NON-PERTURBATIVE DYSON-SCHWINGER EQUATIONS AND NOVEL SYMMETRIES OF QUANTUM FIELD THEORY

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DYSON-SCHWINGER EQUATIONS

INVARIANCE OF (PATH) INTEGRAL

$$\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle = \frac{1}{Z} \int_\Gamma D\Phi \, e^{-\frac{1}{\hbar} S[\Phi]} \, \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n)$$

UNDER "SMALL" DEFORMATIONS
OF THE INTEGRATION CONTOUR

$$\Phi \longrightarrow \Phi + \delta\Phi$$

DYSON-SCHWINGER EQUATIONS

QUANTUM EQUATIONS OF MOTION

$$\langle \mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)\delta S[\Phi]\rangle =$$

$$\hbar\sum_{i=1}^{n}\langle \mathcal{O}_1(x_1)\dots\mathcal{O}_{i-1}(x_{i-1})\delta\mathcal{O}_i(x_i)\mathcal{O}_{i+1}(x_{i+1})\dots\mathcal{O}_n(x_n)\rangle$$

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DYSON-SCHWINGER EQUATIONS

WITH SOME LUCK

$$=$$

GOOD CHOICE OF (POSSIBLY NON-LOCAL) OBSERVABLES

$$\mathcal{O}_i(x)$$

AND IN SOME LIMIT (CLASSICAL, PLANAR, ...)

THE DS EQUATIONS FORM A CLOSED SYSTEM

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FOR EXAMPLE

$$\hbar \longrightarrow 0$$

CLASSICAL LIMIT

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\delta S[\Phi]\rangle = \hbar\,(\ldots) \to 0$$

$$\Leftrightarrow \;\; \delta S[\Phi] = 0$$

GAUGE THEORY

$$\Phi \longrightarrow A = A_\mu dx^\mu \in \mathrm{Lie}\, U(N)$$

$$\frac{1}{\hbar} S[\Phi] \longrightarrow S_{YM}[A] = -\frac{1}{4g^2} \int_{\mathbf{R}^4} \mathrm{tr} F_A \wedge \star F_A$$

$$\mathcal{O}_i(x_i) \longrightarrow W_R(\gamma) = \mathrm{tr}_R\, P\exp \oint_\gamma A$$

$$\mathcal{W}(\gamma) = \frac{1}{N} \langle W_\Box(\gamma) \rangle$$

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GAUGE THEORY: PLANAR LIMIT

$$N \longrightarrow \infty, \qquad g^2 \to 0,$$

$$\textit{FINITE} \qquad \lambda = g^2 N$$

$$\Delta_\gamma \mathcal{W}(\gamma) = \frac{g^2}{N} \langle W_\Box(\gamma) \delta S_{YM}[A] \rangle =$$

$$= \lambda \delta_{\gamma=\gamma_1 \star \gamma_2} \mathcal{W}(\gamma_1) \mathcal{W}(\gamma_2) + \frac{1}{N^2} \textit{ correctons}$$

MAKEENKO-MIGDAL LOOP EQUATIONS

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GAUGE THEORY: MATRIX MODEL

$$\Phi \in \mathrm{Lie}\, U(N)$$

$$\frac{1}{\hbar} S[\Phi] = \frac{1}{\hbar} \mathrm{tr} V(\Phi)$$

$$V(X) = v_p X^p + v_{p-1} X^{p-1} + \ldots + v_1 X + v_0$$

$$\mathcal{O}(x) = \frac{1}{N} \mathrm{tr}_{\square} \left(\frac{1}{x - \Phi} \right)$$

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MATRIX MODEL

PLANAR LIMIT: $\lambda = \hbar N$ FIXED

$$\hbar \to 0, \;\; N \to \infty$$

DS EQUATIONS $\Longrightarrow$ LOOP EQUATIONS

$$y(x)^2 = V'(x)^2 + g_{p-2}(x)$$

$$y(x) = \langle \mathcal{O}(x) \rangle + V'(x)$$

$g_{p-2}(x) =$ DEGREE $p-2$ POLYNOMIAL IN x

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QFT PATH INTEGRAL INVOLVES SUMMATION

OVER TOPOLOGICAL SECTORS

FOR EXAMPLE, IN GAUGE THEORY

$$Z = \sum_{n \in \mathbb{Z}} e^{in\vartheta} \int_{\mathcal{A}_n} \left[\frac{DA}{Vol(\mathcal{G}_n)} \right] e^{-S_{YM}[A]}$$

$$-\frac{1}{8\pi^2} \int \mathrm{tr} F_A \wedge F_A = n, \qquad A \in \mathcal{A}_n$$

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NON-PERTURBATIVE DS EQUATIONS

IDENTITIES DERIVED BY

LARGE “DEFORMATIONS” OF THE PATH INTEGRAL CONTOUR

$$A \in \mathcal{A}_n \longrightarrow A + \delta A \in \mathcal{A}_{n+1}$$

GRAFTING A POINT-LIKE INSTANTON

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COMPATIBILITY OF PERTURBATIVE

expansion in $\hbar$, g^2, . . .

AND NON-PERTURBATIVE CONTRIBUTIONS

expansion in $e^{-\frac{1}{\hbar}}$, $e^{-\frac{1}{g^2}}$, . . .

Resurgence, trans-series, . . . A.Voros, J.Zinn-Justin, . . .

Exact β-functions in SYM, Novikov-Shifman-Vainshtein, Zakharov

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TESTING GROUNDS

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SUPERSYMMETRIC

GAUGE THEORIES

SIGMA MODELS

♢

$\mathcal{N} = 2$ **SUPERSYMMETRIC**

FOUR DIMENSIONAL GAUGE THEORIES

TWO DIMENSIONAL SIGMA MODELS

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$\mathcal{N} = 2$ SUPERSYMMETRIC GAUGE THEORIES

VECTOR MULTIPLETS

$$\mathbf{A} = \psi \quad \begin{matrix} \Phi & & \textit{complex adjoint scalar} \\ & \lambda & \textit{two Weyl adjoint fermions} \\ A & & \textit{gauge field} \end{matrix}$$

$$\text{adjoint} = Lie(G)$$

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$\mathcal{N} = 2$ SUPERSYMMETRIC GAUGE THEORIES

HYPERMULTIPLETS

in representation $R \in Rep(G)$

$$\mathbf{M} = M \quad \begin{matrix} \psi \\ \\ \tilde{\psi}^{\dagger} \end{matrix} \quad \tilde{M}$$

two complex scalars, in R and R^*
two Weyl fermions, in R and R^*

◊

CLASS OF SPECIFIC GAUGE THEORIES

$\mathcal{N} = 2$ SUPERSYMMETRIC GAUGE THEORIES

QUIVER THEORIES

$$G = U(N_1) \times \ldots \times U(N_r)$$

$$R = \bigoplus_{<i,j>} (N_i, \bar{N}_j) \bigoplus_i (N_i, \bar{M}_i)$$

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$\mathcal{N} = 2$ SUPERSYMMETRIC QUIVER THEORIES

MATTER IN FUNDAMENTAL

$$R \ni (N_i, \bar{M}_i), \qquad M_i = \textbf{multiplicity space}$$

MATTER IN BI-FUNDAMENTAL

$$R \ni (N_i, \bar{N}_j), \qquad i \neq j$$

MATTER IN ADJOINT

$$R \ni (N_i, \bar{N}_i)$$

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$\mathcal{N} = 2$ QUIVER THEORIES

DATA

$\gamma =$ oriented graph, $\quad s, t : \mathrm{Edge}_\gamma \to \mathrm{Vert}_\gamma$

$$G = \times_{i \in \mathrm{Vert}_\gamma} U(N_i)$$

$$R = \bigoplus_{e \in \mathrm{Edge}_\gamma} Hom(N_{s(e)}, N_{t(e)}) \bigoplus_{i \in \mathrm{Vert}_\gamma} Hom(M_i, N_i)$$

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$\mathcal{N} = 2$ QUIVER THEORIES

PARAMETERS

MASSES

$$\mathfrak{M}_i \in End(M_i), \qquad i \in \mathrm{Vert}_\gamma$$

$$m_e \in \mathbb{C}, \qquad e \in \mathrm{Edge}_\gamma$$

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$\mathcal{N} = 2$ QUIVER THEORIES

PARAMETERS

GAUGE COUPLINGS

$$\tau_i = \frac{\vartheta_i}{2\pi} + \frac{4\pi \mathrm{i}}{g_i^2}$$

AMPLITUDE OF INSTANTON IN THE $U(N_i)$

$$\mathfrak{q}_i = \exp 2\pi\mathrm{i}\tau_i, \qquad i \in \mathrm{Vert}_\gamma$$

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$\mathcal{N} = 2$ QUIVER THEORIES

$\mathcal{N} = 2$ SUPERCONFORMAL FIXED POINT

IN THE UV

iff $\gamma = A, D, E$ *or* $\widehat{A}, \widehat{D}, \widehat{E}$

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$\mathcal{N} = 2$ ASYMPTOTICALLY CONFORMAL QUIVER THEORIES

$$\gamma = A, D, E \qquad or \qquad \widehat{A}, \widehat{D}, \widehat{E}$$

QUIVER GROUP G_γ

SIMPLE LIE (KAC-MOODY) GROUP

! NOT AN APPARENT SYMMETRY OF THE THEORY !

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WE SHALL NOW SEE THAT

$$G_\gamma$$

MORE PRECISELY ITS QUANTUM DEFORMATION

$$Y(\mathbf{g}_\gamma), U_q(\widehat{\mathbf{g}}_\gamma), \ldots$$

IS PRESENT AS A SYMMETRY PRINCIPLE
ORGANIZING THE DYSON-SCHWINGER EQUATIONS

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$\mathcal{N} = 2$ QUIVER THEORIES

SUBJECT TO DEFORMATIONS:

FOR BETTER CONTROL

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$\mathcal{N} = 2$ QUIVER THEORIES

Ω-DEFORMATION:

$$\Phi \longrightarrow \Phi + v^{\mu} \nabla_{\mu}$$

$$\nabla_{\mu} = \partial_{\mu} + A_{\mu}$$

$$v^{\mu} \partial_{m} = \epsilon_{1} \left(x^{2} \partial_{x^{1}} - x^{1} \partial_{x^{2}} \right) + \epsilon_{2} \left(x^{4} \partial_{x^{3}} - x^{3} \partial_{x^{4}} \right)$$

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$\mathcal{N} = 2$ QUIVER THEORIES

Ω-DEFORMATION:

Generator $\mathcal{R}_3$ of the $SU(2)$ R-symmetry group is shifted

$$\mathcal{R}_3 \longrightarrow \mathcal{R}_3 + \mathcal{J}_3^R$$

where $\mathcal{J}_3^R$ is the generator
of the $SU(2)_R \subset$
rotation group $Spin(4) = SU(2)_L \times SU(2)_R$

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$\mathcal{N} = 2$ QUIVER THEORIES

NONCOMMUTATIVE DEFORMATION

$$\mathbb{R}^4 \longrightarrow \mathbb{R}^4_\theta$$

$$x^\mu \longrightarrow \widehat{x}^\mu$$

$$[\widehat{x}^\mu, \widehat{x}^\nu] = -\mathrm{i}\theta^{\mu\nu}$$

$\widehat{x}^\mu$ ARE OPERATORS IN $\mathcal{H}$

$\mathcal{H} =$ REPRESENTATION OF $\mathbb{R}^4_\theta$

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$\mathcal{N} = 2$ THEORIES

SUBJECT TO Ω, θ-DEFORMATIONS

CAN BE MAPPED TO “MATRIX MODELS”

θ-N.Seiberg, 1999; Ω-NN, A.Okounkov, 2003

$$\widehat{\mathbf{X}}^{\mu} = \widehat{x}^{\mu} + \mathrm{i}\theta^{\mu\nu} A_{\nu}(\widehat{x}), \quad (F_A)_{\mu'\nu'}\theta^{\mu\mu'}\theta^{\nu\nu'} = [\widehat{\mathbf{X}}^{\mu}, \widehat{\mathbf{X}}^{\nu}] + \mathrm{i}\theta^{\mu\nu}$$

$$\Phi(x) \longrightarrow \widehat{\mathbf{\Phi}} = \Phi(\widehat{x}) + \frac{1}{2}\tilde{\Omega}_{\mu\nu}\widehat{\mathbf{X}}^{\mu}\widehat{\mathbf{X}}^{\nu}$$

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$\mathcal{N} = 2$ THEORIES

SUBJECT TO Ω, θ-DEFORMATIONS

CAN BE MAPPED TO "MATRIX MODELS"

θ-N.Seiberg, 1999; Ω-NN, A.Okounkov, 2003

$$S \longrightarrow \frac{1}{4g^2} \sum_{\mu<\nu} \mathrm{Tr}_{\mathcal{H}} \left([\widehat{\mathbf{X}}^{\mu}, \widehat{\mathbf{X}}^{\nu}] + \mathrm{i}\theta^{\mu\nu} \right)^2 +$$

$$\sum_{\mu} \mathrm{Tr}_{\mathcal{H}} \left([\widehat{\mathbf{X}}^{\mu}, \widehat{\mathbf{\Phi}}] + \Omega^{\mu}_{\nu} \widehat{\mathbf{X}}^{\nu} \right) \left([\widehat{\mathbf{X}}^{\mu}, \widehat{\mathbf{\Phi}}^{\dagger}] + \bar{\Omega}^{\mu}_{\nu} \widehat{\mathbf{X}}^{\nu} \right) +$$

$$\mathrm{Tr}_{\mathcal{H}} [\widehat{\mathbf{\Phi}}, \widehat{\mathbf{\Phi}}^{\dagger}]^2 + \textit{fermions}$$

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OBSERVABLES FOR DS EQUATIONS

OBSERVABLE $Y(x)$

IN FOUR DIMENSIONAL $U(N)$ GAUGE THEORY

$$\mathbf{Y(x)} \sim \mathbf{det}_{\mathbb{C}^N}(x - \Phi) \sim \prod_{\alpha=1}^{N}(x - a_\alpha)$$

NAIVELY

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OBSERVABLES FOR DS EQUATIONS

$\mathbf{Y}(\mathbf{x})$ IN FOUR DIMENSIONS

MORE PRECISELY

$$\mathbf{Y}(\mathbf{x}) = \frac{\mathbf{Det}_{\mathcal{H}}(\mathbf{x} - \widehat{\mathbf{\Phi}} - \epsilon_1)\mathbf{Det}_{\mathcal{H}}(\mathbf{x} - \widehat{\mathbf{\Phi}} - \epsilon_2)}{\mathbf{Det}_{\mathcal{H}}(\mathbf{x} - \widehat{\mathbf{\Phi}})\mathbf{Det}_{\mathcal{H}}(\mathbf{x} - \widehat{\mathbf{\Phi}} - \epsilon_1 - \epsilon_2)}$$

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$$\mathbf{Y}(\mathbf{x}) = \frac{\mathbf{Det}_{\mathcal{H}}(\mathbf{x} - \widehat{\mathbf{\Phi}} - \epsilon_1)\mathbf{Det}_{\mathcal{H}}(\mathbf{x} - \widehat{\mathbf{\Phi}} - \epsilon_2)}{\mathbf{Det}_{\mathcal{H}}(\mathbf{x} - \widehat{\mathbf{\Phi}})\mathbf{Det}_{\mathcal{H}}(\mathbf{x} - \widehat{\mathbf{\Phi}} - \epsilon_1 - \epsilon_2)}$$

$$\widehat{\mathbf{\Phi}} \in \mathrm{End}(\mathcal{H})$$

RATIONAL FUNCTION OF DEGREE N

UNLIKE THE NAIVE $\det_{\mathbb{C}^N}(x - \Phi)$ IT HAS POLES

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FOR QUIVER GAUGE THEORY

$$G = U(N_1) \times \ldots \times U(N_r)$$

$$G = \times_{i \in \mathrm{Vert}_\gamma} U(N_i)$$

$$\mathcal{H} = \bigoplus_{i \in \mathrm{Vert}_\gamma} \mathcal{H}_i, \qquad \widehat{\mathbf{\Phi}} = \bigoplus_{i \in \mathrm{Vert}_\gamma} \widehat{\mathbf{\Phi}}_i$$

$$\mathbf{Y}(\mathbf{x}) \longrightarrow \left(\mathbf{Y}_\mathbf{i}(\mathbf{x})\right)_{i \in \mathrm{Vert}_\gamma}$$

MAIN CLAIM

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MAIN CLAIM

THERE EXIST

LAURENT POLYNOMIALS (SERIES FOR AFFINE γ)

$$\mathcal{X}_i(x) = Y_i(x + \epsilon_1 + \epsilon_2) + \ldots$$

in $Y_j(x + \text{linear combinations of } m_e, \epsilon_1, \epsilon_2)$'s, such that

$$\langle\ \mathcal{X}_i(x)\ \rangle = \text{POLYNOMIAL IN } x$$

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MAIN CLAIM

$$\mathcal{X}_i(x) = Y_i(x + \epsilon_1 + \epsilon_2) + \ldots$$

COEFFICIENTS = PRODUCTS OF

$$\mathfrak{q}_j, P_j(x + \text{linear combinations of } m_e, \epsilon_1, \epsilon_2), \qquad j \in \mathrm{Vert}_\gamma$$

$$P_j(x) = \mathbf{det}_{M_j}(x - \mathfrak{M}_j)$$

ENCODE FUNDAMENTAL MASSES

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WE CALL $\mathcal{X}_i(x)$

THE FUNDAMENTAL qq-CHARACTERS

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$\mathcal{X}_i(x)$ BECOME

THE FUNDAMENTAL CHARACTERS OF G_γ

IN THE LIMIT $\epsilon_1, \epsilon_2 \to 0$

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$$\mathcal{X}_i(x) = \mathrm{g}_\infty(x)^{-\lambda_i} \mathrm{Tr}_{V_i} \mathrm{g}(x)$$

$$\mathrm{g}(x) = \prod_{j \in \mathrm{Vert}_\gamma} Y_j(x)^{\alpha_j^\vee} \left(\mathfrak{q}_j P_j(x) \right)^{-\lambda_j^\vee} \in \mathbb{C} G_\gamma$$

$\epsilon_1, \epsilon_2 \to 0$ NN, V.Pestun, 2012

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THE DS EQUATIONS IN THE LIMIT $\epsilon_1, \epsilon_2 \to 0$

BECOME ALGEBRAIC EQUATIONS

$$\mathcal{X}_i(x) = \mathrm{g}_{\infty}(x)^{-\lambda_i} \mathrm{Tr}_{V_i} \mathrm{g}(x) = \text{POLYNOMIAL IN } x$$

SEIBERG-WITTEN GEOMETRY

NN, V.Pestun, 2012

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IN THE LIMIT $\epsilon_2 \to 0$, $\epsilon_1 = \hbar$ FINITE

$\mathcal{X}_i(x)$ BECOME

NN, V.Pestun, S.Shatashvili, 2013

THE FUNDAMENTAL q-CHARACTERS OF $Y(\mathbf{g}_\gamma)$

H.Knight

FIVE DIMENSIONAL THEORY ON $\mathbb{S}^1 \times \mathbb{R}^4$

PRODUCES q-CHARACTERS OF $U_q(\widehat{\mathbf{g}}_\gamma)$

E.Frenkel, N.Reshetikhin

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GENERAL ϵ_1, ϵ_2 CASE

USE NAKAJIMA'S QUIVER VARIETIES

$$\mathfrak{M}_\gamma(\mathbf{w}, \mathbf{v})$$

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NAKAJIMA VARIETY

MATHEMATICALLY

$$\mathfrak{M}_\gamma(\mathbf{w},\mathbf{v}) = T^*\left(\bigoplus_{e\in\mathrm{Edge}_\gamma} Hom(V_{s(e)},V_{t(e)})\bigoplus_{i\in\mathrm{Vert}_\gamma} Hom(V_i,W_i)\right) // \times_{i\in\mathrm{Vert}_\gamma} GL(V_i)$$

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NAKAJIMA VARIETY

PHYSICALLY

$$\mathfrak{M}_\gamma(\mathbf{w}, \mathbf{v}) =$$

HIGGS BRANCH OF THE $\mathcal{N} = 4$
THREE DIMENSIONAL QUIVER GAUGE THEORY
WITH GAUGE GROUP

$$G_v = \times_{i \in \mathrm{Vert}_\gamma} U(V_i)$$

w_i FUNDAMENTALS FOR $U(V_i)$
BI-FUNDAMENTALS IN $(V_{s(e)}, V^*_{t(e)})$

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FORMULA FOR qq-CHARACTERS

$$\mathcal{X}_{\mathbf{w}}(x) = \sum_{\mathbf{v}} \prod_{i \in \mathrm{Vert}_{\gamma}} \mathfrak{q}_i^{v_i} \int_{\mathfrak{M}_{\gamma}(\mathbf{w},\mathbf{v})} e_{-\varepsilon_2}(T\mathfrak{M}_{\zeta}(\mathbf{v},\mathbf{w})) I[\mathscr{F}]$$

WHERE

$$I[\mathscr{F}] = \prod_{i \in \mathrm{Vert}_{\gamma}} \prod_{\kappa} P_i(x + \nu_{i,\kappa}) \frac{\prod_{\kappa_+} Y_i(x + \xi^{+}_{i,\kappa_+})}{\prod_{\kappa_-} Y_i(x + \xi^{-}_{i,\kappa_-})}$$

$$Ch(V_i) = \sum_{\kappa} e^{\nu_{i,\kappa}}, \ Ch(C_i) = \sum_{\kappa_+} e^{\xi^{+}_{i,\kappa_+}} - \sum_{\kappa_-} e^{\xi^{-}_{i,\kappa_-}}$$

C_i = TAUTOLOGICAL COMPLEXES ON $\mathfrak{M}_{\gamma}(\mathbf{w},\mathbf{v})$

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FORMULA FOR qq-CHARACTERS

$\mathcal{X}_{\mathbf{w}}(x)$ = PARTITION FUNCTION

OF A POINT-LIKE DEFECT $\mathscr{D}_{\mathbf{w}}(x)$

FOR SOME (ALL?) THEORIES

$\mathscr{D}_{\mathbf{w}}(x)$ CAN BE ENGINEERED

USING INTERSECTING BRANES

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EXAMPLES: $U(N)$ THEORIES

A_1 CASE: $N_c = N$, $N_f = 2N$

FUNDAMENTAL qq-CHARACTER

$$\mathcal{X}_1(x) = Y(x + \epsilon_1 + \epsilon_2) + \mathfrak{q} P(x) Y(x)^{-1}$$

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EXAMPLES: $U(N)$ THEORIES

A_1 CASE: $N_c = N$, $N_f = 2N$

GENERAL qq-CHARACTER

$$\vec{\nu} = (\nu_1, \ldots, \nu_{\mathrm{w}})$$

$$\mathcal{X}_{\vec{\nu}}(x) = \sum_{[\mathrm{w}] = I \amalg J} \mathfrak{q}^{\# J} \prod_{i \in I, j \in J} \frac{(\nu_i - \nu_j + \epsilon_1)(\nu_i - \nu_j + \epsilon_2)}{(\nu_i - \nu_j)(\nu_i - \nu_j + \epsilon_1 + \epsilon_2)} \times$$

$$\prod_{j \in J} \frac{P(x + \nu_j)}{Y(x + \nu_j)} \times \prod_{i \in I} Y(x + \epsilon_1 + \epsilon_2 + \nu_i)$$

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EXAMPLES: $U(N)$ THEORIES

$\widehat{A}_0$ CASE: $\mathcal{N} = 2^* \; U(N)$ THEORY

μ MASS OF THE ADJOINT HYPER

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FUNDAMENTAL *qq*-CHARACTER

$$\mathcal{X}_1(x) = \sum_{\lambda} \mathfrak{q}^{|\lambda|} \prod_{\square\in\lambda} \frac{(\mu h_\square + (\epsilon_1+\epsilon_2)a_\square + \epsilon_1)(\mu h_\square + (\epsilon_1+\epsilon_2)a_\square + \epsilon_2)}{(\mu h_\square + (\epsilon_1+\epsilon_2)a_\square)(\mu h_\square + (\epsilon_1+\epsilon_2)(a_\square + 1))}$$

$$\times \prod_{\square\in\partial_+\lambda} Y(x+\sigma_\square+\epsilon_1+\epsilon_2)$$

$$\times \prod_{\square\in\partial_-\lambda} Y(x+\sigma_\square)^{-1}$$

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$a_\square, l_\square, h_\square$ are the arm-length, the leg-length and the hook-length of the box $\square = (i, j) \in \lambda$

$$a_\square = \lambda_i - j, \qquad l_\square = \lambda_j^t - i, \qquad h_\square = a_\square + l_\square + 1,$$

$$\sigma_\square = \mu(i - j) + (\epsilon_1 + \epsilon_2)(1 - j)$$

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APPLICATIONS

BPS/CFT CORRESPONDENCE

NN, 2002-2004

Correlators of chiral observables
in four dimensional supersymmetric theories
are **holomorphic blocks (form-factors)**
of some **conformal field theory**
(or a massive integrable deformation thereof)
in two dimensions

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APPLICATIONS

THE NONPERTURBATIVE DS EQUATIONS

CAN BE USED TO DEMONSTRATE

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Z-FUNCTIONS

OF A-TYPE QUIVER THEORIES

FOR SPECIAL MASSES

OBEY THE BPZ-TYPE EQUATIONS

$\Longrightarrow$ Alday-Gaiotto-Tachikawa dictionary

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Z-FUNCTIONS

IN THE PRESENCE OF SURFACE DEFECTS

Y_i-observables fraction: $Y_i \to Y_{i,\alpha}$, $\alpha = 1, \ldots, n_i$

OF A-TYPE QUIVER THEORIES

OBEY THE KZ-TYPE EQUATIONS

Use Nakajima realization, cf. Kanno-Tachikawa

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CONCLUSIONS/SPECULATIONS

THE G_γ SYMMETRY CAN BE SEEN

IN THE II STRING REALIZATION OF GAUGE THEORY

YANGIAN $Y(\mathbf{g}_\gamma)$ AND $U_q(\widehat{\mathbf{g}}_\gamma)$ IN STRING THEORY?
SH^c degenerate DAHA of S. Kanno, Y. Matsuo, H. Zhang?

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CONCLUSION/SPECULATION

GRAVITATIONAL

NON-PERTURBATIVE DS EQUATIONS?

NON-LINEAR WHEELER-DE WIT EQUATION?

SYMMETRY OF THE LANDSCAPE?

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CONCLUSION/SPECULATION

TOPOLOGICAL STRING(M-THEORY)

NON-PERTURBATIVE DS EQUATIONS?

qq-characters vs. q, t-characters

Nakajima, Frenkel-Hernandez?

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