Spaces of $\mathcal{N} = 1$ QFTs

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SR; SR, Vafa work in progress

Strings 2016

August 4, 2016 - 北京

Motivation

- What is the most general (S)CFT we can consider?

- S(CFT) as endpoint of RG flow of a theory with semi-classical description

- Useful geometric structure for space of (S)CFTs

- Systematic understanding of non-trivial (S)CFT phenomena (duality, RG flows)

- Here we will be in *4D* and have $\mathcal{N} \geq 1$

Spaces of QFTs from dimensional reduction

- Reduce D dimensional SCFTs to D' ($< D$) dimensions in different ways

- Example, N M5 branes probing A_{k-1} singularity reduced on Riemann surface $\mathcal{C}_{g,\mathbf{s}}$

- Theories in $4D$ labeled by (N, k) and $(g, \mathbf{s})$

- $k = 1$ is well studied

  [$\mathcal{N} = 2$ Gaiotto 09, $\mathcal{N} = 1$ Benini-Tachikawa-Wecht 10, Beem-Bah-Bobev-Wecht 11]

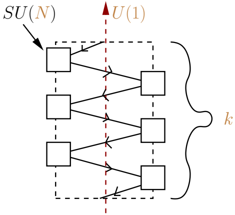
- Today, crank up k, have the discussion completely in $4D$

The strategy

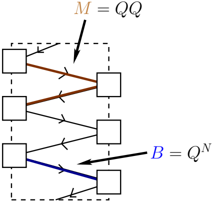
- Will discuss how to build spaces of theories labeled by geometric data
- Start from spheres with few punctures as building blocks and build theories corresponding to more complicated spheres and tori
- Use dualities to construct more sophisticated building blocks from which arbitrary Riemann surfaces can be built
- Will conjecture existence of new $\mathcal{N} = 1$ SCFTs and will discuss their properties

“Matter” building blocks

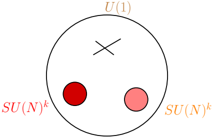
- $2N^2k$ free chiral fields
- $SU(N)^k$ – maximal puncture
- One $U(1)$ – minimal puncture
- Additional $2k-1$ $U(1)$ “internal” symmetries



- Mesons M charged under $SU(N)^k$
- Baryons B charged under $U(1)$
- k types of maximal punctures

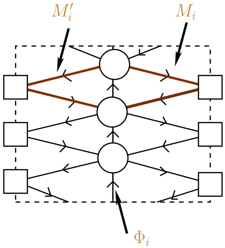


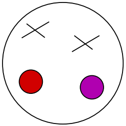
Label the theory by sphere with three punctures (free trinion), two maximal (of two different types for $k > 1$), and one minimal

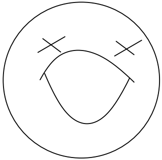


Gluing; Spheres and Tori

- "Glue" trinions together by gauging diagonal $SU(N)^k$
- Introduce bifundamental fields Φ_i and couple them through superpotential $M_i\Phi_i - \Phi_i M'_i$
- Sphere with 2 maximal and 2 minimal punctures
- 1dim conformal manifold with a duality group acting on it (Seiberg+$\mathcal{N}=2$) (complex structure modulus $\to$ coupling)
- Glue many trinions together $\to$ 2 maximal and many minimal punctures
- Glue 2 maximal punctures to each other $\to$ torus with minimal punctures
- Same max. punctures glued $\to$ internal symmetries preserved



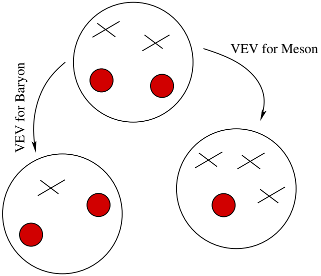




Flows

Access more theories by turning on vevs

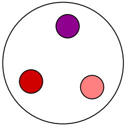
- VEVs for a baryon → close minimal punctures
- VEVs for a meson → reduce symmetry associated to maximal puncture to $SU(N)^{k-2} \times U(1)$



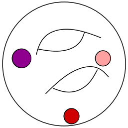
- $e.g$, $k = 2$, $N = 2$ sphere with 2 max. and 2 min., both flows lead to $SU(2)$ $N_f = 4$ SYM albeit with different singlet fields and superpotentials
- Have many baryons and mesons, vevs to different combinations lead to different physical theories

Higher genus; Interacting Trinions

To go to higher genus (or more maximal punctures) need to introduce new objects

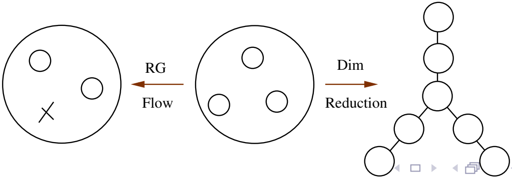


- Spheres with three maximal punctures
- Gluings and flows then lead to arbitrary surfaces



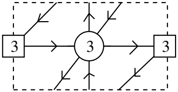
- Requirements: Couplings to moduli; Duality
- $k = 1$ $N > 2$: trinions are new SCFTs with no obvious Lagrangians

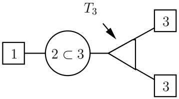
Reductions in degrees of freedom can lead to Lagrangians.

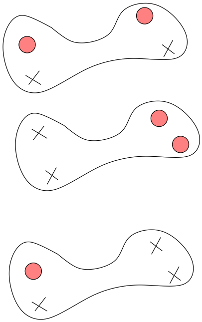


Case study A: $N = 3$ $k = 1$

- Simplest interacting theory is $SU(3)$ $\mathcal{N} = 2$ SYM with 6 flavors
- AS dual to an SCFT T_3 with $SU(2)$ gauging [Argyres, Seiberg 07]
- T_3 is the rank 1 SCFT with E_6 flavor symmetry [Minahan, Nemeschansky 96]
- Gluing together T_3s and giving VEVs to mesons (moment maps) theories corresponding to arbitrary Riemann surface with arbitrary punctures can be built
- T_3 has "no known Lagrangian description"
- A posteriori motivation for AS: VEV to meson in $SU(3)$ SYM leads $\mathcal{N} = 2$ $SU(2)$ SYM, should be true in any duality frame, $SU(2)$ superconformal tail

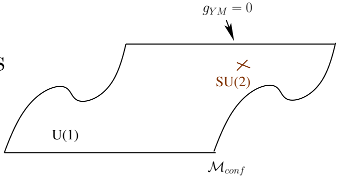






"Lagrangian" from Duality

- The SYM has 1D conf. manifold $\mathcal{M}_{conf}$
- In $\mathcal{N}=1$ language $\mathcal{M}_{conf}$ in general has global symmetry involving a $U(1)$
- At zero coupling in AS frame the $U(1)$ enhances to $SU(2)$

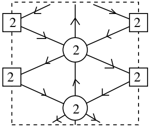


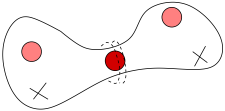
- We can couple the theory to dynamical $\mathcal{N}=1$ vector fields and extra chiral fields
- The dynamics of this theory ($SU(2)$ sector with $N_f=2$) leads to the T_3 SCFT in the IR
- In the $SU(3)$ frame the enhancement is at infinite coupling
- The procedure on SYM side provides $\mathcal{N}=1$ "Lagrangian" for T_3 albeit with infinite couplings
- Such Lagrangian is useful to extract any protected information about T_3 (anomalies, supersymmetric partition functions)

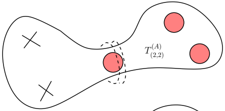
[Gadde,SR,Willett 15] ([Gadde,Rastelli,SR,Yan 10],[Putrov,Song,Yan 15])

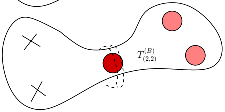
Case study B: $N = 2$ $k = 2$

- Simplest interacting theory is $SU(2)^2$ SYM, six flavors for each node
- 1D conformal manifold $\mathcal{M}_{conf}$
- AS-like duality frames; $SU(2)$ supersymmetric tails after VEV to meson
- Two types of maximal punctures, two types of trinions with maximal punctures, $T^{(A/B)}_{(2,2)}$ [Gaiotto,SR 15]
- The dualities are IR dualities here
- Gluing and flowing using the trinions theories corresponding to arbitrary Riemann surface with arbitrary punctures can be built
- What are these trinions?



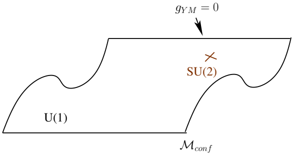






“Lagrangian” from Duality

- Almost identical procedure to $N = 3$ $k = 1$
- $\mathcal{M}_{conf}$ in general has global symmetry involving a $U(1)$
- At zero coupling in dual frame the $U(1)$ enhances to $SU(2)$



- Couple the theory to dynamical $\mathcal{N} = 1$ vector fields and extra chiral fields
- The dynamics of this theory ($SU(2)$ sector with $N_f = 2$) leads to the $T^{(A/B)}_{(2,2)}$ SCFT in the IR
- In the SYM frame the enhancement is at infinite coupling
- The procedure on SYM side provides “Lagrangian” for $T^{(A/B)}_{(2,2)}$ albeit with infinite couplings,
- which is useful to extract any protected information about $T^{(A/B)}_{(2,2)}$ (anomalies, supersymmetric partition functions)

Anomalies

- The $\mathcal{N}=1$ Lagrangian has $2k-1$ abelian symmetries which can be admixed with $U(1)_R$
- Compute trial a and extremize

$k=1,\ N=3$

$$R_{sc}=R+Sq_t$$

$$a_{trial}=-\frac{3}{32}(3S^3+27S^2-88S+44)$$

$$a=\frac{41}{24}\,,\qquad c=\frac{13}{6}$$

$k=2,\ N=2,\ (A)$

$$R_{sc}=R+Sq_t+S_\beta q_\beta+S_\gamma q_\gamma$$

$$S_\beta=S_\gamma=0.0689$$

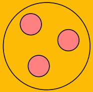
$$a=2.1153\,,\qquad c=2.56004$$

$k=2,\ N=2,\ (B)$

$$R_{sc}=R+Sq_t+S_\beta q_\beta+S_\gamma q_\gamma$$

$$S_\beta=-S_\gamma=0.0985$$

$$a=2.0621\,,\qquad c=2.61997$$

Index, example of 

The supersymmetric index of $T^{(A)}_{(2,2)}$

$$1+\left(\frac{2}{\beta^2\gamma^2 t^2}+\frac{\beta^2\gamma^2}{t^2}+t(\mathbf{2}_{u_1}\mathbf{8}_v+\mathbf{2}_{v_1}\mathbf{8}_s+\mathbf{2}_{w_1}\mathbf{8}_c)+ t\beta^2\gamma^2\mathbf{2}_{u_1}\mathbf{2}_{v_1}\mathbf{2}_{w_1}\right)(pq)^{\frac{2}{3}}+\left(\frac{1}{\beta^2\gamma^2}\mathbf{28}+ \beta^2\gamma^2(\mathbf{3}_{u_1}+\mathbf{3}_{v_1}+\mathbf{3}_{w_1})-\mathbf{28}-\mathbf{3}_{u_1}-\mathbf{3}_{v_1}-\mathbf{3}_{w_1}-1-1\right)(pq)+\dots$$

- $(pq)^{\Delta}$ term in the index

  [Beem,Gadde 12]

$\Delta<1$ Relevant

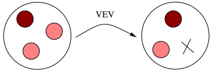
$\Delta=1$ Marginal – Currents

- Expect $(SU(2)^2)^3\times U(1)^3$ symmetry though not manifest in the Lagrangian
- Expected symmetry enhances to $SO(8)\times SU(2)^3\times U(1)^2$

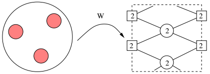
Flows

The SCFTs $T^{(A/B)}_{(2,2)}$ do not have a known standard Lagrangian, but are related to (each other and to) standard theories through RG flows.

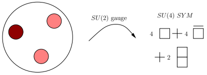
- VEV to meson of $T^{(B)}_{(2,2)}$ $\rightarrow$ free trinion



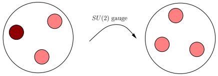
- Relevant superpotential deformations, E_7 surprize [Dimofte,Gaiotto 12]



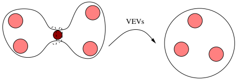
- $SU(2)$ gauging of $T^{(B)}_{(2,2)}$ $\rightarrow$ $SU(4)$ SYM with $N_f = 4$ and two antisymmetrics [Csaki,Schmaltz,Skiba,Terning 97]



- Different $SU(2)$ gauging of $T^{(B)}_{(2,2)}$ $\rightarrow$ $T^{(A)}_{(2,2)}$ (and vice versa)

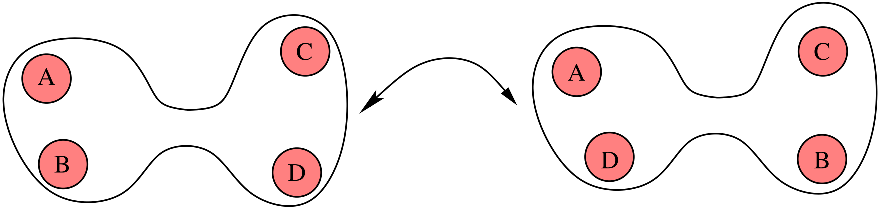


- Two $T^{(B)}_{(2,2)}$ glued, VEV to meson, VEV to baryon $\rightarrow$ $T^{(A)}_{(2,2)}$



Dualities

For the Riemann surface label of the theories to make sense they have to be invariant under dualities



The supersymmetric partition functions turn out to be invariant under such transformations

Conformal manifolds

We can glue the trinions together to form theories corresponding to arbitrary Riemann surfaces

The index of theory corresponding to genus g Riemann surface with s and $\widetilde{s}$ maximal punctures of the two types glued from $T^{(B)}_{(2,2)}$ and $T^{(A)}_{(2,2)}$

$$1+\cdots+\left(3g-3+\widetilde{s}+s+3g-1-1-1-\sum_{j=1}^{\widetilde{s}+s}(\mathbf{3}_{u_1^{(j)}}+\mathbf{3}_{u_2^{(j)}})\right)(pq)+\cdots$$

Have $3g-3+\widetilde{s}+s$ exactly marginal operators corresponding to complex structure moduli and additional deformations associated to flat connections for different symmetries on the surface.

[SR, Vafa *in progress*]

Summary

- New theories and new dualities
- "Lagrangians" for SCFTs which are useful to extract protected information
- From the novel SCFTs spaces of $\mathcal{N}=1$ theories are built with different theories labeled by Riemann surfaces

Open questions

- More evidence for existence of the theories (3d constructions, coupling dependent quantities)
- Generalization of the quantitative procedure beyond $k = 2$ $N = 2$
- Connecting directly to $6d$ constructions [SR,Vafa *in progress*] [del Zotto,Vafa,Xie 15, Ohmori,Shimizu,Tachikawa,Yonekura 15]
- The index is related to integrable models generalizing elliptic RS [Gorsky,Nekrasov 94, ...]with affine structure and has TQFT structure, $2d/4d$ correspondence [Gaiotto,SR 15, Maruyoshi,Yagi 16, Ito,Yoshida 16]

$$\mathcal{N} = 2 \quad \rightarrow \quad \mathcal{N} = 1$$

rational $\rightarrow$ algebraic

谢谢