State Dependent Operators and the Information Paradox in AdS/CFT

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Collaborators and References

- This talk is based on work with Kyriakos Papadodimas.
  - An Infalling Observer in AdS/CFT, arXiv:1211.6767
  - The Black Hole Interior in AdS/CFT and the Information Paradox, arXiv:1310.6334
  - State-Dependent Bulk-Boundary Maps and Black Hole Complementarity, arXiv:1310.6335
- And also on work in progress with Kyriakos, Prashant Samantray (postdoc at ICTS-TIFR, Bangalore) and Souvik Banerjee (postdoc at U. Groningen)

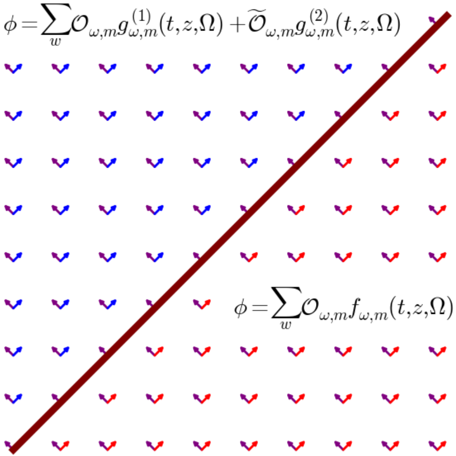
Summary

- Effective field theory predicts that quantum gravity effects are confined to a Planck-scale region near the singularity.
- Recent work suggests that to resolve the information paradox, one must drop this robust assumption: “quantum effects radically alter the structure of the horizon.”

  [Mathur, Almheiri, Marolf, Polchinski, Sully, Stanford, Bousso]
- I will describe how our construction of the black hole interior in AdS/CFT(see talk by Kyriakos) successfully addresses all these recent arguments.
- Then I will discuss the “state dependence” of our proposal, and describe work in progress.

Outline

Need for Mirror Operators



Apart from usual single-trace operators, new modes are required to construct a local field behind the horizon.

Properties of the Mirror Operators

- More precisely, the condition for smoothness of the horizon is that there should exist new operators $\widetilde{\mathcal{O}}(t, \Omega)$, satisfying

$$\langle \Psi | \mathcal{O}(t_1, \Omega_1) \ldots \widetilde{\mathcal{O}}(t'_1, \Omega'_1) \ldots \widetilde{\mathcal{O}}(t'_l, \Omega'_l) \ldots \mathcal{O}(t_n, \Omega_n) | \Psi \rangle$$
$$= Z_\beta^{-1} \mathrm{Tr} \Big[e^{-\beta H} \mathcal{O}(t_1, \Omega_1) \ldots \mathcal{O}(t_n, \Omega_n) \mathcal{O}(t'_l + i\frac{\beta}{2}, \Omega'_l)$$
$$\ldots \mathcal{O}\left(t'_1 + i\frac{\beta}{2}, \Omega'_1 \right) \Big].$$

- In Fourier space, we need $\widetilde{\mathcal{O}}_\omega$ satisfying

$$\langle \Psi | \mathcal{O}_{\omega_1} \ldots \widetilde{\mathcal{O}}_{\omega'_1} \ldots \widetilde{\mathcal{O}}_{\omega'_l} \ldots \mathcal{O}_{\omega_n} | \Psi \rangle$$
$$= e^{-\frac{\beta}{2}(\omega'_1 + \ldots \omega'_l)} \langle \Psi | \mathcal{O}_{\omega_1} \ldots \mathcal{O}_{\omega_n} (\mathcal{O}_{\omega'_l})^\dagger \ldots (\mathcal{O}_{\omega'_1})^\dagger | \Psi \rangle.$$

- This equation is deceptively simple. On the RHS, the tilde-operators have been moved to the right and reversed.

Construction of the Mirror Operators

- Given a basis equilibrium state, $|\Psi\rangle$, we can construct the mirror operators to satisfy the following linear equations.

$$\widetilde{\mathcal{O}}_{\omega}\mathcal{O}_{\omega_1}\ldots\mathcal{O}_{\omega_n}|\Psi\rangle = e^{\frac{-\beta\omega}{2}}\mathcal{O}_{\omega_1}\ldots\mathcal{O}_{\omega_n}(\mathcal{O}_{\omega})^{\dagger}|\Psi\rangle.$$

- Denote all products of $\mathcal{O}_{\omega_i}$ that appear above as $A_1 \ldots A_D$. This constitutes all reasonable low energy excitations of $|\Psi\rangle$.

- Clearly $D \ll \dim(H) = e^{N^2}$, and so for generic states we can solve these equations.

- Explicitly, with

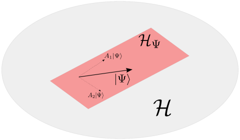
$$|v_m\rangle = A_m|\Psi\rangle; \quad |u_m\rangle = A_m e^{\frac{-\beta H}{2}}(\mathcal{O}_{\omega})^{\dagger}e^{\frac{\beta H}{2}}|\Psi\rangle, \quad g_{mn} = \langle v_m|v_n\rangle,$$

define

$$\widetilde{\mathcal{O}}_{\omega} = g^{mn}|u_m\rangle\langle v_n|.$$

State Dependence

- To fix these operators, we need to fix the "base state" $|\Psi\rangle$ and then consider reasonable experiments about this state.



- After this, these operators act as ordinary linear operators. One can multiply them, take expectation values etc.

$$\langle\Psi|\widetilde{\mathcal{O}}_{\omega_1} O_{\omega_2} \widetilde{\mathcal{O}}_{\omega_3} \ldots O_{\omega_n}|\Psi\rangle$$

- However, if we make a big change in the state, then one has to use different operators on the boundary to describe the field "at the same point" behind the horizon.

- Somewhat unusual, but perhaps to be expected.

Using Mirrors to Remove the Firewall

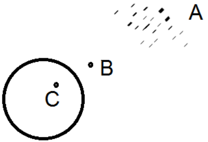
- Our explicit construction contradicts arguments in support of the structure at the BH horizon which can be sharply paraphrased as follows.

General reasoning (from counting, strong subadditivity of entropy, genericity of commutators etc.) suggest that the $\widetilde{\mathcal{O}}$ do not exist in the CFT

- I will now discuss how our explicit construction of the $\widetilde{\mathcal{O}}$ sidesteps all of these arguments.

- This is useful both to understand the hidden assumptions in these arguments and to understand some intriguing facets of our construction.

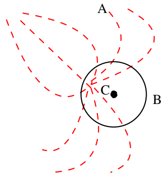
Resolving the Strong Subadditivity Paradox



- The first argument for structure at the BH horizon was based on strong subadditivity of entropy.
- For an “old black hole”, $S_{AB} < S_A$.
- For a smooth horizon, $S_{BC} = 0$. But, thermality of Hawking radiation implies $S_B = S_C > 0$.
- Seems to violate Strong Subadditivity at O(1)!

$$S_A + S_C \leq S_{AB} + S_{BC}.$$

Resolution to the SSE Paradox



- Our resolution is that A, B, C are not independent.
- Explicitly, in our construction

$$[\mathcal{O}_{\omega}, \widetilde{\mathcal{O}}_{\omega'}] \neq 0.$$

- This is consistent with old notions of complementarity: dof in the interior of the black hole have an overlap with the dof far away. Called $A = R_B$ by some authors.

[Verlinde^2, Bousso, Maldacena, Susskind]

[Nomura, Weinberg, Varela]

The Generic Commutator

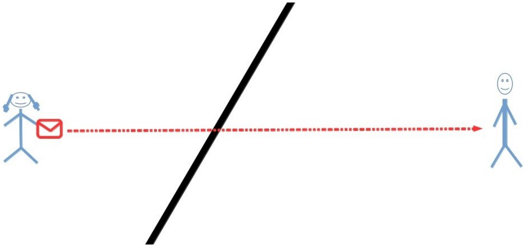
- For generic embeddings of the interior in the exterior, the non-zero commutator is easily measurable at $\mathrm{O}[1]$.

- More precisely, consider some operator $\mathcal{O}_\omega$, and try and define $\widetilde{\mathcal{O}}_\omega = U^\dagger \mathcal{O}_\omega^\dagger U$, for a randomly selected U.

- Since the Hilbert space is e^{N^2} dimensional, the matrix elements of $[O_\omega, \widetilde{\mathcal{O}}_{\omega'}]$ will be very small ($e^{\frac{-N^2}{2}}$).

- But
$$\langle \Psi | [\mathcal{O}_\omega, \widetilde{\mathcal{O}}_{\omega'}][\mathcal{O}_\omega, \widetilde{\mathcal{O}}_{\omega'}]^\dagger | \Psi \rangle = \mathrm{O}(1),$$
because the exponential suppression of the matrix elements is offset by the size of the matrix ($e^{N^2} \times e^{N^2}$).

The Commutator and Superluminal Propagation



- This suggests an unacceptable loss of locality.
- With such commutators, one could send messages across the horizon.
- The generic order 1 commutator was a powerful argument against the use of complementarity to remove the firewall.

Suppressing the Commutator

- Our construction resolves this in a clever way.
- Within low point correlators,

$$[\mathcal{O}_\omega, \widetilde{\mathcal{O}}_{\omega'}] A_p |\Psi\rangle = e^{\frac{-\beta\omega'}{2}} \mathcal{O}_\omega A_p (\mathcal{O}_{\omega'})^\dagger |\Psi\rangle - e^{\frac{-\beta\omega'}{2}} \mathcal{O}_\omega A_p (\mathcal{O}_\omega)^\dagger |\Psi\rangle = 0!$$

- While the commutator does not vanish, it is undetectable in low point correlators. We denote this by

$$[\mathcal{O}_\omega, \widetilde{\mathcal{O}}_{\omega'}] \doteq 0.$$

- Resolves a central objection to the use of complementarity!

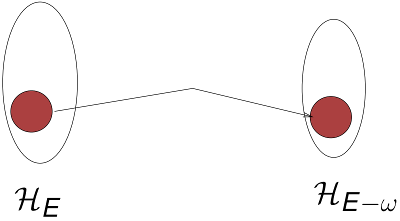
The Counting Argument

- Set $\widetilde{c}_\omega^\dagger = G_\omega^{\frac{-1}{2}} \widetilde{\mathcal{O}}_\omega^\dagger$: the normalized creation operator behind the horizon. Then,
$$[\widetilde{c}_\omega, \widetilde{c}_\omega^\dagger] A_p |\Psi\rangle = A_p |\Psi\rangle,$$
and so
$$\left(\frac{\widetilde{c}_\omega}{1 + \widetilde{c}_\omega^\dagger \widetilde{c}_\omega} \right) \widetilde{c}_\omega^\dagger = 1?$$

- But creating a particle behind the horizon in the Hartle-Hawking state is like destroying a particle in front of it.
$$[H_{\text{cft}}, \widetilde{c}_\omega^\dagger] = -\omega \widetilde{c}_\omega^\dagger.$$

- Since the growth of number of states with energy in the CFT is monotonic, $\widetilde{c}_\omega^\dagger$ cannot have a left inverse?

Resolving the Counting Argument



- The action of $\widetilde{c}_\omega, \widetilde{c}_\omega^\dagger$ is correct only on $|\Psi\rangle$ and its descendants produced by excitations with bounded energy and insertions.

$$[\widetilde{c}_\omega, \widetilde{c}_\omega^\dagger] \doteq 1$$
$$\Rightarrow [\widetilde{c}_\omega, \widetilde{c}_\omega^\dagger] A_p |\Psi\rangle = A_p |\Psi\rangle,$$

  for any light operator A_p.

- No contradiction with Linear Algebra!

The $N_a \neq 0$ Paradox

- General arguments suggest that for a fixed operator $\widetilde{\mathcal{O}}_\omega$, the microcanonical expectation of the number operator, $\langle N_a \rangle$, for the infalling observer is $\mathrm{O}[1]$.

  [Marolf, Polchinski]

- But,

$$G_\omega N_a = (1 - e^{-\beta\omega_n})^{-1} \Big[\left(O_\omega^\dagger - e^{-\frac{\beta\omega}{2}} \widetilde{\mathcal{O}}_\omega \right) \left(\mathcal{O}_\omega - e^{-\frac{\beta\omega}{2}} \widetilde{\mathcal{O}}_\omega^\dagger \right) + \left(\widetilde{\mathcal{O}}_\omega^\dagger - e^{-\frac{\beta\omega}{2}} \mathcal{O}_\omega \right) \left(\widetilde{\mathcal{O}}_\omega - e^{-\frac{\beta\omega}{2}} O_\omega^\dagger \right) \Big].$$

- However, our operators satisfy

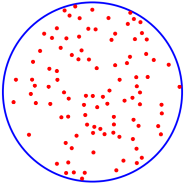
$$\widetilde{\mathcal{O}}_\omega |\Psi\rangle = e^{\frac{-\beta\omega}{2}} (\mathcal{O}_\omega)^\dagger |\Psi\rangle; \widetilde{\mathcal{O}}_\omega^\dagger |\Psi\rangle = e^{\frac{\beta\omega}{2}} \mathcal{O}_\omega |\Psi\rangle.$$

- Therefore $N_a|\Psi\rangle = 0$! Our construction has the explicit property that the infalling observer measures no particles at the horizon.

Interim Summary

The use of an appropriately state-dependent mapping between boundary operators and local bulk operators addresses all the recent information theoretic arguments for structure at the horizon.

Implications for Locality



- Now, we turn to some potential bugs/features of our construction.
- Our construction suggests that for connected N-point correlators, locality breaks down completely.
- Is there independent evidence for this?

[Mathur]

Locality and Perturbation Theory

- The CFT permits a dual local description only for quantities that have a good $\frac{1}{N}$ expansion.
- Consider the bulk Feynman path integral

$$\mathcal{Z} = \int e^{-S} \mathcal{D} g_{\mu\nu}.$$

  A semi-classical spacetime is a saddle point of this path-integral, about which we can do a $\frac{1}{N}$ expansion.

- So locality breaks down $\sim \frac{1}{N}$ perturbation theory breaks down for N-point correlators.
- Possible to do by crude counting of Feynman diagrams.

Combinatorics of High Point Correlators

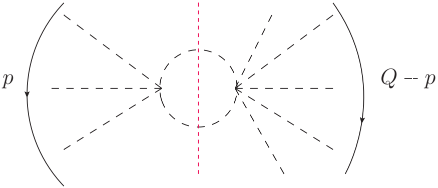
- A tree-level bulk connected Q-point amplitude scales like

$$M_{\text{tree}} \sim \left(\frac{1}{N}\right)^{Q-2} (Q-3)!$$

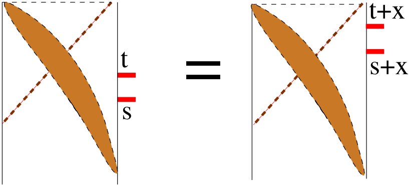
- But, at one-loop, we get a contribution from

$$M_{1l} \sim \left(\frac{1}{N}\right)^{Q} \sum_{p=1}^{Q-1} \binom{Q}{p} (p-1)!(Q-p-1)! \sim \left(\frac{1}{N}\right)^{Q} (Q-1)!,$$

$$\frac{M_{1l}}{M_{\text{tree}}} \sim \frac{Q^2}{N^2}.$$



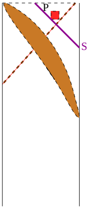
Criterion for Equilibrium



- The formalism must be improved for states out of equilibrium. [Bousso, van Raamsdonk]
- A necessary condition for equilibrium is time-independence of correlators.
- More precisely, with $\chi_p(t) = \langle \Psi | e^{iHt} A_p e^{-iHt} | \Psi \rangle$ an equilibrium state satisfies

$$\nu_p = \omega_{\text{min}} \int_0^{\omega_{\text{min}}^{-1}} |(\chi_p(t) - \chi_p(0))| dt = \text{O}\left[e^{-\frac{S}{2}}\right], \ \forall p.$$

Mirrors for Near Equilibrium States



- Consider a class of near equilibrium states

$$|\Psi'\rangle = U|\Psi\rangle, \quad U = e^{iA_p}.$$

- Can detect U by using time-invariance criterion, and identify it.

- Now, improve mirror operators to

$$\widetilde{\mathcal{O}}_\omega A_p|\Psi'\rangle = A_p U e^{-\frac{\beta\omega}{2}} (\mathcal{O}_\omega)^\dagger U^\dagger |\Psi'\rangle.$$

- Again reproduces semi-classical expectations.

Potential Ambiguity in Equilibrium States

- Given an equilibrium state $|\Psi\rangle$, consider another state

$$|\widetilde{\Psi}\rangle = e^{i\widetilde{\mathcal{O}}_\omega}|\Psi\rangle.$$

- $\langle\widetilde{\Psi}|\mathcal{O}(t_1)\ldots\mathcal{O}(t_n)|\widetilde{\Psi}\rangle$ is also time-translationally invariant.

[van Raamsdonk]

- However, consider inserting the Hamiltonian

$$C_{OH} = -i\langle\widetilde{\Psi}|\mathcal{O}_\omega H|\widetilde{\Psi}\rangle.$$

For an equilibrium state, this correlator is exponentially small.

- However, here we have

$$C_{OH} = \frac{\omega e^{\frac{-\beta\omega}{2}}}{1 - e^{-\beta\omega}}.$$

So measuring the Hamiltonian helps us detect these perturbations behind the horizon.

Another Ambiguity

- However, it is possible to define different operators $\widetilde{\mathcal{O}}'_{\omega}$, which satisfy

$$[\widetilde{\mathcal{O}}'_{\omega'}, \mathcal{O}_{\omega}] \doteq 0, \quad [\widetilde{\mathcal{O}}'_{\omega'}, H] \doteq 0.$$

[Harlow]

- These cannot be defined on an energy eigenstate. Moreover, $\widetilde{\mathcal{O}}'_{\omega}$ are not natural candidates for building the field inside the black-hole since they create particles inside the black hole without a change in energy.

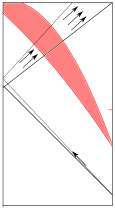
- Important to understand how to classify

$$|\widetilde{\Psi}'\rangle = e^{i\widetilde{\mathcal{O}}'_{\omega}}|\Psi\rangle,$$

because we cannot detect that it is out of equilibrium using either $\mathcal{O}_{\omega}$ or the Hamiltonian.

More on the Ambiguity

- This question is independent of our proposal.
- Before the recent fuzz/fire/complementarity arguments, everyone would agree that an exponentially small fraction of microstates have excitations behind the horizon.
- How does one know if a given CFT state falls in this class or not?
- Even from bulk, very hard to tell because of the trans-Planckian problem.



State Dependence

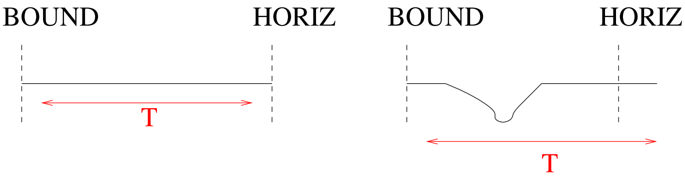
- Adding general state-dependent operators to the Hamiltonian can allow one to send superluminal signals through EPR pairs or communicate between “branches of the wave-function.”
  [Gisin, Polchinski, 1990–91]

- Important difference in our case: one might imagine, based on this old work, that the bulk theory could have uncontrolled properties but we have an autonomous and well defined CFT in this case.

- Need to understand better what happens when the CFT is entangled with other systems in various ways. But, so far, no thought experiment that produces a concrete contradiction.

- Moreover, local operators are unusual in quantum gravity.

Positioning Local Operators



- Should one expect to be able to "position" the bulk operator in a state-independent manner? Attempting a relational procedure from the boundary is difficult.

[Susskind, Motl]

- In fact, effects of the firewall can be mimicked by incorrectly positioning local operators. So, a funny two-point function "across the horizon" may mean that the geometry is perfectly regular but the bulk probes are not positioned where one thinks they are

Background independent local operators?

- Consider
$$\phi(x) = \int \mathcal{D}g \left(\sum_{\omega} \mathcal{O}_{\omega} f_{\omega,g}(x) \right) P_g.$$
where P_g projects onto coherent states corresponding to the semi-classical metric g, and the sum is over all such metrics.

- Coherent state projectors are not orthogonal. [Motl]

- Therefore, difficult to prove that this operator above is "local":
$$\lim_{x \to x'} \langle g | \phi(x) \phi(x') | g \rangle = \left(g^{\mu\nu} (x_{\mu} - x'_{\nu})(x_{\mu} - x'_{\nu}) \right)^{-\Delta} ?$$

- If this works outside the BH, should it also work inside?

- Consistent with the lore that there are no background independent local operators in quantum gravity.

Local Operators in Quantum Gravity

- So, perhaps one is forced to use a reference state to define a background and then place operators in this background.

- This needs to be understood better!

This necessity of state-dependent bulk-boundary maps to smoothen the horizon of the black hole seems to be a key lesson of the firewall debate. Leads to a question of "how do we really describe local bulk observables in AdS/CFT?"

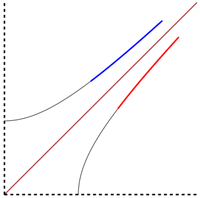
- Seems to be a very broad and interesting question that has arisen out of this discussion.

Hopefully, we will have more to say on this by Strings 2015, which is at our new campus in Bangalore!

Appendix

Interactions with an environment

- Simply adding interactions with an environment is not a problem for the construction.



- Prescription is not obtained by manually identifying “entanglement.”
- Rather, the action of an operator inside the horizon can be represented by an operator outside. (see figure.)
- Very robust against interactions with the CMB etc. that do not modify the horizon within EFT.

Small Corrections

- A theorem of Mathur (2009) states that "small corrections cannot unitarize Hawking radiation".

- This theorem implicitly disallows the state-dependent and non-local $\widetilde{\mathcal{O}}_\omega$ operators that we have used.

$$\langle\Psi|\phi_{\text{CFT}}(t_1, z_1)\ldots\phi_{\text{CFT}}(t_n, z_n)|\Psi\rangle = \langle\phi(t_1, z_1)\ldots\phi(t_n, z_n)\rangle_{\text{bulk}} + \mathrm{O}\left(\frac{1}{N}\right),$$

  where on the LHS, our operators are sandwiched in a typical state, and the RHS is calculated by Feynman diagrams in the bulk QFT.

- In particular, the two point function across the horizon is smooth

- So, small corrections to bulk correlators are consistent with unitarity and no information loss.

Literally doing the AMPS experiment

- What if someone really collects the outgoing Hawking radiation, performs a quantum computation and gives the infalling observer the bit that is entangled with the inside dof?
- This is a non geometric process; involves measuring a N-point correlator.
- Mathematically, it is like adding some operator $A^1_{\text{ng}} \ldots A^p_{\text{ng}}$ to the set of observables, so that $A^i_{\text{ng}}|\Psi\rangle = 0$.
- Then the operators in the ideal

$$\mathcal{I}\left(A^1_{\text{ng}} \ldots A^p_{\text{ng}}\right)$$

cannot be doubled.

- In a sense, there is a firewall for "these observables", but other observables still see a smooth horizon.

Other thermal systems

- As Kyriakos explained yesterday, other chaotic systems also see doubling in typical pure states.
- However, the existence of mirror operators is not sufficient for there to be an "interior."
- We have to be able to put the mirror and ordinary operators together in a local quantum field.
- Relies on properties of correlators outside the horizon, which are not met in other cases.