Soft Hair on Black Holes - Part 2

Malcolm Perry

University of Cambridge

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Some Key points from Part One

There are an infinity of charges Q_ϵ^+ defined at the past end-point of future null infinity, $\mathcal{I}_-^+$.

$$Q_\epsilon^+ = \int_{\mathcal{I}_-^+} d\Omega \; \epsilon * F.$$

where ϵ is any spherical harmonic and F is the elctromagnetic 2-form field strength.

There are an infinity of charges Q_ϵ^- defined at the future end-point of past null infinity, $\mathcal{I}_+^-$.

$$Q_\epsilon^- = \int_{\mathcal{I}_+^-} d\Omega \; \epsilon * F.$$

Although spacelike infinity is singular, Christodoulou and Klainerman showed that under certain conditions

$$Q_\epsilon^+ = Q_\epsilon^-$$

where the antipodal map between $\mathcal{I}_-^+$ and $\mathcal{I}_+^-$ is used in the evaluation of the integrals.

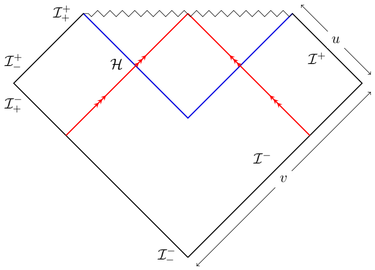
This gives an infinite number of conservation laws!

Each integral can be, by Gauss' theorem, extended into an integral over a Cauchy surface. In Minkowski space for example, assuming only massless degrees of freedom so that ι^+ can be neglected,

$$Q_\epsilon^+ = \int_{\mathcal{I}^+} d\epsilon * F + \epsilon * j$$

The first term is interpreted as due to soft photons passing through $\mathcal{I}^+$. The second term is due currents passing through $\mathcal{I}^+$. Let $|0\rangle$ be a vacuum state. Then $|0'\rangle = Q_\epsilon^+|0\rangle$ is a different vacuum state, differing from the oriiginal by the addition of a soft photon. This is a large gauge transformation. The vacuum is thus infinitely degenerate.
Picking a particular vacuum state breaks this symmetry leaving the photon as the Nambu-Goldstone boson.

Eternal Black Holes



Define an horizon charge at the future end-point of the Horizon $\mathcal{H}^+$.

$$Q_\epsilon^{\mathcal{H}} = \int_{\mathcal{H}} \epsilon * F.$$

Extend this into an integral over the horizon just as we did for the integral over null infinity. The two terms represent a soft photon on the horizon and current flowing through the horizon.
The action of $Q_\epsilon^{\mathcal{H}}$ is again a large gauge transformation on the horizon.
Classically, the no-hair theorem gives us that $Q_\epsilon^{\mathcal{H}} = 0$ except for the case $\epsilon = 1$ or $l = 0$. Black holes can carry classical electric charge.

$$|BH'\rangle = Q_\epsilon^{\mathcal{H}}|BH\rangle \neq |BH\rangle.$$

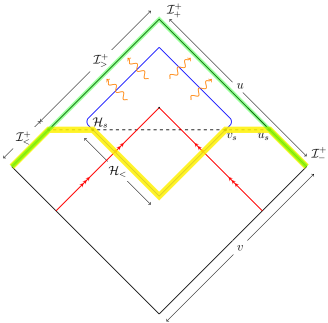
Thus $Q_\epsilon^{\mathcal{H}}$ is an non-trivial operator even if its exepctation valeu is zero.

Conservation law

$$Q_\epsilon^- = Q_\epsilon^+ + Q_\epsilon^{\mathcal{H}}.$$

This is in clear contradiction to classical ideas where there is only conservation of total electric charge.

Evaporating Black Holes



For evaporating black holes, an horizon forms and then disappears. Charges can be defined by integrals over the green surface, a Cauchy surface in the distant future on null infinity, Q_ϵ^{green}. Charges can also be defined by the Cauchy surface consisting of part of the horizon, part of future null infinity, and a spacelike segment joining the two, indicated in yellow Q_ϵ^{yellow}.

$$Q_\epsilon^- = Q_\epsilon^{green} = Q_\epsilon^{yellow}.$$

Hair Implants

Throw a shockwave into a black hole at a constant advanced time $v = v_0$.

$$J_v = \frac{Y_{lm}(z, \bar{z})}{r^2}\delta(z - z_0), \quad l > 0.$$

Suppose intially no photons.
Then as $r \to \infty$, $F_{vr} = Y_{lm}\theta(v - v_0) + \ldots$ This gives soft photons on $\mathcal{I}^+$ with polarization vector $\partial_z Y_{lm}$ and on $\mathcal{H}$ similar soft photons.
So this way you can change the soft photon content of the horizon.

Not all modes can be excited. There is a maximum value of l. The size of a particle is given by its Compton wavelngth $\frac{1}{m}$. But if m is increased, the size will shrink. Eventually at m_{Planck}, the size is l_{Planck} but this is described by a black hole. Beyonfd this, one gets bigger black holes.
The smallest area on the horizon to be excited has area $\sim l_{Planck}^2$. Thus the number of soft photon degrees of freedom is $\sim A$, the black hole area.

Supertranslations are gravitational analogs of the elctromagnetic gauge transformations. They form an infinite dimensional Abelian group contained in the BMS group.

At null infinity, the diffeomorphism

$$f\partial_u - \frac{1}{2}D^2 f\partial_r + \frac{1}{r}D^A f\partial_A$$

is a supertranslation.

The charge and the past endpoint of future null infinity is given by

$$Q_f^+ = \frac{1}{8\pi}\int_{\mathcal{I}_-^+} f m_B$$

where m_B is the Bondi mass apsect.

Asymptotically

$$ds^2 = -(1 - \frac{2m_b}{r})du^2 - 2dudr + \dots$$

define the Bondi mass apsect.

There are similar flux formulae (but more complicated) and an extension to the horizon.

Issues

- Super-rotations
- Is this too much hair?
- Is this enough hair
  - Species problem
  - Entropy Calculation
- How does all the information get recorded
  - Global Symmetries
- How does all the information get retrieved in the Hawking radiation
- Black Hole Complemntarity
  - No cloning theorem
  - The edge of spacetime
- The singularity at the end of the horizon

Conclusions

Black Holes have an infinite amount of soft hair.
A step towards the resolving the information paradox.

Action to be taken