Deciphering AdS/CFT

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Discussion Session - "Proving Dualities",

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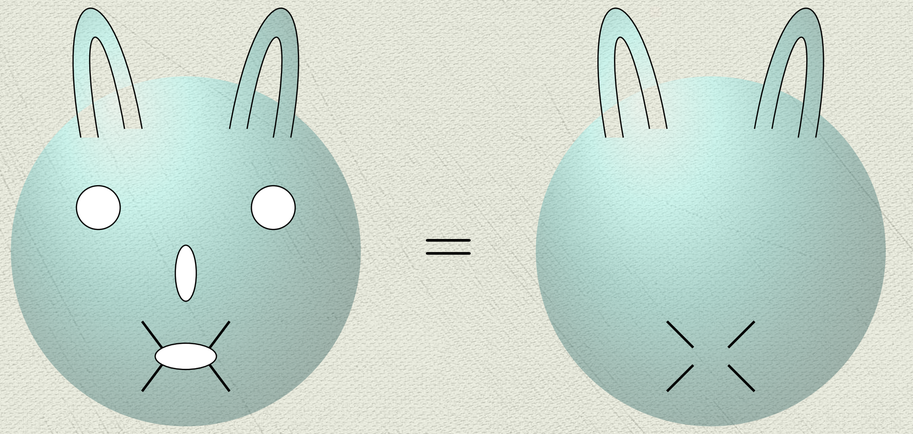
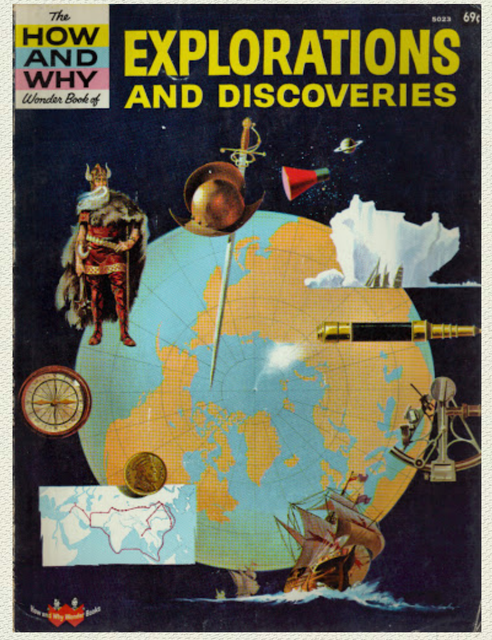
$\sum_{\text{holes}}$  $=$

How and Why?

- Why derive Gauge-String Duality?
- What can we learn from deciphering AdS/CFT?
- How can we derive Gauge-String Duality?
- A particular perspective & strategy.



Other Approaches: Berkovits, Ooguri, Vafa, Polyakov, H.Verlinde, Costello, Gaiotto, BMN/Integrability, OSFT, H-Spin, Bootstrap…

Building a Bridge

From Strings to Fields

From Fields to Strings

Pick a good spot: Tensionless AdS String $\leftrightarrow$ Free $(\lambda \to 0)$ Gauge Theory

From Strings to Fields (how holes open up)

- Open-closed string duality.
- How do D-branes emerge from a closed string worldsheet?
- Two phases: `Higgs' (H) and `Coulomb' (C) coexist as $\lambda \to 0$.
- Linear Sigma Model for conifold
  $x_1x_2 - x_3x_4 = 0.$ $(x_i = a_\alpha b_\beta)$
- H: $\langle a_\alpha \rangle \neq 0$; C: $\langle a_\alpha \rangle = 0$;
  $\langle \sigma \rangle = 0.$ $\langle \sigma \rangle \neq 0.$ Holes!

hep-th/9811131

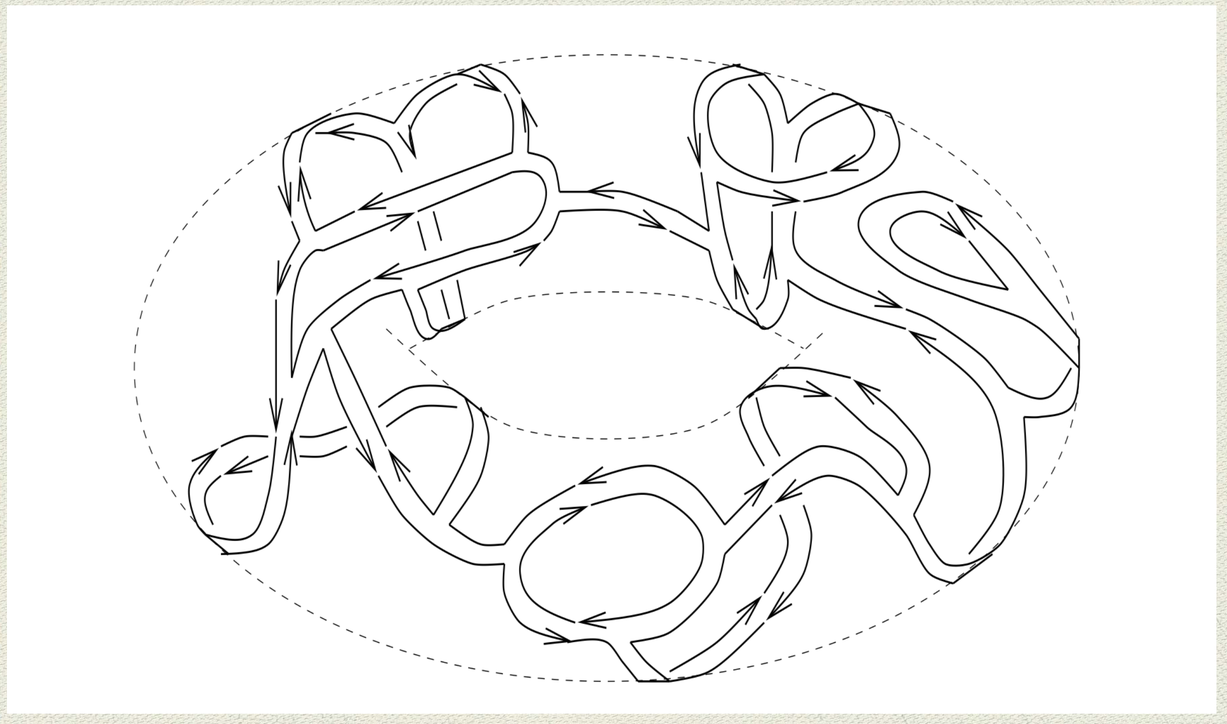
On the Gauge Theory/Geometry Correspondence

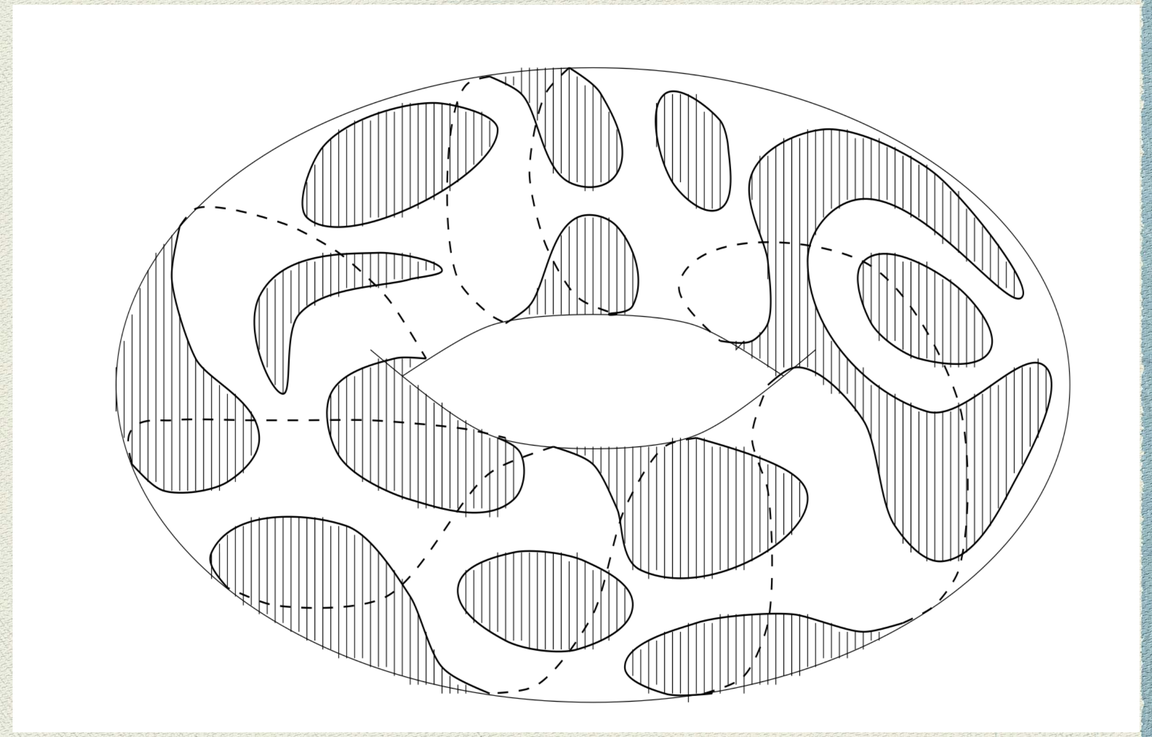
Rajesh Gopakumar [1] and Cumrun Vafa [2]

The 't Hooft expansion of $SU(N)$ Chern-Simons theory on S^3 is proposed to be exactly dual to the topological closed string theory on the S^2 blow up of the conifold geometry. The B-field on the S^2 has magnitude $Ng_s = \lambda$, the 't Hooft coupling. We are able to make a number of checks, such as finding exact agreement at the level of the partition function computed on *both* sides for arbitrary λ and to all orders in $1/N$. Moreover, it seems possible to derive this correspondence from a linear sigma model description of the conifold. We propose a picture whereby a perturbative D-brane description, in terms of holes in the closed string worldsheet, arises automatically from the coexistence of two phases in the underlying $U(1)$ gauge theory. This approach holds promise for a derivation of the AdS/CFT correspondence.

[Fuller derivation for CS case - Ooguri-Vafa'02]

Coulomb Phase / Face

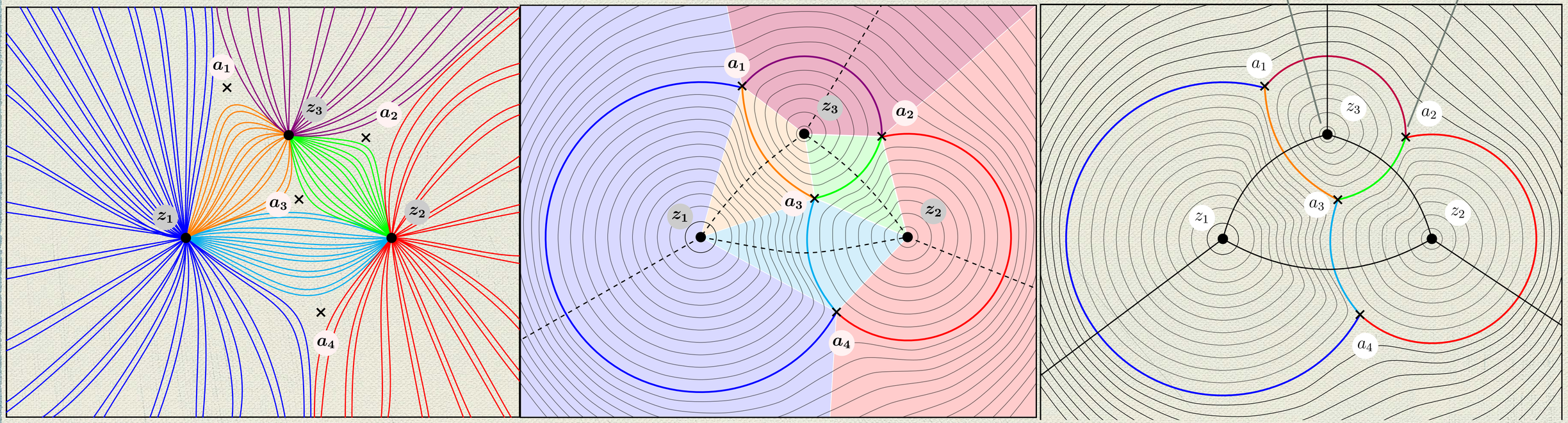




[From Ooguri-Vafa'02]

From Fields to Strings (how holes close up)

Open-Closed Triptych



A) 'tHooft Graphs

B) Edges glued up

C) Strebel Riemann Surface

[R.G. '03-'05]

Pictures: Pronobesh Maity

Hexagonalisation

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle =$$

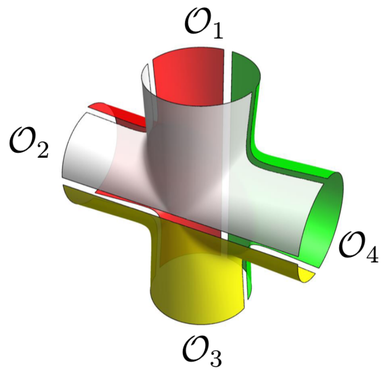
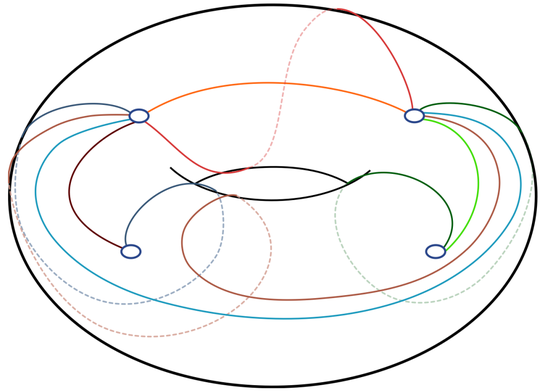
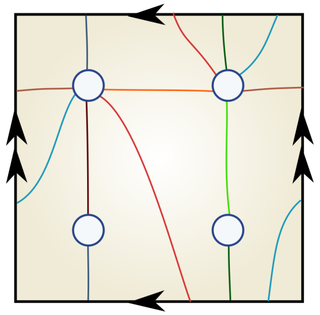
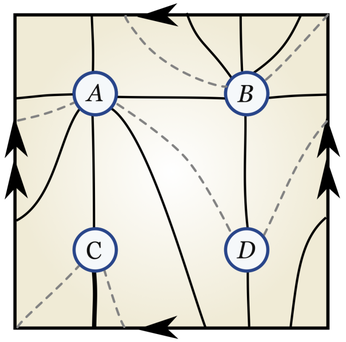


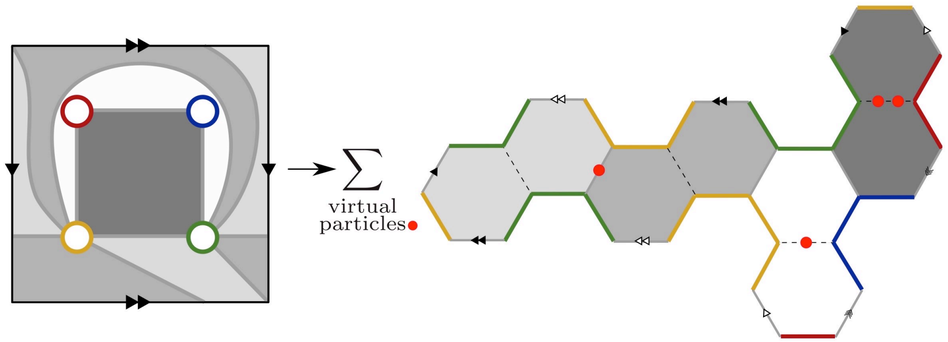
Figure 1: Hexagonalization of a four-point function: A planar four-point function can be represented as a surface with four holes. The idea of hexagonalization is to cut it into four hexagonal patches as depicted above. The contribution from each patch is given by a hexagon form factor. It is conceptually different from the usual operator product expansion.







Genus zero correlator [from Fleury-Komatsu'16]



Non-planar correlator

[Bargheer, Caetano, Fleury, Komatsu, Vieira '17-'18]

The String Dual to Free $\mathcal{N} = 4$ SYM

Twistorial Gauged Linear Sigma Model for
$AdS_3 \times S^3 : Y_I = (\eta_\alpha, \chi_\beta);\ Z^I = (\xi^\alpha, \psi^\beta)\,.$
[Eberhardt-Gaberdiel-R.G.'18,'19; Dei-Gaberdiel-R.G.-Knighton'20]

Twistorial Gauged Linear Sigma Model for
$AdS_5 \times S^5 : Y_I = (\mu^\dagger_\alpha, \lambda^\dagger_{\dot\alpha}, \psi^\dagger_a);\ Z^I = (\lambda^\alpha, \mu^{\dot\alpha}, \psi^a)\,.$
[Gaberdiel-R. G. '21; Matthias' talk]

Ambitwistor Open String Theory (Y_I, Z^I)
[Berkovits'04; Mason-Skinner'13; cf. Mason's talk]

BMN Light Cone Gauge String Theory
[Berenstein-Maldacena-Nastase '02]

From Strings to Fields (Once More)

- GLSM description of $AdS_5 \times S^5$, $AdS_3 \times S^3$ reminiscent of GV Duality.
- Twistor fields have vortex like behaviour $\sim (z - z_i)^{\pm \frac{w_i}{2}}$ near closed string insertions - signature of Coulomb phase?
- Twistor open string = description of D-branes at $R_{AdS} = 0$?
- Worldsheet is rigid except for (w) dynamical string beads on a necklace.
- Localisation on moduli space like topological string (A-model).

From Fields to Strings (again!)

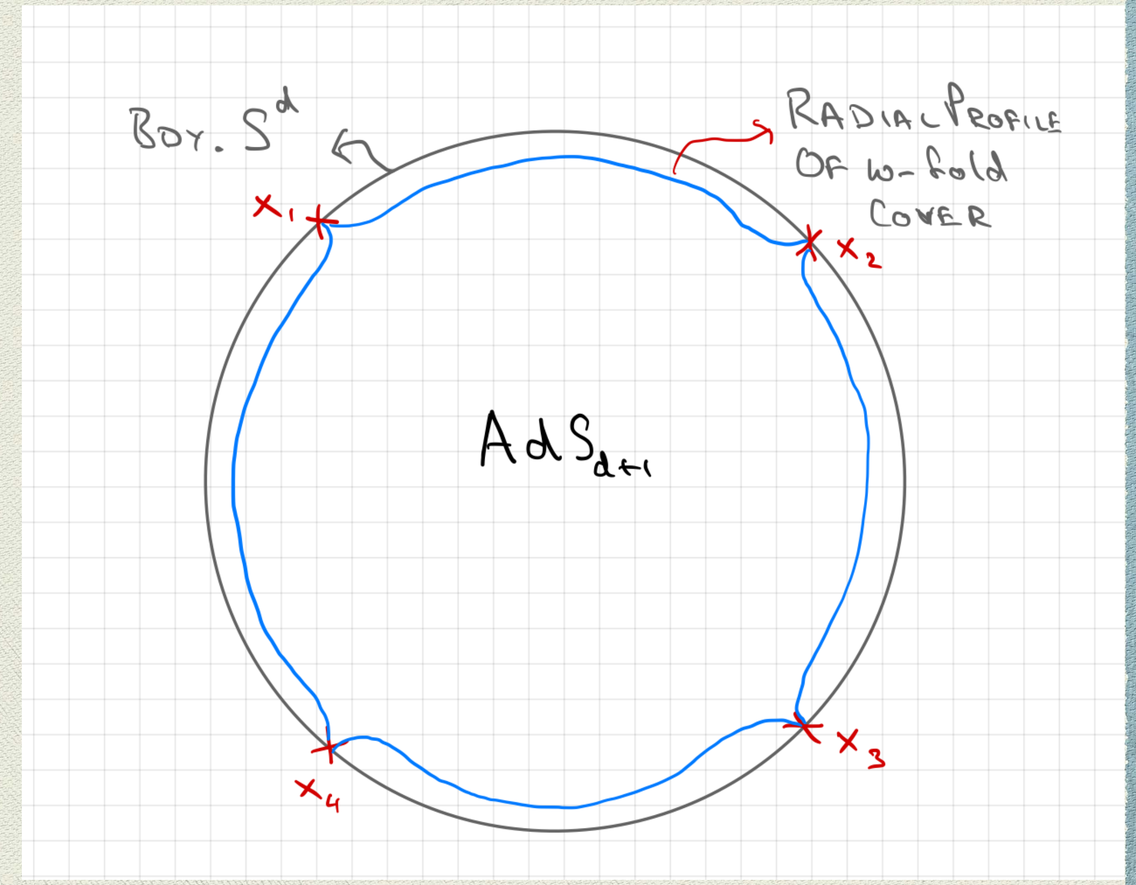
- Double lines of $\mathrm{Tr}\Phi^w \leftrightarrow$ rigid worldsheet with w string bits: multiple cover.
- Feynman diagrams of symm-product CFT $\leftrightarrow$ discrete points on moduli space - admit covering.

[Gaberdiel-RG-Knighton-Maity'20]

- Strebel surfaces (integer length, for large w).

[R.G.'05, Razamat'08]

- Covering maps in twistor space for 4d YM.
- Glued Feynman worldlines $\rightarrow$ worldsheet hugging the *AdS* boundary - radial profile.



[Bhat-R.G.-Maity-Radhakrishnan'21] [Gaberdiel-R.G.-Knighton-Maity'21] (In Progress)

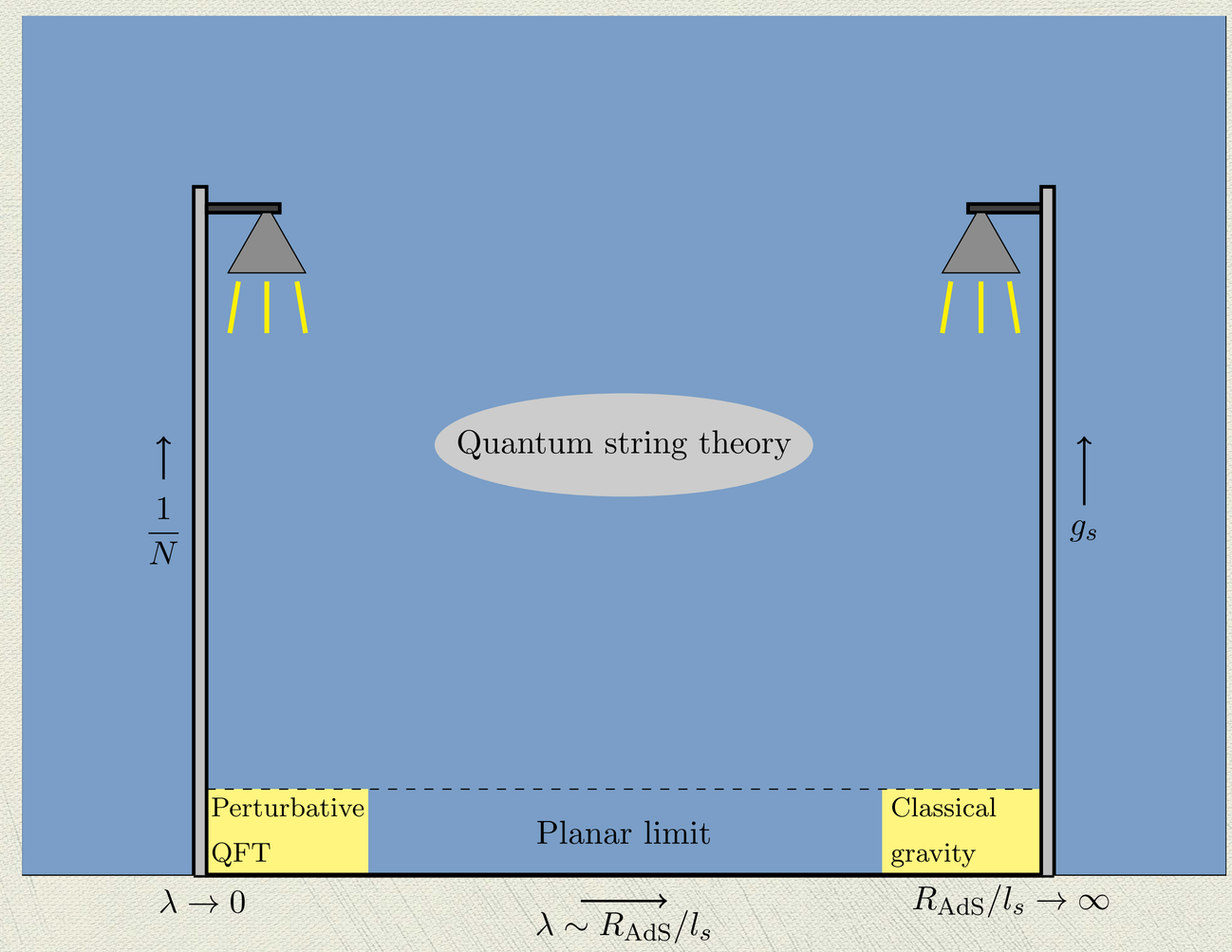
What can we hope to learn?

- Twistorial AdS/CFT (correlators)? [cf. Adamo-Skinner-Williams'16].
- Topological string formulation? Emergence of space-time at large radius?
- String Bits and holographic encoding of Quantum Information?
- Worldsheet ↔ Spacetime interplay (A CFT_2/CFT_d correspondence?)
- $\lambda \neq 0$: Coulomb-Higgs phases of GLSM? Connect with D-branes/scattering amplitudes.
- $\lambda \neq 0$: The role of higher spin symmetry and its higgsing.
- $\lambda \neq 0$: Covariant spin chain picture and integrability? [cf. Komatsu review] [cf. Aharony review]
- Worldsheet dual to planar Yang-Mills/QCD? Bosonic vacua in an N=4 critical theory?

Perhaps time to walk across the Bridge

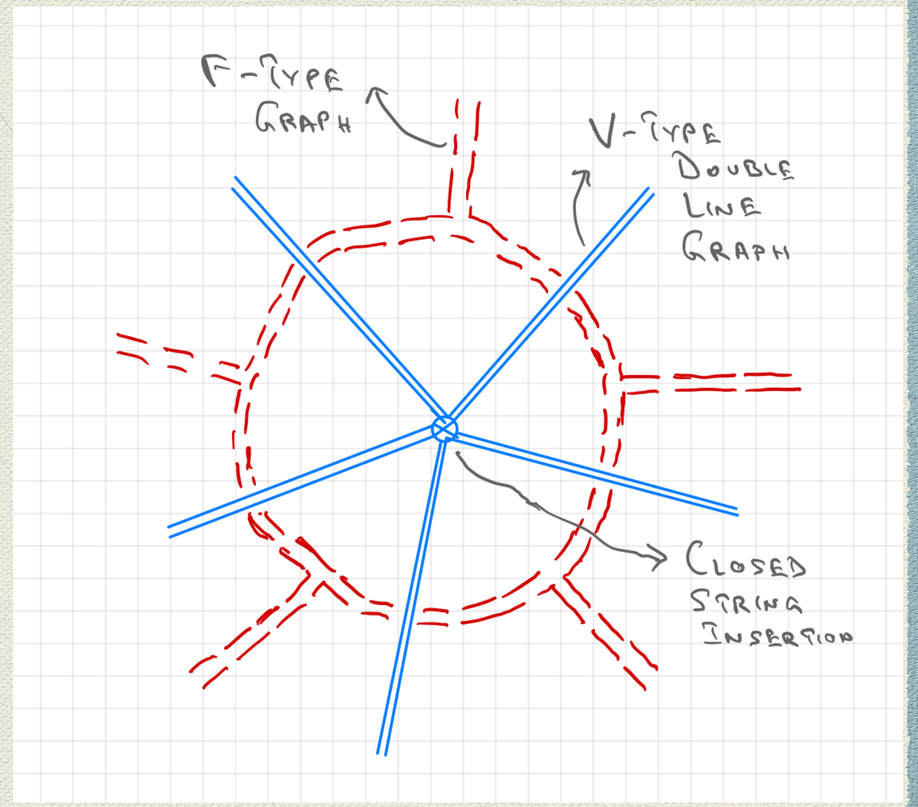
Additional Slides

Another Lamp Post



Open-Closed-Open Triality

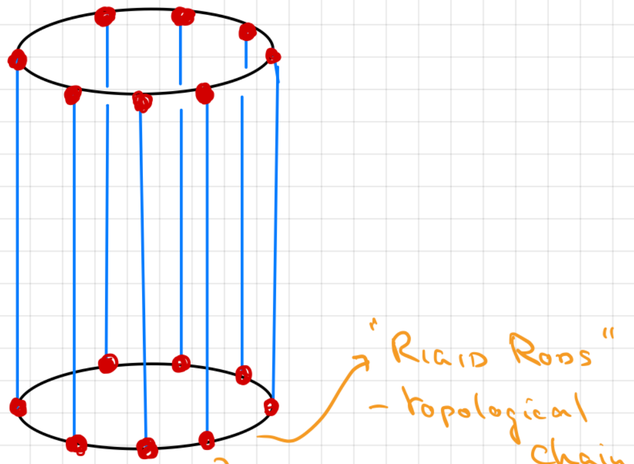
- Chern-Simons Duality of F-type.
- Faces (holes) = closed string insertions.
- Usual AdS/CFT dictionary: V-type.
- Vertices = closed string insertions.



- Two different open string pictures, related by graph duality. D-branes at `large radius' (V) vs. `small radius' (F)?

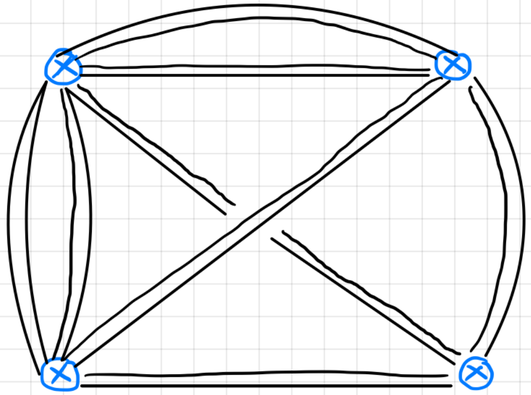
[R.G.'10 (Jo'burg talk)]

Takeaway Pictures



Covariant BMN-like Picture w/ Twistor Bits

- Forming a spin chain



SYM Correlators Arise From These Rigid Rods Glued Together

- Worldsheets On Discrete Pts. On $M_{g,n}$ (via Strebel Differentials)

Another View

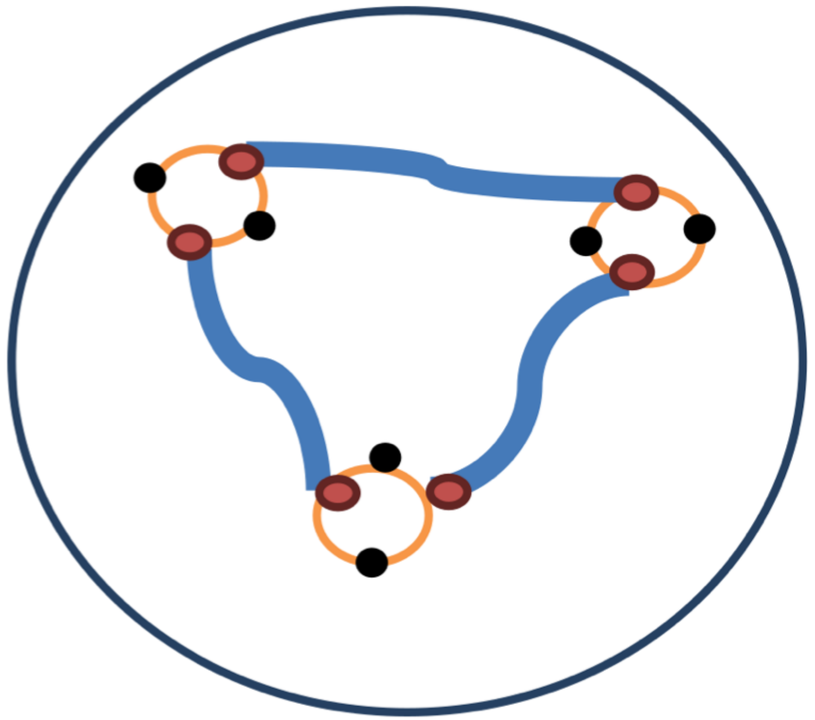
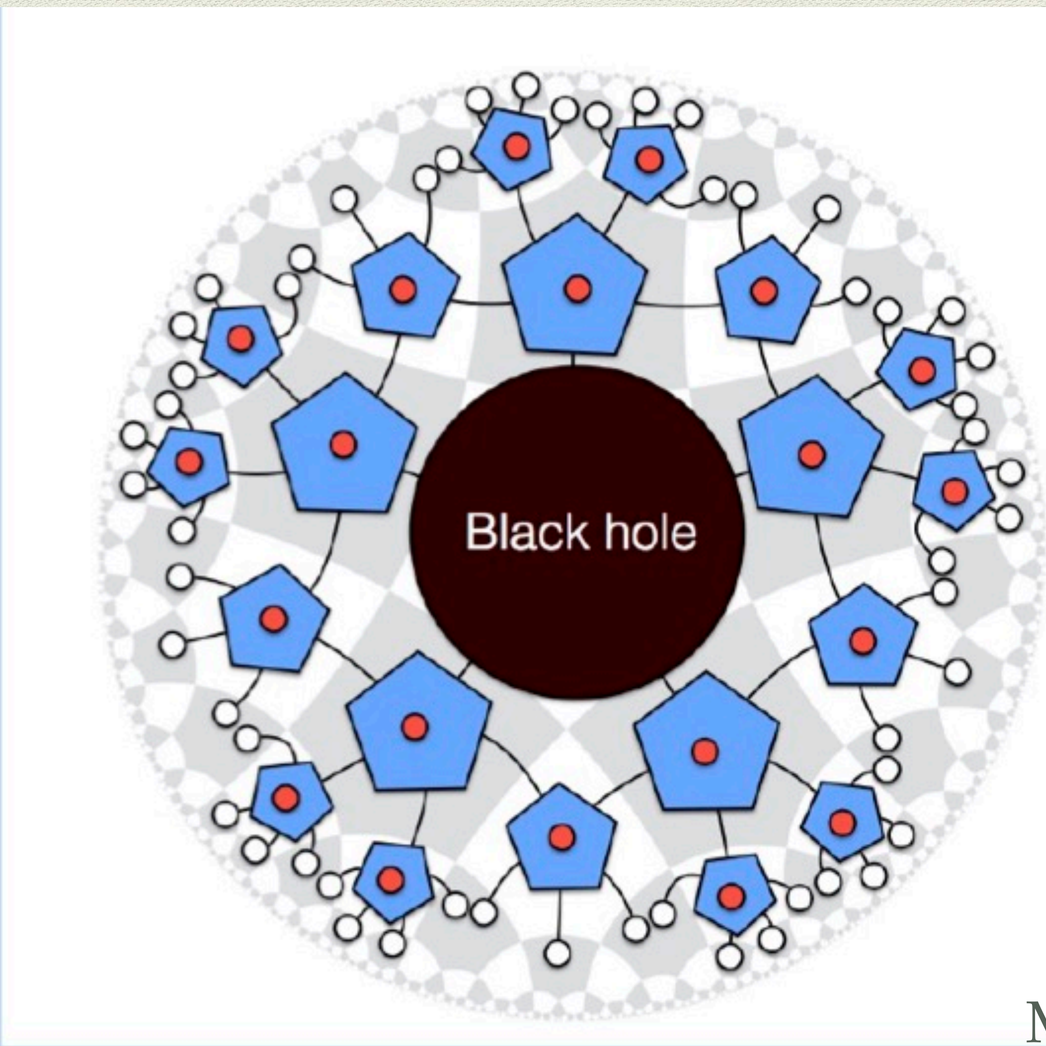


Fig. 1: 3-point amplitude on sphere where orange circles are closed string vertex operators, blue strips are thickened propagators near the AdS boundary, white regions are D3-brane holes near the AdS horizon, black dots are picture-raising operators, and red dots are beads E on the closed strings

[From Berkovits'19]

Building a Spacetime (bit by bit)



Maldacena Strings'21 Slides