

The black hole interior in AdS/CFT

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CERN and University of Groningen

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based on work with Suvrat Raju: [1211.6767](#), [1310.6334](#), [1310.6335](#)

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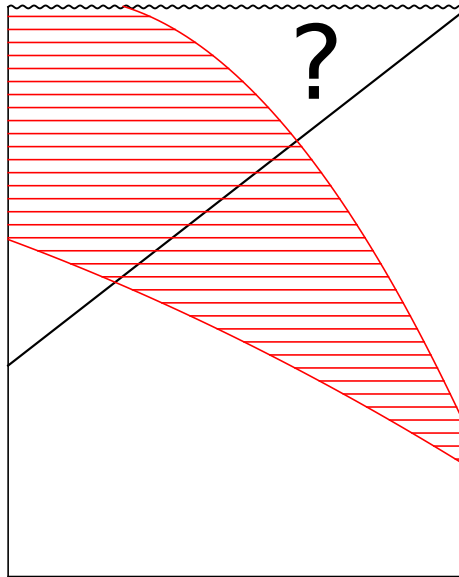
work in progress, with Souvik Banerjee (postdoc at University of Groningen)
Prashant Samantray (postdoc at ICTS Bangalore)
and S. Raju

First Part: I will give overview of our proposal

Second Part: **Suvrat Raju**, Wednesday at 16:00, will address Joe's
objections

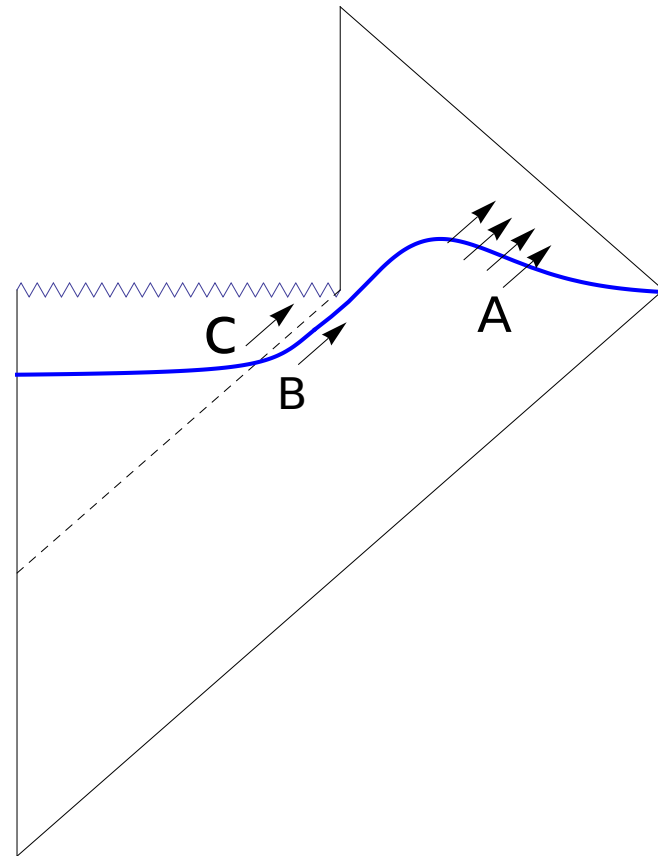
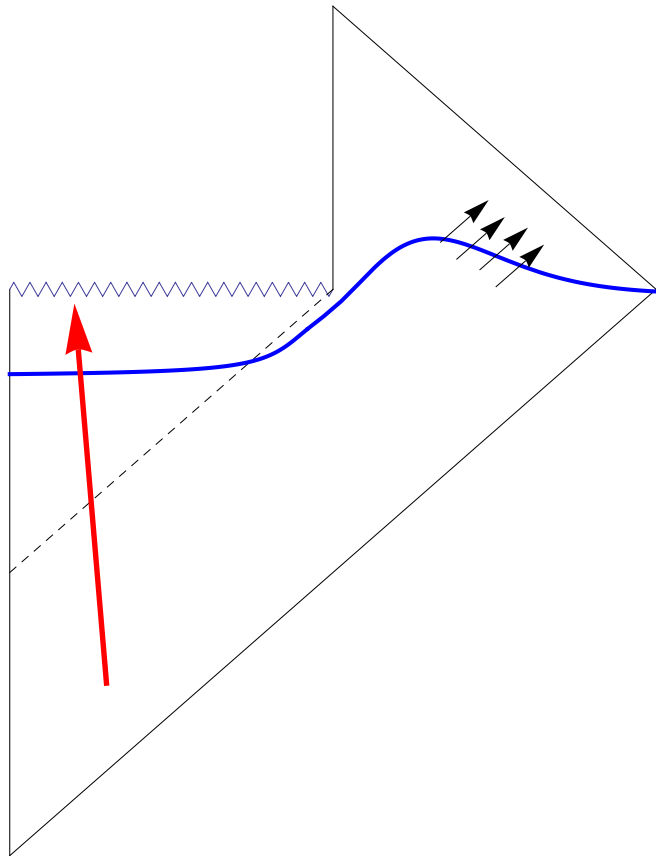
Black Hole interior in AdS/CFT

Does a big black hole in AdS have an interior and can the CFT describe it?



Smooth BH interior \Rightarrow harder to resolve the information paradox

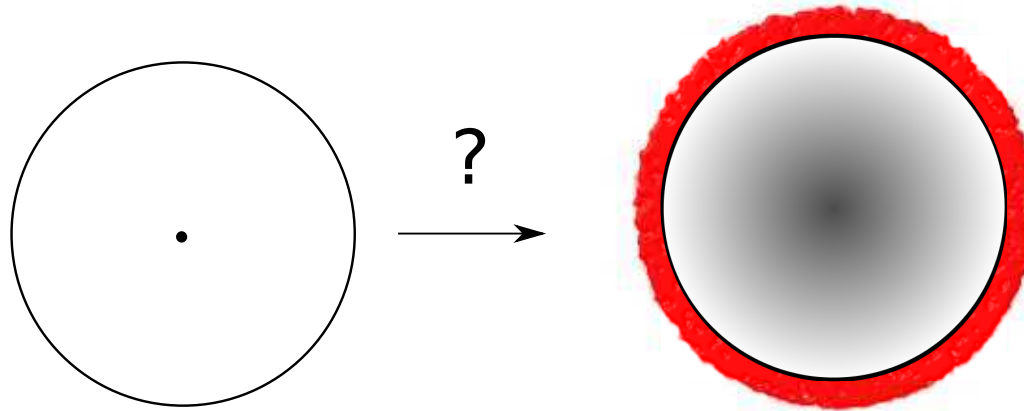
Black Hole information paradox



Quantum cloning on nice slices Strong subadditivity paradox [Mathur],
[Almheiri, Marolf, Polchinski, Sully (AMPS)]

Black Hole information paradox

Should we give up smooth interior? Firewall, fuzzball,...



Alternative: limitations of locality

In Quantum Gravity locality is emergent (large N , strong coupling) \Rightarrow it cannot be exact

Cloning/entanglement paradoxes rely on unnecessarily strong assumptions about locality

Resolution: Complementarity

The Hilbert space of Quantum Gravity does not factorize into interior \times exterior

['t Hooft, Susskind, Thorlacius, Uglum, Bousso, Nomura, Varela, Weinberg, Verlinde \times 2, Maldacena...]

BH interior is a scrambled copy of exterior

This would resolve cloning/subadditivity paradoxes

Questions:

1. Is there a precise mathematical realization of complementarity?
 2. Is complementarity consistent with locality in effective field theory?
-

Resolution: Complementarity

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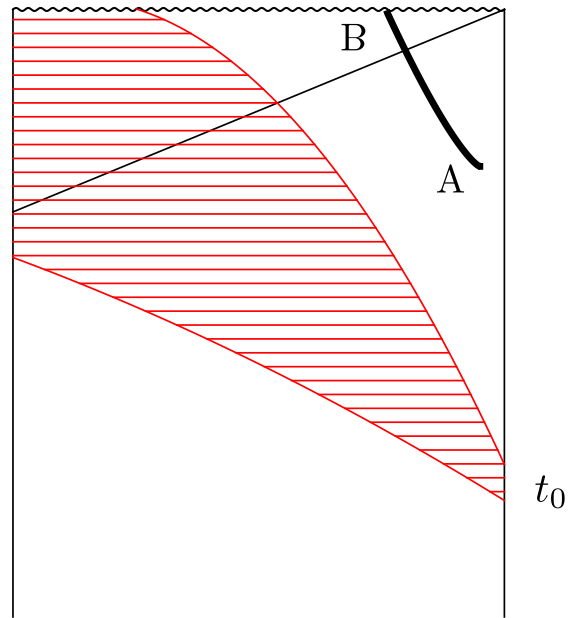
Questions:

1. Is there a precise mathematical realization of complementarity?
2. Is complementarity consistent with locality in effective field theory?

Our work:

1. **Progress towards a mathematical framework for complementarity**
 2. **Evidence that complementarity is consistent with locality in EFT**
-

Setup



Consider the $\mathcal{N} = 4$ SYM on $S^3 \times \text{time}$, at large N , large λ .
and typical pure state $|\Psi\rangle$ with energy of $\mathcal{O}(N^2)$.

What is experience of infalling observer? \Rightarrow Need local bulk observables

Reconstructing local observables in empty AdS

Large N factorization allows us to write local* observables in empty AdS as non-local observables in CFT (smeared operators)

$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int_{\omega > 0} d\omega d\vec{k} \left(\mathcal{O}_{\omega, \vec{k}} f_{\omega, \vec{k}}(t, \vec{x}, z) + \text{h.c.} \right)$$

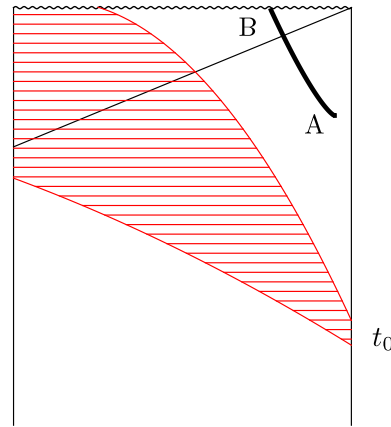
where ϕ_{CFT} obeys EOMs in AdS, and $[\phi_{\text{CFT}}(P_1), \phi_{\text{CFT}}(P_2)] = 0$, if points P_1, P_2 spacelike with respect to AdS metric

(based on earlier works: Banks, Douglas, Horowitz, Martinec, Bena, Balasubramanian, Giddings, Lawrence, Kraus, Trivedi, Susskind, Freivogel Hamilton, Kabat, Lifschytz, Lowe, Heemskerk, Marolf, Polchinski, Sully...)

* Locality is approximate:

1. (Plausibly) true in $1/N$ perturbation theory
 2. Unlikely that $[\phi_{\text{CFT}}(P_1), \phi_{\text{CFT}}(P_2)] = 0$ to e^{-N^2} accuracy
 3. Locality may break down for high-point functions (perhaps no bulk spacetime interpretation)
-

Black hole in AdS



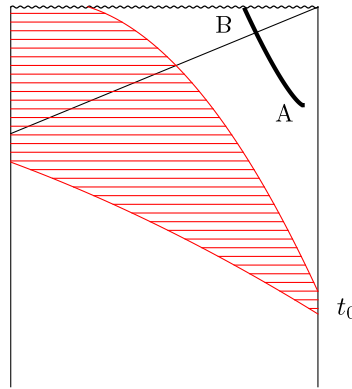
Consider typical QGP pure state $|\Psi\rangle$ (energy $O(N^2)$). Single trace correlators **still factorize** at large N

$$\langle \Psi | \mathcal{O}(x_1) \dots \mathcal{O}(x_n) | \Psi \rangle = \langle \Psi | \mathcal{O}(x_1) \mathcal{O}(x_2) | \Psi \rangle \dots \langle \Psi | \mathcal{O}(x_{n-1}) \mathcal{O}(x_n) | \Psi \rangle + \dots$$

The 2-point function in which they factorize is the thermal 2-point function, which is hard to compute, but obeys KMS condition

$$G_\beta(-\omega, k) = e^{-\beta\omega} G_\beta(\omega, k)$$

Black hole in AdS



Local bulk field outside horizon of AdS black hole*

$$\phi_{\text{CFT}}(t, \Omega, z) = \sum_m \int_0^\infty d\omega \mathcal{O}_{\omega, m} f_{\omega, m}^\beta(t, \Omega, z) + \text{h.c.}$$

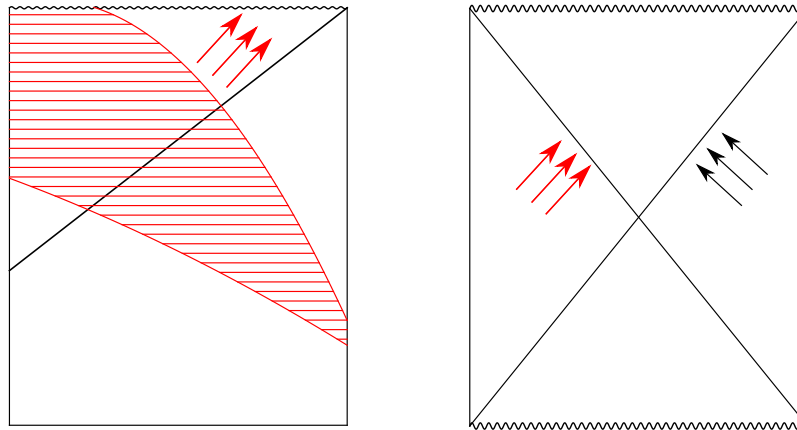
At large N (and late times) the correlators

$$\langle \Psi | \phi_{\text{CFT}}(t_1, \Omega_1, z_1) \dots \phi_{\text{CFT}}(t_n, \Omega_n, z_n) | \Psi \rangle$$

reproduce those of semiclassical QFT on the BH background (in AdS-Hartle-Hawking state).

* We have clarified confusions about the convergence of the sum/integral

Behind the horizon



Need new modes

For free infall we expect

$$\phi_{\text{CFT}}(t, \Omega, z) = \sum_m \int_0^\infty d\omega \left[\mathcal{O}_{\omega, m} e^{-i\omega t} Y_m(\Omega) g_{\omega, m}^{(1)}(z) + \text{h.c.} \right. \\ \left. + \boxed{\tilde{\mathcal{O}}_{\omega, m}} e^{-i\omega t} Y_m(\Omega) g_{\omega, m}^{(2)}(z) + \text{h.c.} \right]$$

where the modes $\tilde{\mathcal{O}}_{\omega, m}$ must satisfy certain conditions

Conditions for $\tilde{\mathcal{O}}_{\omega,m}$

The $\tilde{\mathcal{O}}_{\omega,m}$'s (*mirror* or *tilde* operators) must obey the following conditions, in order to have smooth interior:

1. For every \mathcal{O} there is a $\tilde{\mathcal{O}}$
2. The algebra of $\tilde{\mathcal{O}}$'s is isomorphic to that of the \mathcal{O} 's
3. The $\tilde{\mathcal{O}}$'s commute with the \mathcal{O} 's
4. The $\tilde{\mathcal{O}}$'s are “correctly entangled” with the \mathcal{O} 's

Equivalently:

Correlators of all these operators on $|\Psi\rangle$ must reproduce (at large N) those of the thermofield-double state

$$|TFD\rangle = \sum_i \frac{e^{-\beta E_i/2}}{\sqrt{Z}} |E_i, \tilde{E}_i\rangle$$

$$\langle \Psi | \mathcal{O}(t_1) \dots \tilde{\mathcal{O}}(t_k) \dots \mathcal{O}(t_n) | \Psi \rangle \approx \frac{1}{Z} \text{Tr} \left[\mathcal{O}(t_1) \dots \mathcal{O}(t_n) \mathcal{O}(t_k + i\frac{\beta}{2}) \dots \mathcal{O}(t_m + i\frac{\beta}{2}) \right]$$

MAIN QUESTION: does a single CFT contain operators $\tilde{\mathcal{O}}$ with the desired properties?

If so, then black hole has smooth interior, and interior is visible in the CFT.

Construction of the mirror operators

Exterior of AdS black hole \Rightarrow Described by “algebra of (products of) single trace operators \mathcal{O} ”

Why do we get a second **commuting** copy $\tilde{\mathcal{O}}$?



Construction of the mirror operators

Exterior of AdS black hole \Rightarrow Described by “algebra of (products of) single trace operators \mathcal{O} ”

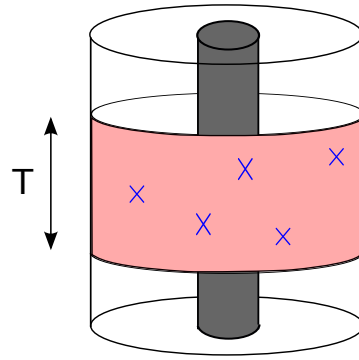
Why do we get a second **commuting** copy $\tilde{\mathcal{O}}$?

The doubling of the observables is a general phenomenon whenever we have:

- A large (chaotic) quantum system in a typical state $|\Psi\rangle$
- We are probing it with a small algebra \mathcal{A} of observables

Under these conditions, the small algebra \mathcal{A} is effectively “doubled”.

Construction of the mirror operators



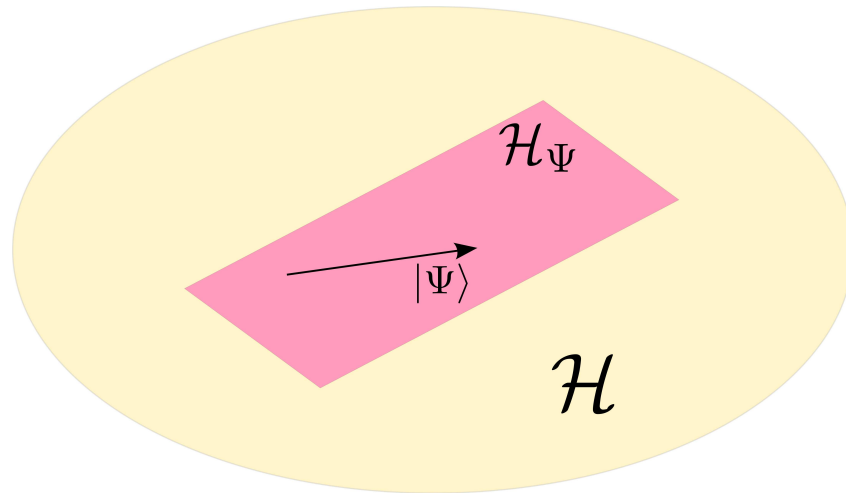
For us, $|\Psi\rangle =$ BH microstate (typical QGP state of $E \sim \mathcal{O}(N^2)$)

$\mathcal{A} =$ “algebra” of small (i.e. $\mathcal{O}(N^0)$) products of single trace operators

$$\mathcal{A} = \text{span of } \{ \mathcal{O}(t_1, \vec{x}_1), \mathcal{O}(t_1, \vec{x}_1) \mathcal{O}(t_2, \vec{x}_2), \dots \}$$

Here T is a long time scale and also need some UV regularization.

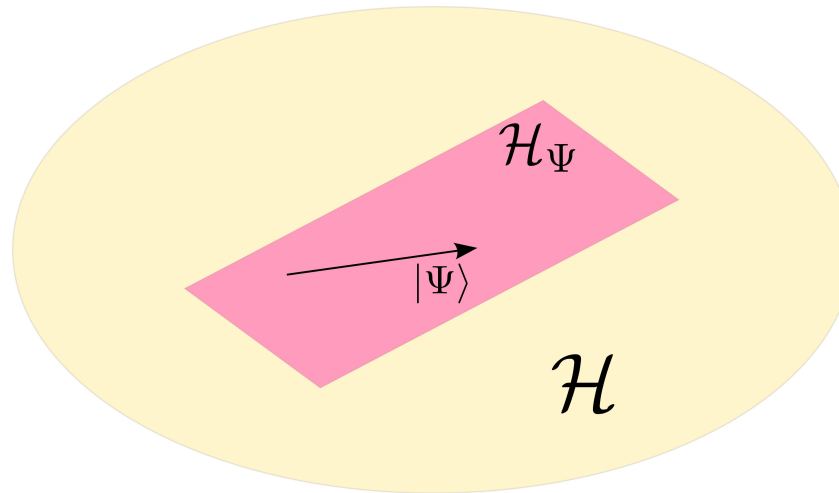
The Hilbert space \mathcal{H}_Ψ



For any given microstate $|\Psi\rangle$ consider the linear subspace \mathcal{H}_Ψ of the full Hilbert space \mathcal{H} of the CFT

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle = \{\text{span of } : \mathcal{O}(t_1, \vec{x}_1) \dots \mathcal{O}(t_n, \vec{x}_n) : |\Psi\rangle\}$$

The Hilbert space \mathcal{H}_Ψ



- \mathcal{H}_Ψ depends on $|\Psi\rangle$
 - $\mathcal{H}_\Psi \Rightarrow$ Contains states of higher and lower energies than $|\Psi\rangle$
 - Bulk EFT experiments around BH $|\Psi\rangle$ take place within \mathcal{H}_Ψ (bulk observer cannot easily see outside \mathcal{H}_Ψ)
-

Reducibility of representation of \mathcal{A}

The “doubling” follows from the important property:

$$A|\Psi\rangle \neq 0 \quad \text{if} \quad A \neq 0, \quad \forall A \in \mathcal{A}$$

(we cannot annihilate the QGP microstate by the action of a few single trace operators)

Physical interpretation of this property:

“The state $|\Psi\rangle$ *appears to be entangled when probed by the algebra \mathcal{A}* ”.

Example: two spins

Two spins, small algebra $\mathcal{A} \equiv$ operators acting on the first spin.

1. If no entanglement:

$$|\Psi\rangle = |\uparrow\uparrow\rangle$$

$$s_+^{(1)}|\Psi\rangle = 0 \quad \text{while} \quad s_+^{(2)} \neq 0$$

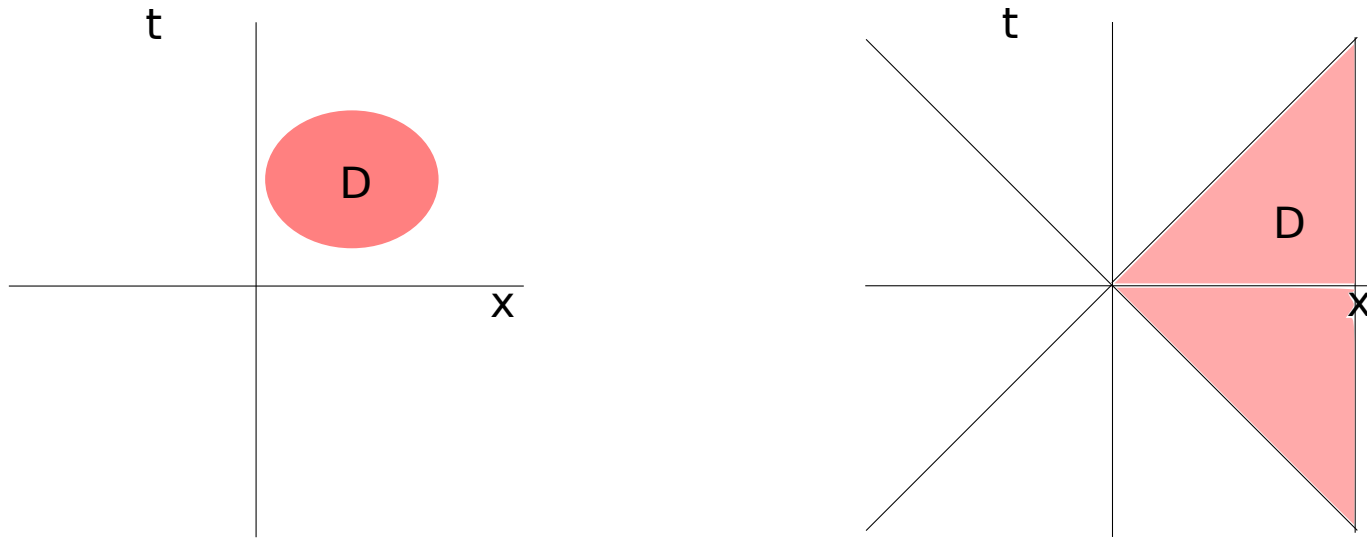
2. If state is entangled:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

can check that

$$A^{(1)}|\Psi\rangle \neq 0 \quad A^{(2)} \neq 0$$

Example: Relativistic QFT in ground state



Reeh-Schlieder theorem: Minkowski vacuum $|0\rangle_M$ cannot be annihilated by acting with local operators in D .

\Rightarrow

In $|0\rangle_M$ local operator algebras are entangled — (though, no proper factorization of Hilbert space due to UV divergences)

Why doubling?

Remember the important condition

$$A|\Psi\rangle \neq 0 \quad \text{for} \quad A \neq 0 \quad (1)$$

Suppose that

$$\dim \mathcal{A} = n$$

Then from (1) follows that

$$\dim \mathcal{H}_\Psi = \dim (\text{span} \mathcal{A} |\Psi\rangle) = n$$

However the algebra $\mathcal{L}(\mathcal{H}_\Psi)$ of *all* operators that can act on \mathcal{H}_Ψ has dimensionality

$$\dim \mathcal{L}(\mathcal{H}_\Psi) = n^2$$

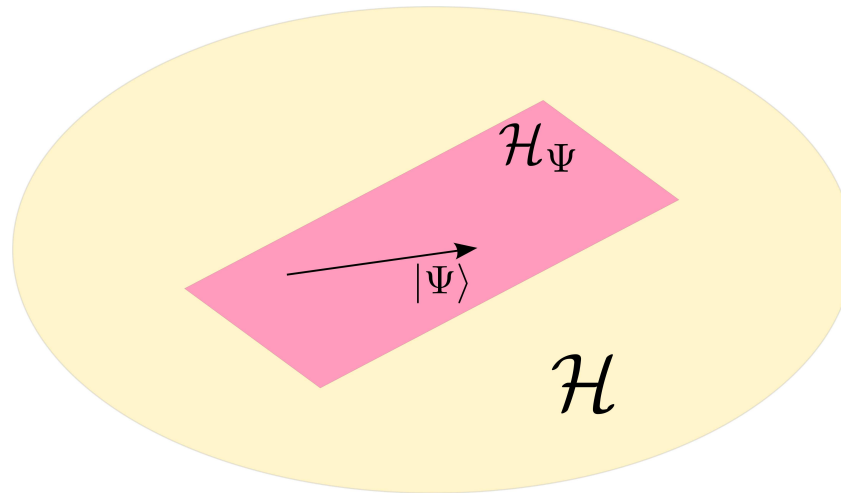
while the original algebra \mathcal{A} had only $\dim \mathcal{A} = n$.

This suggests that

$$\mathcal{L}(\mathcal{H}_\Psi) = \mathcal{A} \otimes \tilde{\mathcal{A}}$$

where $\tilde{\mathcal{A}}$ is a “second copy” of \mathcal{A} . We can choose basis so that $[A, \tilde{A}] = 0$

Summary of the problem



- $|\Psi\rangle =$ BH microstate (QGP microstate)
- $\mathcal{A} =$ “algebra” of small products of single trace operators
- Black Hole interior operators $\tilde{\mathcal{O}}$ must commute with $\mathcal{A} \Rightarrow$ They are elements of the “commutant” \mathcal{A}' of the algebra.

What is \mathcal{A}' for the algebra of single trace operators \mathcal{A} acting on a typical QGP state?

Mathematical aspects of the problem

Consider a von-Neumann algebra \mathcal{A} acting on a Hilbert space \mathcal{H} .

Question: what is the commutant \mathcal{A}' ?

In general, question is difficult. \mathcal{A}' could be trivial. However, if \exists a state $|\Psi\rangle$ in \mathcal{H} for which

- i) States $\mathcal{A}|\Psi\rangle$ span \mathcal{H}
- ii) $A|\Psi\rangle \neq 0$ for all $A \neq 0$

then

Theorem: (Tomita-Takesaki) The commutant \mathcal{A}' is isomorphic to \mathcal{A} (**doubling!**). There is a canonical isomorphism J acting on \mathcal{H} such that

$$\tilde{\mathcal{O}} = J\mathcal{O}J$$

Constructing the mirror operators

On the subspace \mathcal{H}_Ψ we define the *antilinear* map S by

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle$$

This is well defined *because* of the condition $A|\Psi\rangle \neq 0$ for $A \neq 0$.

We manifestly have

$$S|\Psi\rangle = |\Psi\rangle$$

and

$$S^2 = 1$$

For any operator $A \in \mathcal{A}$ acting on \mathcal{H}_Ψ we define a new operator acting on the same space by

$$\hat{A} = SAS$$

Constructing the mirror operators

The hatted operators commute with those in \mathcal{A} :

$$\hat{B}A|\Psi\rangle = SBSA|\Psi\rangle = SBA^\dagger|\Psi\rangle = (BA^\dagger)^\dagger|\Psi\rangle = AB^\dagger|\Psi\rangle$$

and also

$$A\hat{B}|\Psi\rangle = ASBS|\Psi\rangle = AB^\dagger|\Psi\rangle$$

hence

$$[A, \hat{B}]|\Psi\rangle = 0$$

The “hatted” operators $\hat{A} = SAS$ satisfy:

- Their algebra is isomorphic to \mathcal{A}
- They commute with \mathcal{A}

they are almost the mirror operators, but not quite (the mixed A - \hat{A} correlators are not “canonically” normalized)

Constructing the mirror operators

The mapping S is not an isometry. We define the “magnitude” of the mapping

$$\Delta = S^\dagger S$$

and then we can write

$$J = S\Delta^{-1/2}$$

where J is (anti)-unitary. Then the correct mirror operators are

$$\boxed{\tilde{O} = JOJ}$$

The operator Δ is a positive, hermitian operator and can be written as

$$\Delta = e^{-K}$$

where

$$K = \text{“modular Hamiltonian”}$$

For entangled bipartite system $A \times B$ this construction would give $K_A \sim \log(\rho_A)$ i.e. the usual modular Hamiltonian for A .

Constructing the mirror operators

In the large N gauge theory and using the KMS condition for correlators of single-trace operators we find that for equilibrium states

$$K = \beta(H_{CFT} - E_0)$$

To summarize, we have

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle$$

and

$$\Delta = e^{-\beta(H_{CFT} - E_0)}$$

We define the J by

$$J = S\Delta^{-1/2}$$

Finally we define the mirror operators by

$$\boxed{\tilde{O} = JOJ}$$

Constructing the mirror operators

Putting everything together we **define** the mirror operators by the following set of linear equations

$$\tilde{\mathcal{O}}_\omega |\Psi\rangle = e^{-\frac{\beta\omega}{2}} \mathcal{O}_\omega^\dagger |\Psi\rangle$$

and

$$\tilde{\mathcal{O}}_\omega \mathcal{O} \dots \mathcal{O} |\Psi\rangle = \mathcal{O} \dots \mathcal{O} \tilde{\mathcal{O}}_\omega |\Psi\rangle$$

These conditions are self-consistent because $\mathcal{A}|\Psi\rangle \neq 0$, which in turns relies on

1. The algebra \mathcal{A} is not too large
 2. The state $|\Psi\rangle$ is complicated (this definition would not work around the ground state of CFT)
-

Constructing the mirror operators

These “mirror operators” $\tilde{\mathcal{O}}$ obey the desired conditions mentioned several slides ago, i.e. at large N they lead to

$$\langle \Psi | \mathcal{O}(t_1) \dots \tilde{\mathcal{O}}(t_k) \dots \mathcal{O}(t_n) | \Psi \rangle \approx \frac{1}{Z} \text{Tr} [\mathcal{O}(t_1) \dots \mathcal{O}(t_n) \mathcal{O}(t_k + i\frac{\beta}{2}) \dots \mathcal{O}(t_m + i\frac{\beta}{2})]$$



Reconstructing the interior

Using the \mathcal{O}_ω 's and $\tilde{\mathcal{O}}_\omega$'s we can reconstruct the black hole interior by operators of the form

$$\begin{aligned} \phi_{\text{CFT}}(t, \Omega, z) = & \sum_m \int_0^\infty d\omega \left[\mathcal{O}_{\omega,m} e^{-i\omega t} Y_m(\Omega) g_{\omega,m}^{(1)}(z) + \text{h.c.} \right. \\ & \left. + \tilde{\mathcal{O}}_{\omega,m} e^{-i\omega t} Y_m(\Omega) g_{\omega,m}^{(2)}(z) + \text{h.c.} \right] \end{aligned}$$

Low point functions of these operators reproduce those of effective field theory in the interior of the black hole

⇒

∃ **Smooth interior**

Nothing dramatic when crossing the horizon

Realization of Complementarity

The operators $\tilde{\mathcal{O}}$ seem to commute with the \mathcal{O} 's

This is only approximate: the commutator $[\mathcal{O}, \tilde{\mathcal{O}}] = 0$ only inside low-point functions (by construction)

If we consider N^2 -point functions, then we find that the construction cannot be performed since we will violate

$$A|\Psi\rangle \neq 0, \quad \text{for} \quad A \neq 0$$

or equivalently, in spirit, we will find that

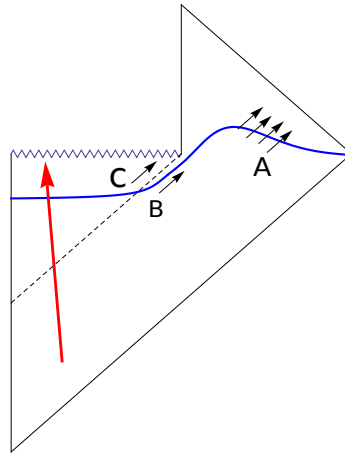
$$[\mathcal{O}, \tilde{\mathcal{O}}] \neq 0$$

inside complicated correlators.

Relatedly, we can express the $\tilde{\mathcal{O}}$'s as very complicated combination of \mathcal{O} 's.

Evaporating black hole

Black Hole interior is not independent Hilbert space, but highly scrambled version of exterior

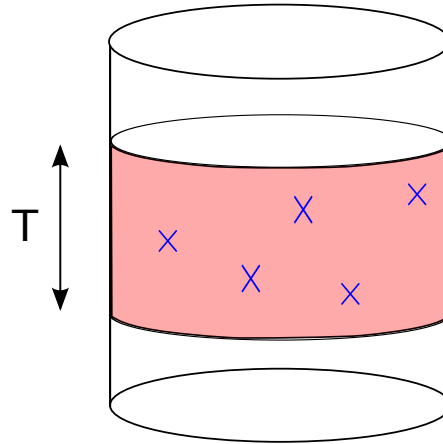


- Exterior of black hole \Rightarrow operators $\phi(x)$
- Interior of black hole \Rightarrow operators $\tilde{\phi}(y)$
- In low-point correlators $\phi, \tilde{\phi}$ seem to be independent and $[\phi, \tilde{\phi}] \approx 0$
- If we act with too many (order S_{BH}) of ϕ 's we can “reconstruct” the $\tilde{\phi}$'s

Complementarity can be realized consistently with locality in effective field theory— Suvrat's talk

Large N gauge theory

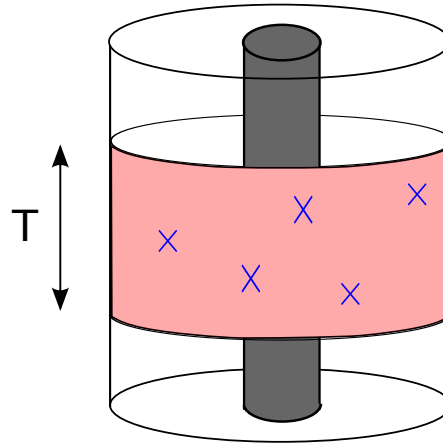
In large N gauge theory, \mathcal{A} = “algebra of products of few single trace operators”, CFT in state $|\Psi\rangle$



$|\Psi\rangle$ is “simple” \Rightarrow Representation of \mathcal{A} is **irreducible**, trivial commutant \mathcal{A}'
(no independent interior)

Large N gauge theory

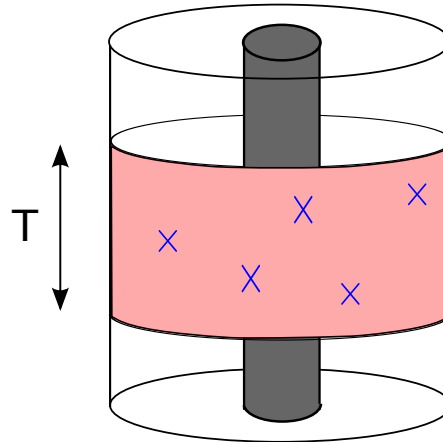
In large N gauge theory, \mathcal{A} = “algebra of products of few single trace operators”, CFT in state $|\Psi\rangle$



$|\Psi\rangle$ in deconfined phase \Rightarrow Representation of \mathcal{A} is **reducible**, non-trivial commutant \mathcal{A}' , isomorphic to $\mathcal{A} \Rightarrow \exists$ Black hole interior

Large N gauge theory

In large N gauge theory, \mathcal{A} = “algebra of products of few single trace operators”, CFT in state $|\Psi\rangle$



$|\Psi\rangle$ in deconfined phase \Rightarrow Representation of \mathcal{A} is **reducible**, non-trivial commutant \mathcal{A}' , isomorphic to $\mathcal{A} \Rightarrow \exists$ Black hole interior

But: If we enlarge \mathcal{A} too much (by allowing $O(N^2)$ -point functions), representation becomes again irreducible, and then there is no commutant. What used to be the commutant (BH interior) for the original smaller \mathcal{A} , can be expressed in terms of enlarged \mathcal{A} (complementarity)

State dependence

- Our operators were defined to act on \mathcal{H}_Ψ (they are *sparse* operators).
 - For given BH microstate and for an EFT observer placed near the BH $|\Psi\rangle$, this part of the Hilbert space is the only relevant (for simple experiments)
 - For different microstate $|\Psi'\rangle$ the “**same physical observables**” will be acting on a different part of the Hilbert space $\mathcal{H}_{\Psi'}$ and (a priori) will be different **linear** operators
 - Is it possible to define the $\tilde{\mathcal{O}}_\omega$ globally on the Hilbert space?
-

State dependence

Why it seems unlikely that \tilde{O} can be defined to act on all microstates:

- There are certain arguments against the existence of globally defined \tilde{O} operators [Bousso, Almheiri, Marolf, Polchinski, Stanford, Sully]
- State-dependence could explain why we automatically get “correct entanglement” for typical states
- It may be that in Quantum Gravity all local observables are state-dependent

More about state dependence in Suvrat's talk tomorrow

Some further questions

- Identification of equilibrium states [Bousso, Harlow, Maldacena, Marolf, Polchinski, Raamsdonk, Verlinde $\times 2$,...]]
 - $1/N$ corrections, HH state? [Harlow]
 - 2-sided black hole, relation to ER/EPR [Maldacena, Susskind, Shenker, Stanford]
 - Interaction of Hawking radiation with environment [Bousso, Harlow]
 - Can we understand $\tilde{\mathcal{O}}$ operators at small 't Hooft coupling? (hard to study thermalization at weak coupling) [Festuccia, Liu]
 - ...
-

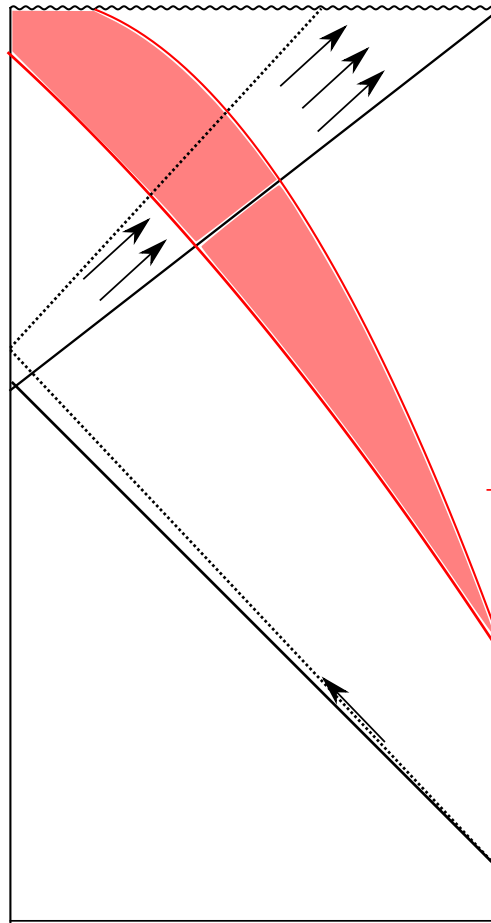
Summary of our understanding

1. Big AdS black holes have smooth interior, CFT can describe it
2. An infalling observer does not see any deviations from what is predicted by semiclassical GR (cannot detect firewall/fuzzball)
3. By extrapolation, we conjecture the same for flat space black holes
4. Information paradox resolved by exponentially small corrections to EFT
5. Entanglement/cloning related paradoxes resolved by complementarity
6. Progress towards a mathematically precise realization of complementarity
7. Evidence that complementarity and locality in EFT are compatible

Important point to settle: state dependence and observables in Quantum Gravity

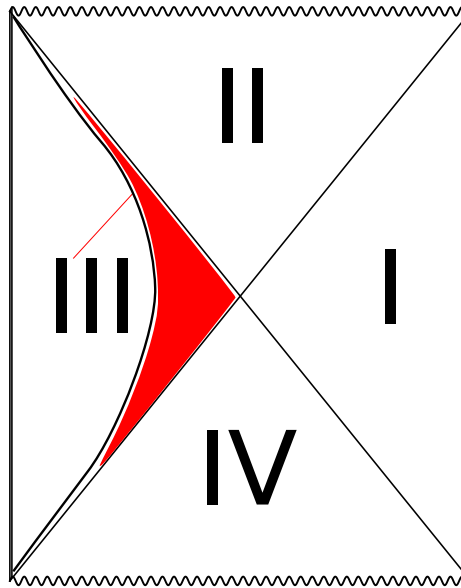
THANK YOU

Behind the horizon



Using bulk EFT evolution to find the $\tilde{\mathcal{O}}$? \Rightarrow Trans-planckian problem...(?)

On reconstructing “Region III”?



- $[H_{\text{CFT}}, \tilde{O}] \neq 0$
 - Blueshift issues?
 - Notice that $\langle \Psi | O_\omega^\dagger O_\omega | \Psi \rangle \sim \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}}$. Our condition $A|\Psi\rangle \neq 0$ becomes **exponentially** close to being violated as we increase $\omega \Rightarrow$ hard to reconstruct “UV” of region III [Maldacena]
-