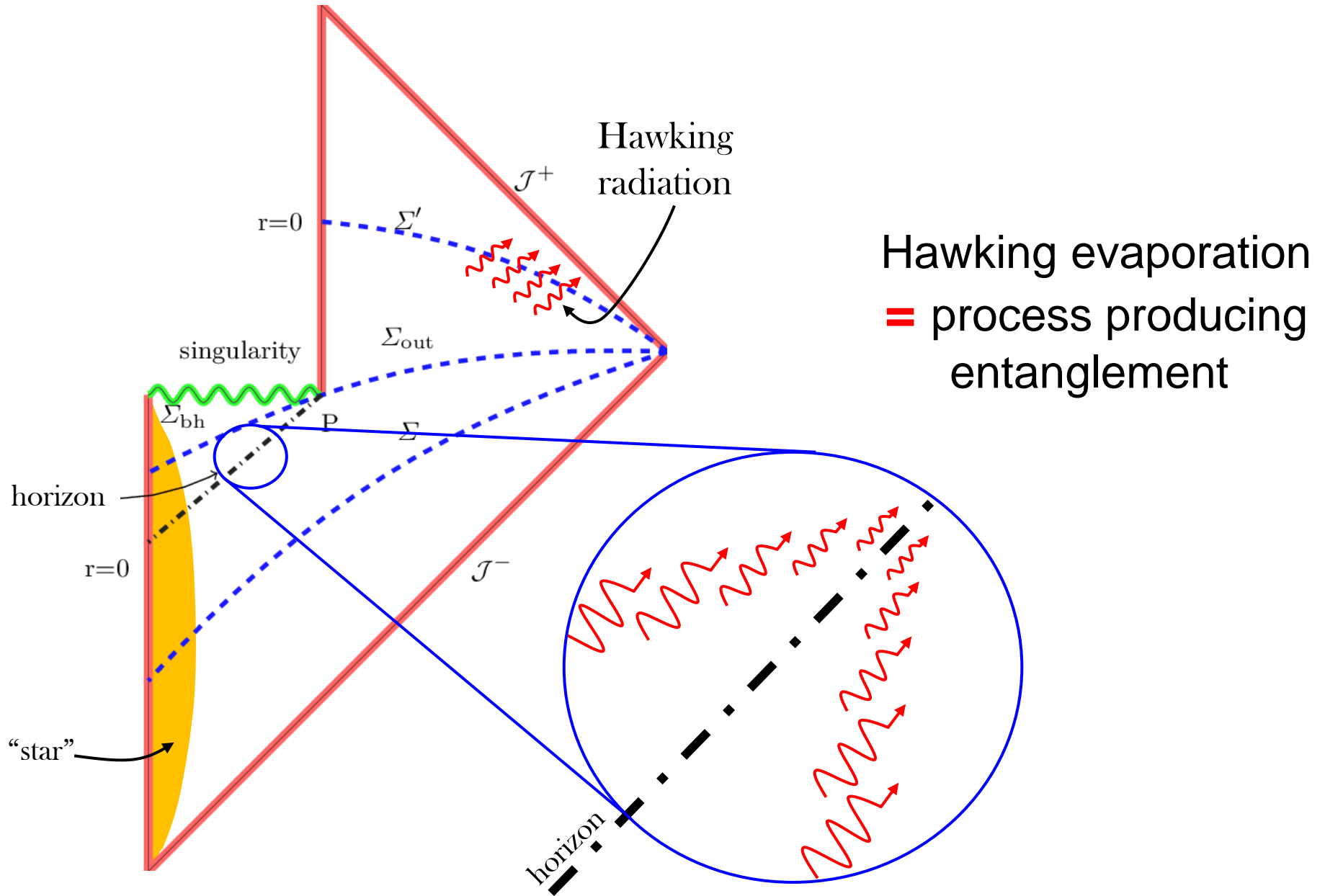
Hawking radiation

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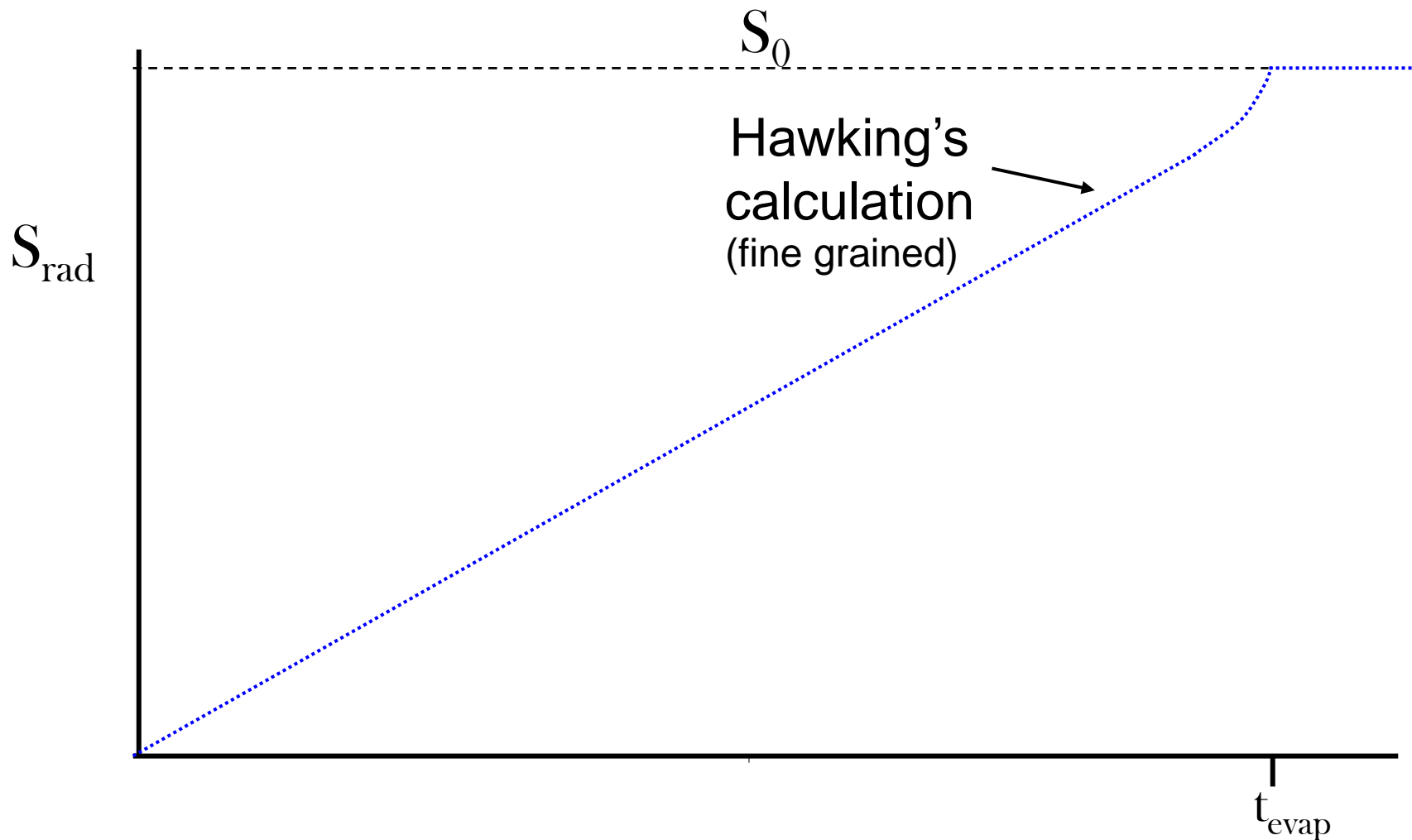
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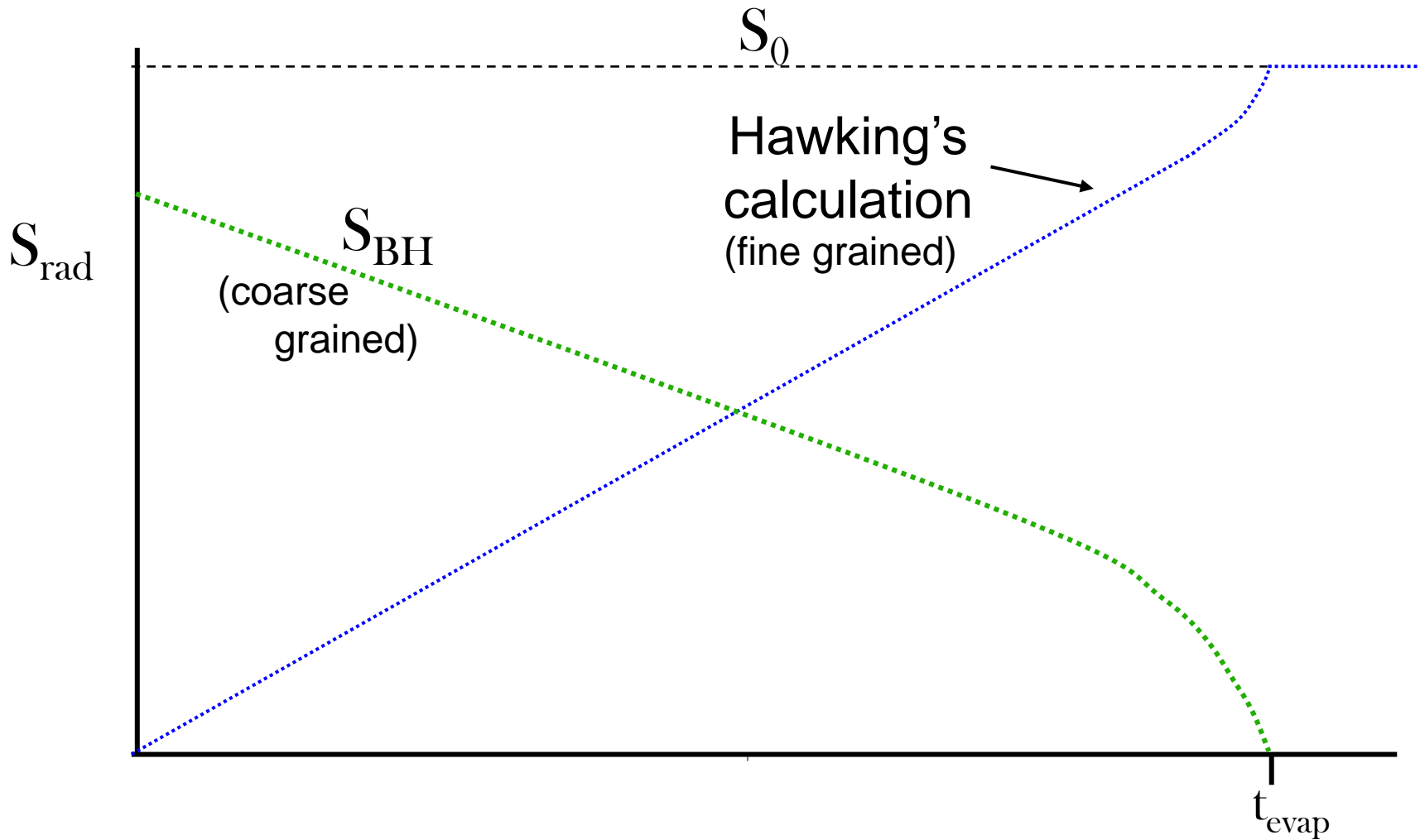
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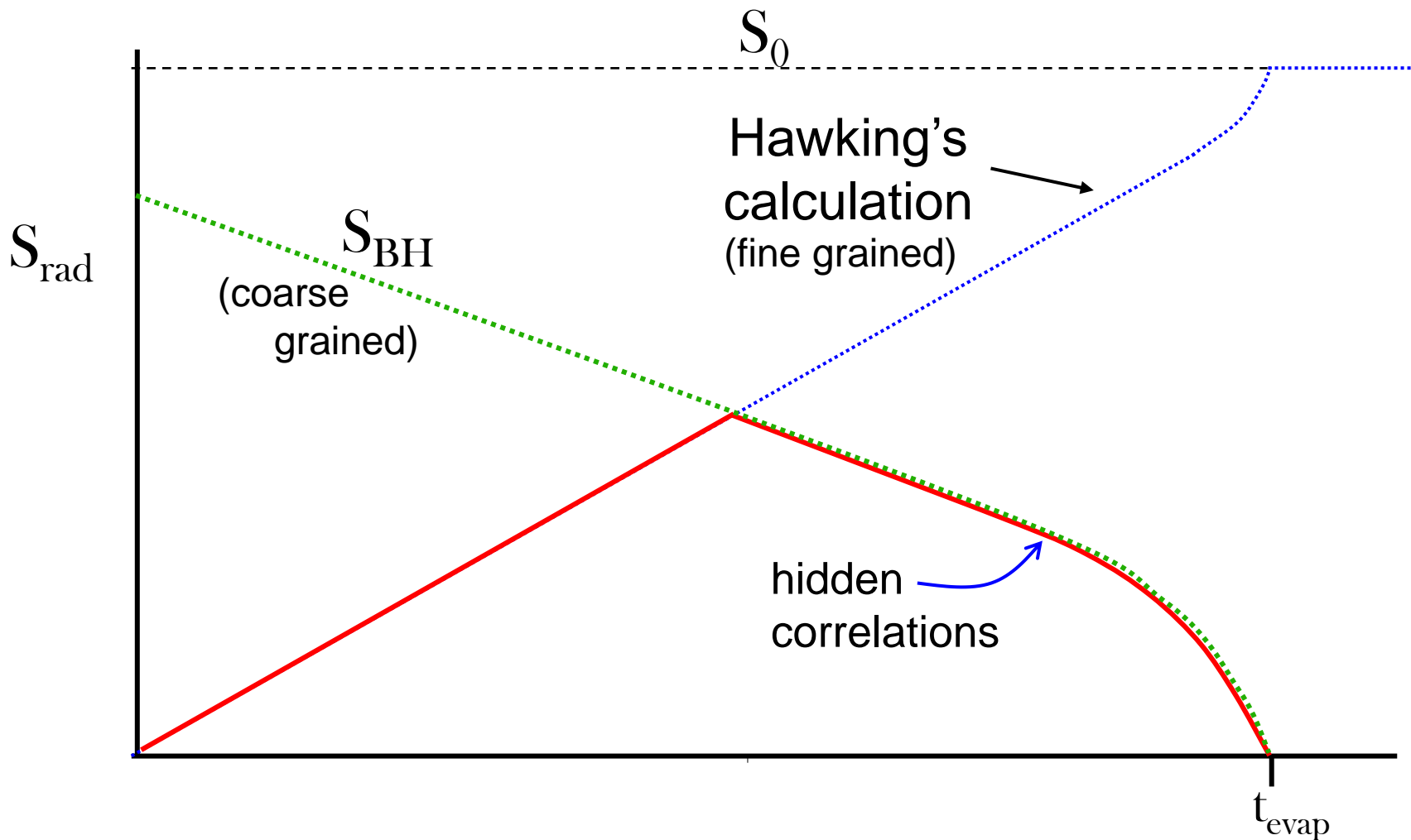
**Page Curve:** unitarity requires radiation entropy bounded by BH entropy  $S_{\text{rad}} \leq S_{\text{BH}}$



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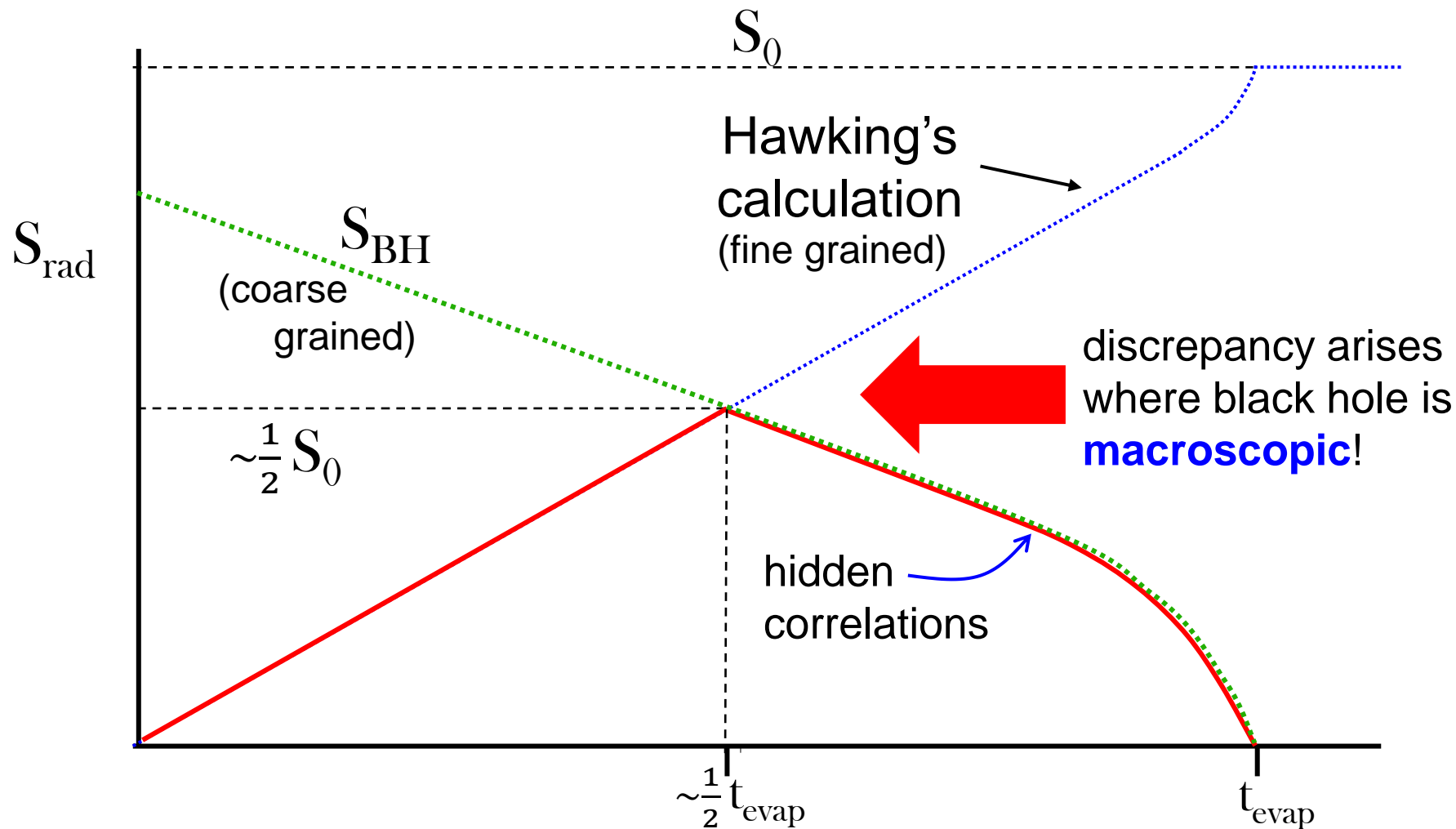
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**Page Curve:** unitarity requires radiation entropy bounded by

$$S_{\text{rad}} \leq S_{\text{BH}}$$



not corrected by small modifications of Hamiltonian or state near the horizon

Mathur; Almheiri, Marolf, Polchinski & Sully

## New insight:

- with recent progress, it is possible to compute the Page curve in a controlled manner!

[Penington \[arXiv:1905.08255\]](#)

[Almheiri, Engelhardt, Marolf & Maxfield \[arXiv:1905.08762\]](#)

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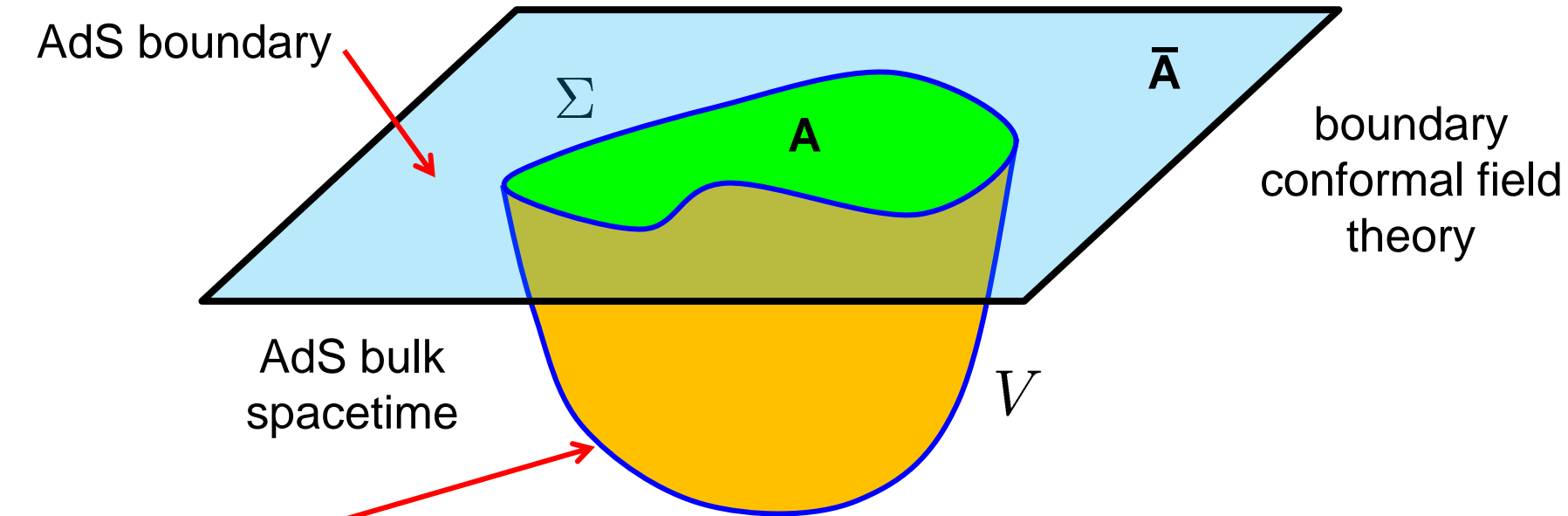
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(Almheiri, Mahajan, Maldacena, Zhao, Akers, Harlow, Rozali, Raamsdonk, Sully, Wadell, Wakeham, Nomura, Moitra, Sake, Trivedi, Vishal, Akers, Engelhardt, Harlow, Chen, Fisher, Hernandez, Ruan, Bousso, Tomasevic, Almheiri, Mahajan, Santos, Penington, Shenker, Stanford, Yang, Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, Blommaert, Mertens, Verschelde, Chen, Qi, Zhang, Verlinde, Kim, Tang, Preskill, Balasubramanian, Kar, Parrikar, Sárosi, Ugajin, Piroli, Sünderhauf, Qi, Marolf, Maxfield, Liu, Vardhan, Pollack, Rozali, Sully, Wakeham, Agarwal, Bao, Akers, Engelhardt, Penington, Usatyuk, Chen, Zhao, Brown, Gharibyan, Penington, Susskind, Chen, Myers, Neuenfeld, Reyes, Sandor, Banks, Alishahiha, Astanceh, Naseh, Krishnan, Patil, Pereira, Agón, Lokhande, Pedraza, Hollowood, Kumar, Hartman, Shaghoulian, Strominger, Fitkevich, Levkov, Zenkevich, Sully, Raamsdonk, Wakeham, Zhang, Hashimoto, Iizuka, Matsuo, Jana, Loganayagam, Rangamani, Giddings, Turiaci, Anegawa, Iizuka, Bhattacharya, Gautason, Schneiderbauer, Sybesma, Thorlacius, Chen, Gorbenko, Maldacena, Gomez, Karlsson, Krishnan, Engelhardt, Fischetti, Maloney, Hollowood, Kumar, Legramandi, Dong, Qi, Shangnan, Yang, Anous, Kruthoff, Mahajan, Bousso, Wildenhain, Bak, Kim, Yi, Yoon, Chandrasekaran, Miyaji, Rath, Li, Chu, Zhou, Geng, Karch, Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, . . . . .)

# Key Ingredient: RT $\rightarrow$ Quantum Extremal Surfaces

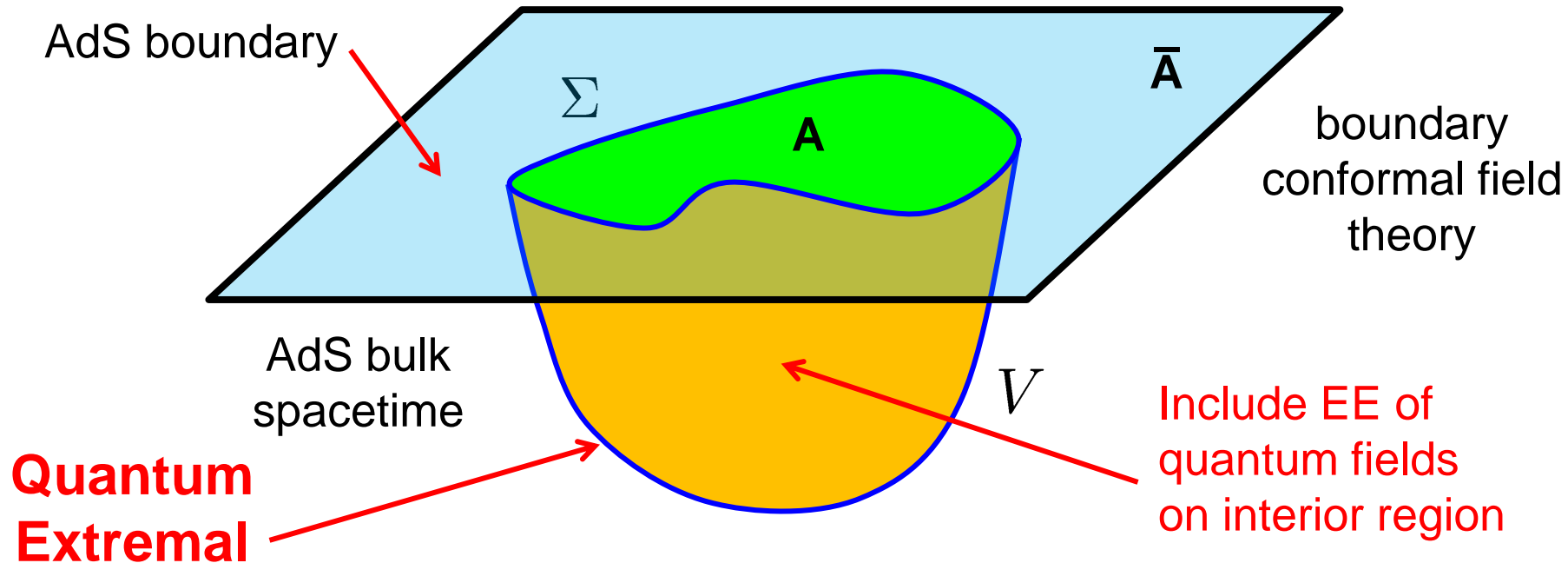


**Extremal Surface**

$$S(A) = \text{ext}_{\partial V \sim \Sigma} \frac{A_V}{4G_N}$$

(Ryu & Takayanagi; Hubeny, Rangamani & Takayanagi; Lewkowycz & Maldacena; ....)

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**Quantum Extremal Surface**

$$S(A) = \text{ext}_{\partial V \sim \Sigma} \frac{A_V}{4G_N} \quad (\text{Ryu \& Takayanagi; Hubeny, Rangamani \& Takayanagi; Lewkowycz \& Maldacena; \dots})$$

$$S(A) = \text{ext}_{\partial V \sim \Sigma} \left[ \frac{A_V}{4G_N} + S_{QFT}(\Sigma) \right]$$

(Faulkner, Lewkowycz & Maldacena; Engelhardt & Wall)

## → Island Rule:

- entropy of the Hawking radiation is given by

$$S_{EE}(\mathbf{R}) = \min \left\{ \text{ext}_{\text{islands}} \left( S_{QFT}(\mathbf{R} \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_N} \right) \right\}$$

- evaluate the generalized entropy of radiation region  $\mathbf{R}$  combined with various space-like subregions in the gravitating region, ie, islands

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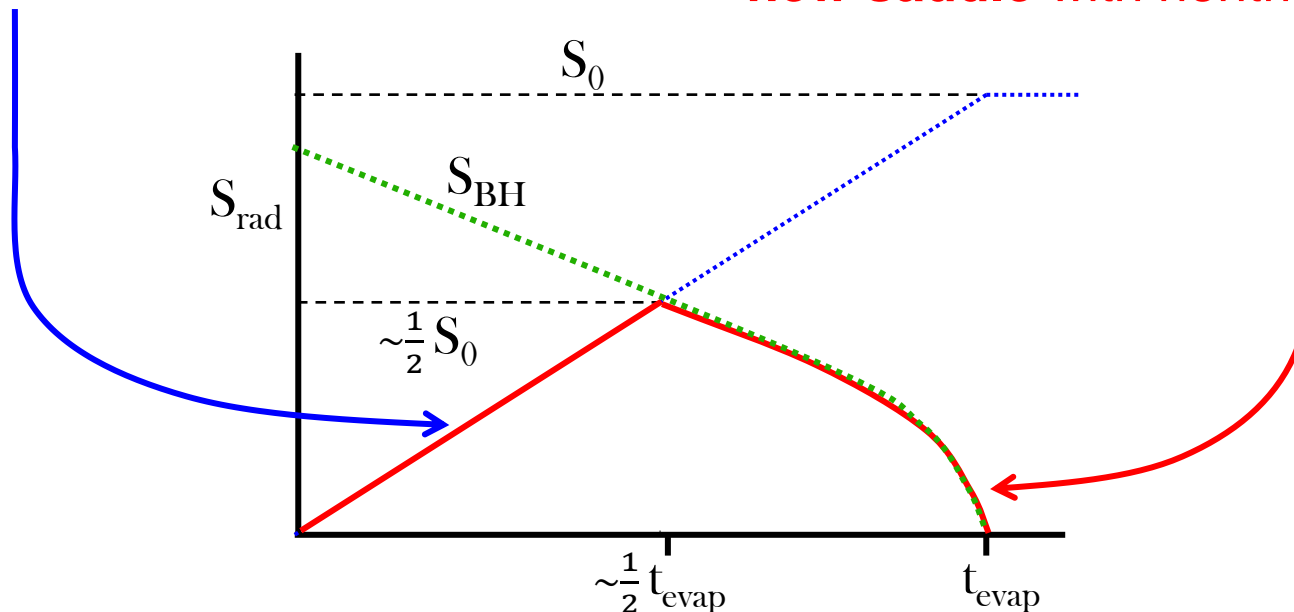
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- ➔ evaluate the generalized entropy of radiation region  $\mathbf{R}$  combined with various space-like subregions in the gravitating region, ie, islands

Early: island is the empty set;  
agrees with Hawking's calculation

Late: large entanglement between  
radiation and region behind horizon;  
**new saddle** with nontrivial island



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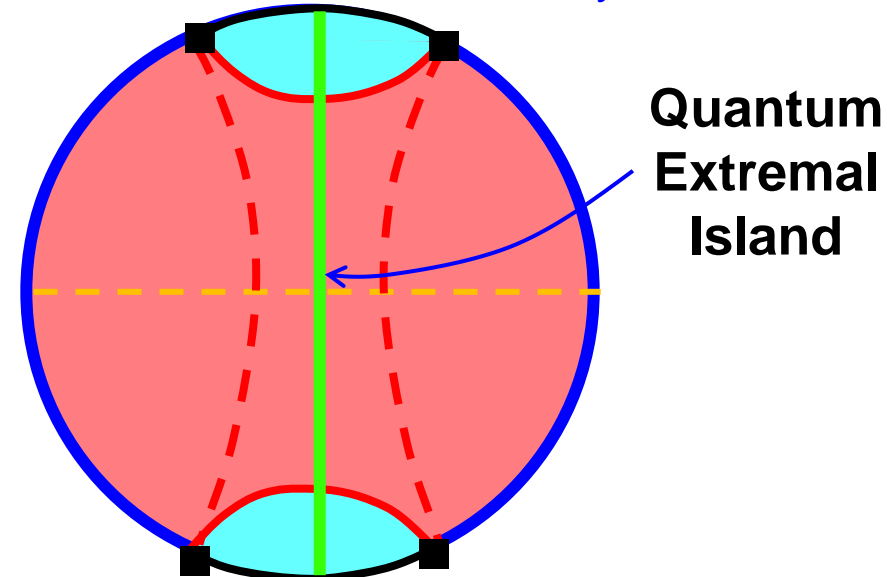
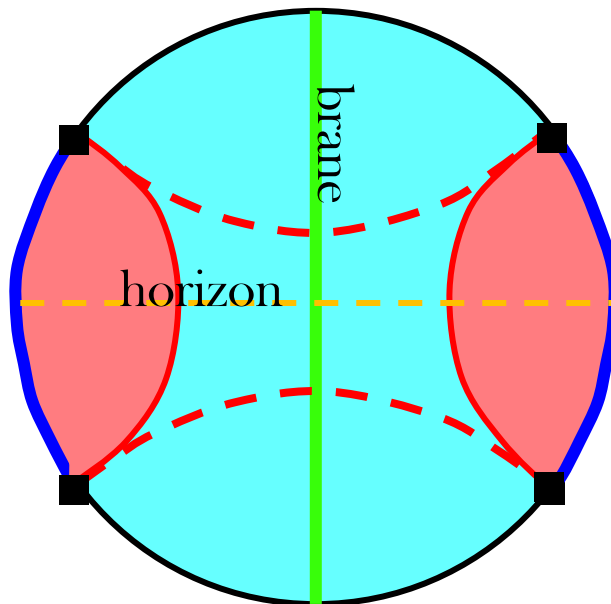
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- ➔ evaluate the generalized entropy of radiation region  $\mathbf{R}$  combined with various space-like subregions in the gravitating region, ie, islands

- competition of saddles, eg, doubly holographic models

Sully, Raamsdonk & Wakeham

Chen, RM, Neuenfeld, Reyes & Sandor





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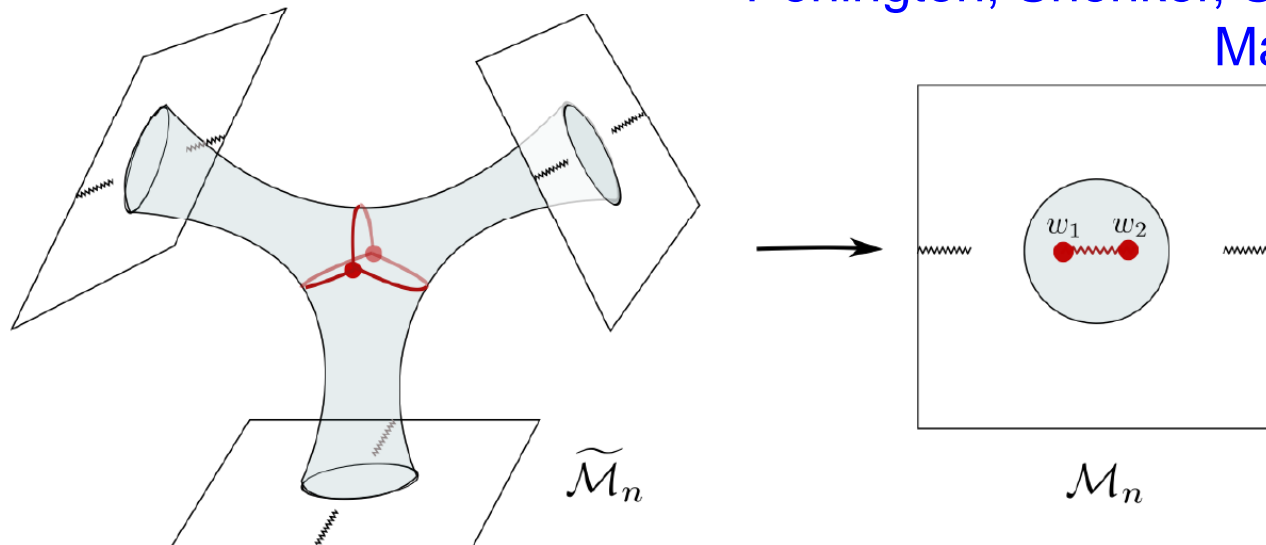
- evaluate the generalized entropy of radiation region  $\mathbf{R}$  combined with various space-like subregions in the gravitating region, ie, islands

- justified by replica wormhole calculations (Shenker review)

Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini

Penington, Shenker, Stanford & Yang

Maxfield & Marolf





# BEYOND ENTROPY

## COMPLEXITY IN THE INFORMATION PARADOX

Status check:

Done: derived the Page curve!

Not done: resolved the information paradox.

Entropy is not enough?<sup>TM</sup> (Susskind)

Harlow-Hayden: Decoding the Hawking radiation is exponentially complex because it requires conditioning on an outcome (postselection). Followed by Aaronson and clarified and developed further by Kim-Tang-Preskill.



# COMPLEXITY IN THE INFORMATION PARADOX

Any full resolution of the paradox would need to explain how to compute the Page curve from first principles, and consequently also explain what Hawking missed.

Comment: seems that the *production* of exponential complexity is a critical aspect of the information problem, and the ability to *access* exponentially complex operators appears to be crucial in deriving unitarity.

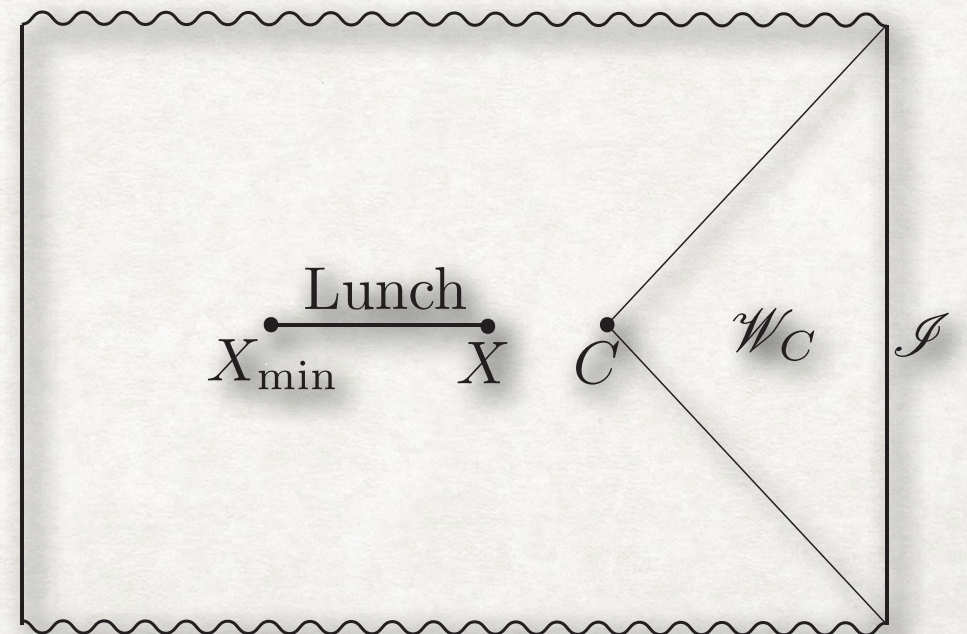
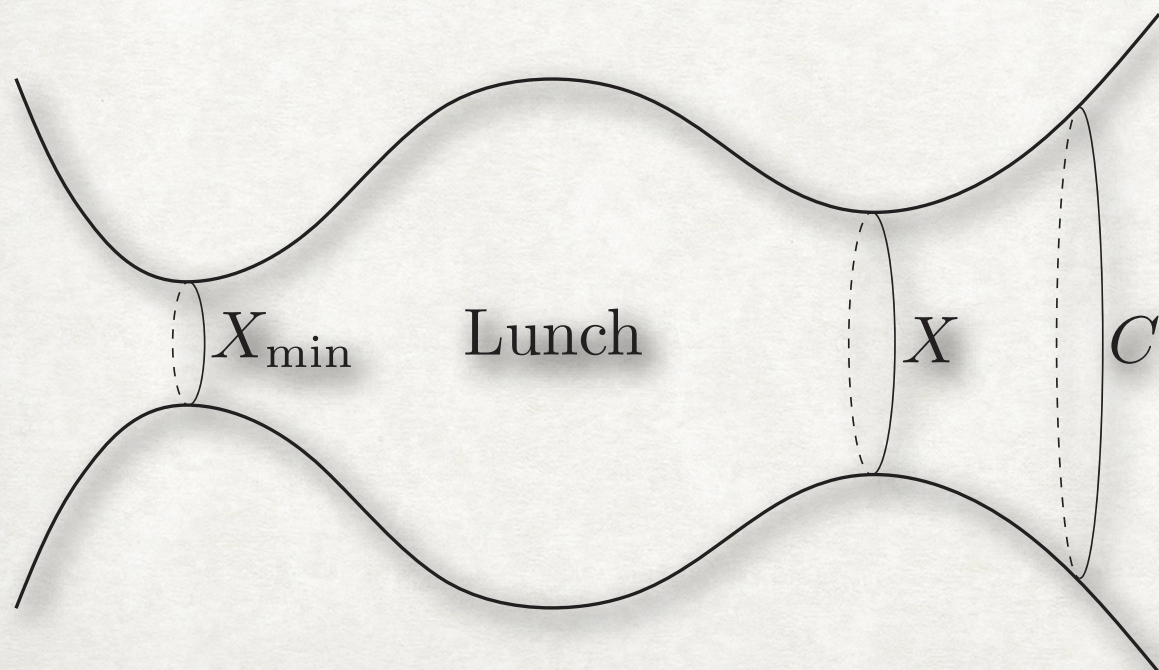
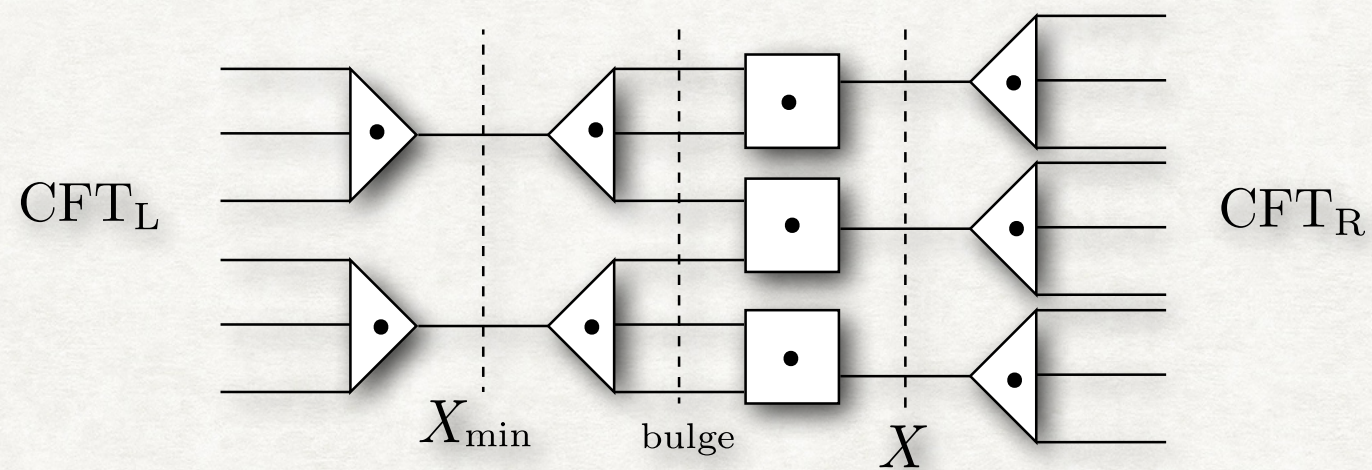
The geometric picture has done wonders for us on the Page curve. Can it help us with this?



# GEOMETRIZATION OF RECONSTRUCTION COMPLEXITY

## THE PYTHON'S LUNCH

In the QES picture, the geometric avatars of exponential complexity are the non-minimal QESs: Brown, Gharibyan, Penington, Susskind



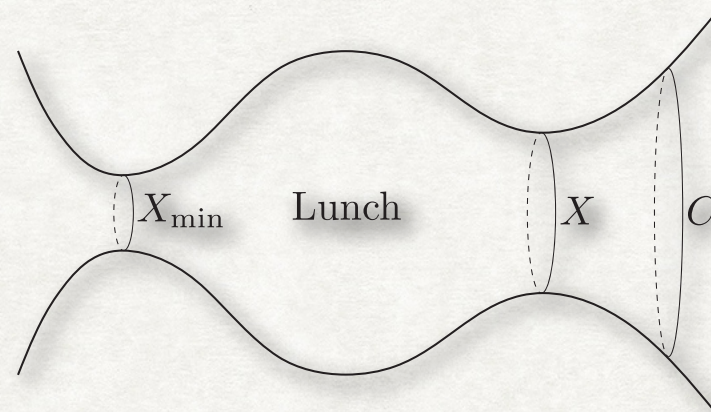


# PYTHON'S LUNCH

Python's Lunch Proposal:

$$C \sim \text{Exp} \left[ \frac{1}{2} \left( S_{\text{gen}}(\text{bulge}) - S_{\text{gen}}(X) \right) \right]$$

cf Geoff's talk: nonminimal QESs are the one and only source of exponential complexity in the AdS/CFT dictionary.



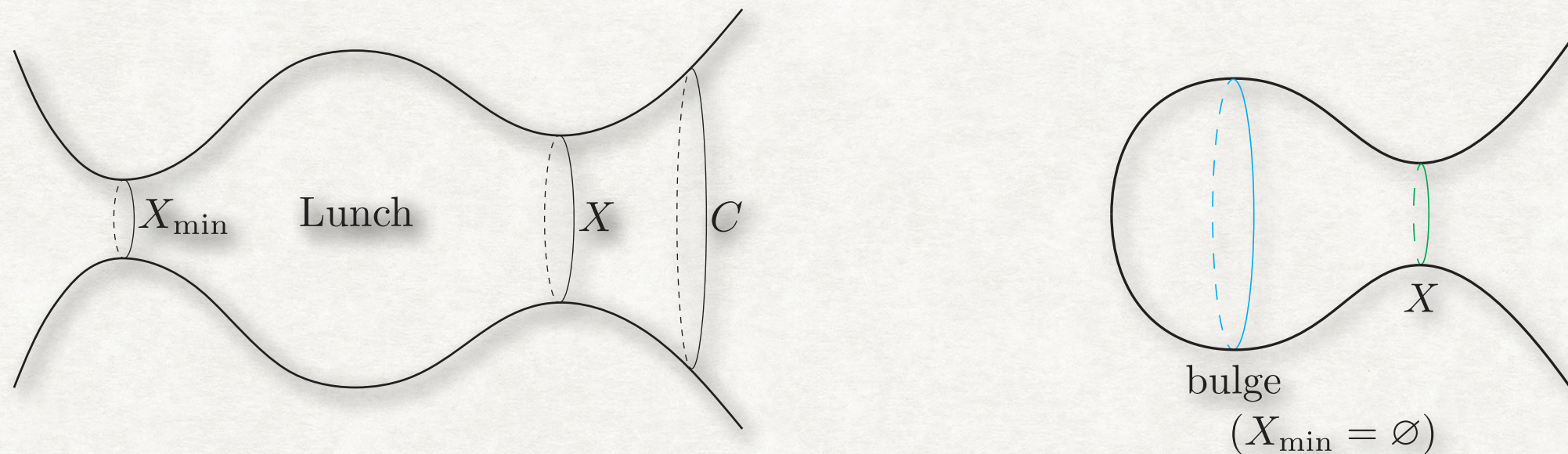
Consistency with gravitational path integral saddle calculations: if you use the "Hawking saddle", i.e. the wrong (subdominant) saddle — the nonminimal QES — then you don't know about the exponentially complex data and can't reconstruct the Hawking radiation.



# CODE SUBSPACE AND RECONSTRUCTION

Whether a QES is minimal or not depends on the code subspace of bulk states under consideration for reconstruction.

When the code subspace is very large, less of the bulk admits a state-independent reconstruction Hayden-Penington; Akers-Leichenauer-Levine

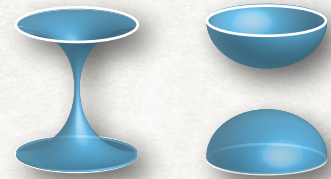


**Comment:** What about very large code subspaces: typical state black holes? What about typical state arguments for firewalls?

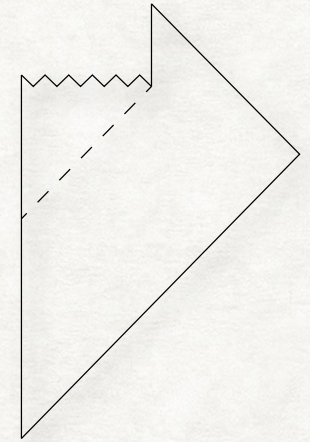


# QUESTIONS AND COMMENTS

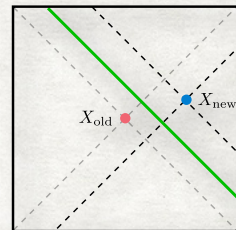
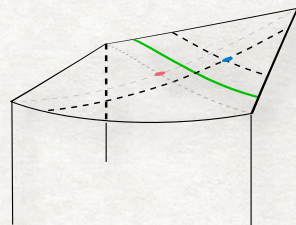
Why is semiclassical gravity - specifically the gravitational path integral - so smart?



Beyond AdS?



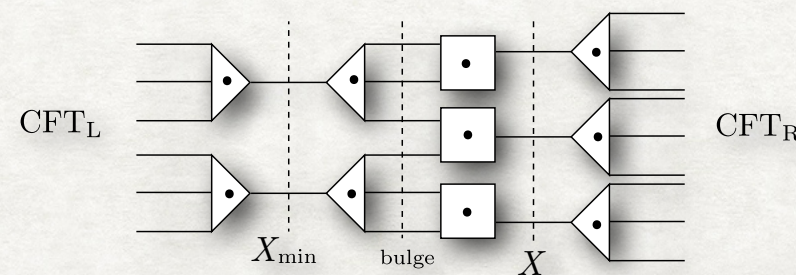
What more can we learn from doubly holographic holography?



Soft modes?  
Postselection?  
Edge modes?

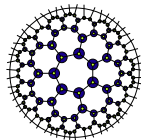
What about typical states?

What is the role of complexity in the resolution of the information paradox?





## QES: Hilbert space perspective

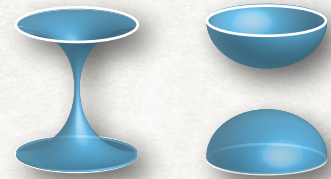


1. We want to understand the QES formula!
2. It's derived from the Euclidean gravitational path integral, but that doesn't explain everything. E.g. computes  $S_{BH} = A/4G$  but doesn't tell us the microstates we're counting.
3. We want a *Hilbert space* understanding of QES, microstates manifest.
4. One approach: understand as a consequence of  $\mathcal{H}_{\text{bulk}}$  embedded into  $\mathcal{H}_{\text{boundary}}$  like a quantum error-correcting code [Almheiri-Dong-Harlow].
5. Progress! Started in [Harlow] "Ryu-Takayanagi from QEC." Generalized to quantum minimal surfaces in [CA-Penington to appear].
6. Some lessons: Area is extra entanglement used to encode. Minimality from limited capacity for encoding.

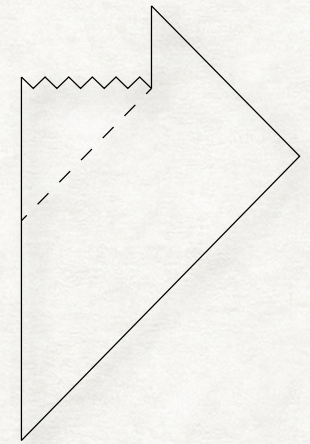


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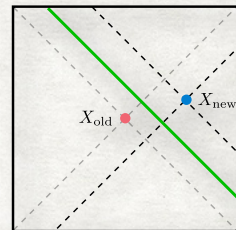
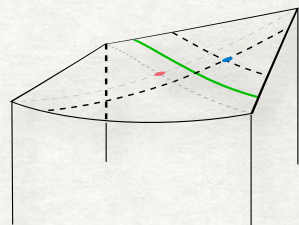
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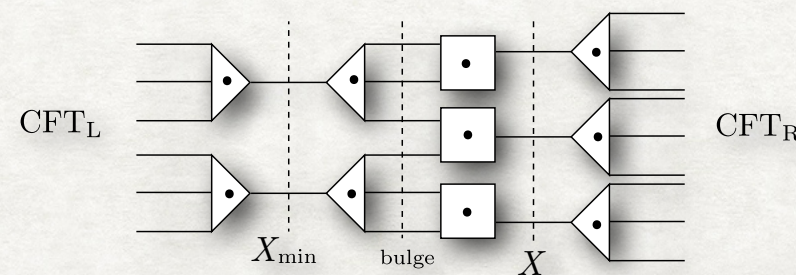
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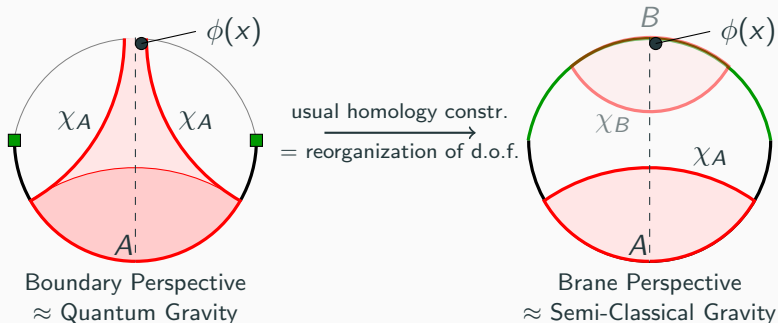




# (More) Lessons from Double Holography

What role does semi-classical gravity play?

How to access the black hole interior in semi-classical gravity?



We still cannot access BH interior from semi-classical gravity.

Reorganization looks like coarse graining from the outside.