BPS News(Half)Hour

(따끈 따끈한 BPS 30분 긴급뉴스)

Gregory Moore, Rutgers University

Strings2013, Seoul, June 28

Today I will report on the headlines for a few projects

The common theme in all these projects is the BPS spectrum of $\mathcal{N}$=2 supersymmetric field theories in four- and in two-dimensions.

``Making the World a Stabler Place''

The BPS Times

Late Edition

Today, BPS degeneracies, wall-crossing formulae. **Tonight**, Sleep. **Tomorrow**, K3 metrics, BPS algebras, p.B6

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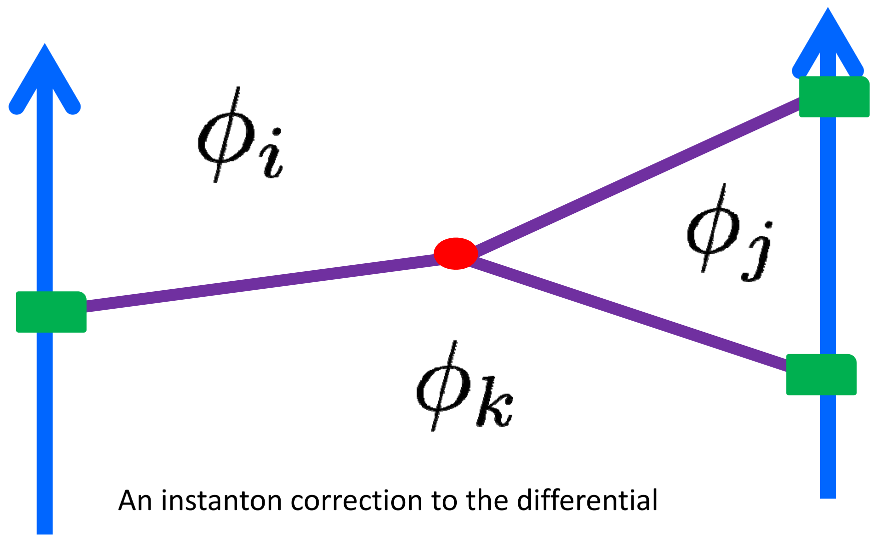

An instanton correction to the differential

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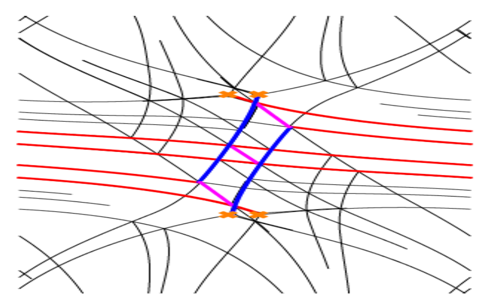

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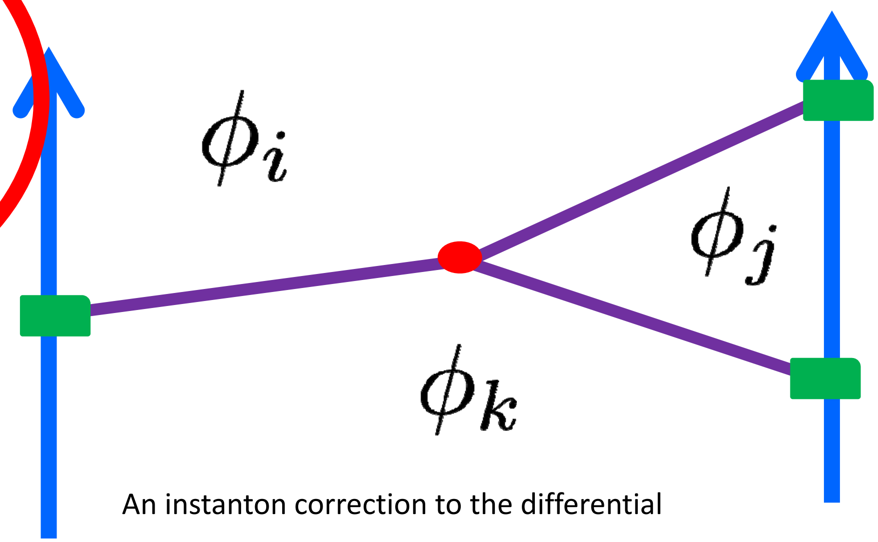

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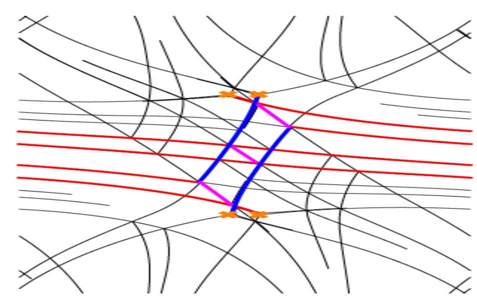


BPS States

$$\mathcal{H} = \oplus_{\gamma\in\Gamma}\mathcal{H}_\gamma \qquad E \geq |Z_\gamma|$$

$$\mathcal{H}_\gamma^{\rm BPS} = \{\psi : E\psi = |Z_\gamma|\psi\}$$

These states are in a rep of $SO(3)_{rot}$ x $SU(2)_R$

$$\mathcal{H}_\gamma^{\rm BPS} = \underbrace{((2,1)\oplus(1,2))}_{C.O.M.}\otimes\mathfrak{h}_\gamma^{\rm BPS}$$

No-exotics conjecture

In field theories with good UV fixed point
$SU(2)_R$ acts trivially on $\mathfrak{h}_{\gamma}^{BPS}$.

``Exotics'' would violate this conjecture.

Motivated by positivity properties of line defects GMN

This has been proven (pretty) rigorously for pure SU(N) gauge theory in:

Geometric engineering of (framed) BPS states

W.-y Chuang, E. Diaconescu, J. Manschot, G. Moore, Y. Soibelman, 1301.3065

Geometric Engineering

Recall geometric engineering of pure SU(N) gauge theory

(Aspinwall; Katz,Morrison,Plesser; Katz, Klemm, Vafa)

Family of resolved A_{N-1} singularities $X_N \longrightarrow \mathbb{P}^1$

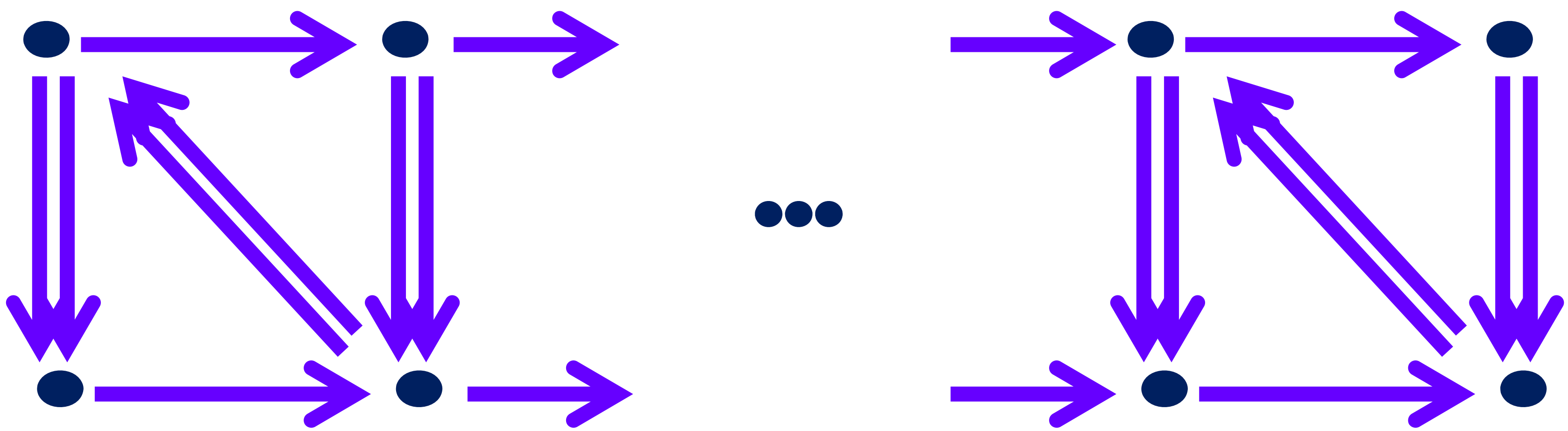
Take a scaling limit of Type IIA on $X_N \times \mathbb{R}^{1,3}$

⟹ Recover $\mathcal{N}=2$ SU(N) SYM

Exceptional Collections & Quivers

Take a strong exceptional collection $\mathcal{L}_\alpha$ on X_N

Compute $\mathrm{Ext}^0 (\oplus \mathcal{L}_\alpha)$ algebra to get quiver:



(Fiol, 2000; Cecotti,Vafa, et. al. ; Diaconescu et. al.)

Outline of Proof

The no-exotics conjecture follows from the statement that the ``motive'' of the moduli space is a function of $[\mathbb{P}^1]$.

"Motives" : A method of telling when spaces are related to each other by cutting and pasting and taking products.

Three ideas in the proof:

Use recent math results on motivic structure of <u>framed</u> quiver moduli spaces in an ``infinite B-field" chamber.

Recursion relations between framed and unframed degeneracies only involve $[\mathbb{P}^1]$.

Motivic KSWCF only adds Laurent polynomials in $[\mathbb{P}^1]$.

Review Framed BPS States

A ***line defect*** L (say along $\mathbb{R}_t$ x {0 }) is ***of type ζ*** if it preserves the susys:

$$Q_\alpha^A + \zeta \sigma^0_{\alpha\dot\beta} \bar{Q}^{\dot\beta A}$$

Example: $$L_\zeta = \exp \int_{\mathbb{R}_t \times \vec{0}} \left(\frac{\varphi}{2\zeta} + A + \frac{\zeta}{2} \overline{\varphi} \right)$$

$$\mathcal{H}_L = \oplus_{\gamma \in \Gamma} \mathcal{H}_{L,\gamma}$$

$$E \geq -\mathrm{Re}(Z_\gamma / \zeta)$$

Framed BPS States saturate this bound. They have proven to be very useful.

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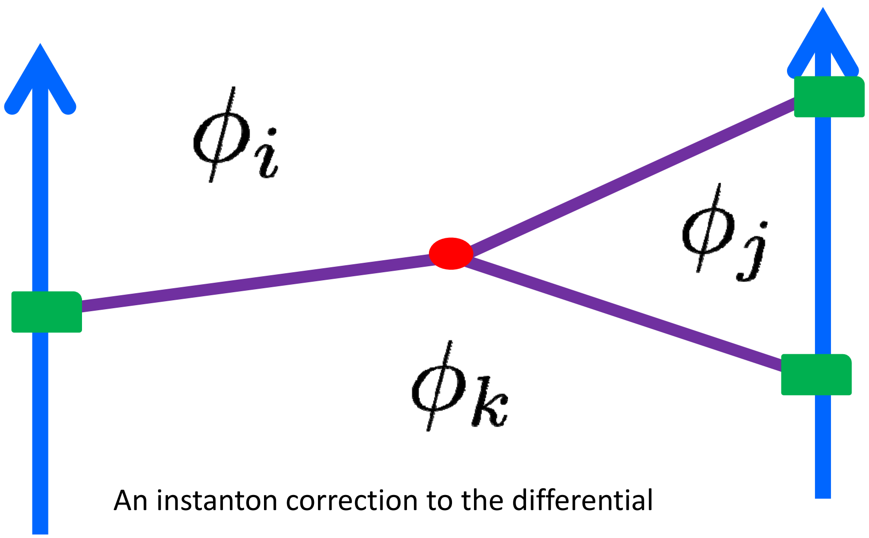

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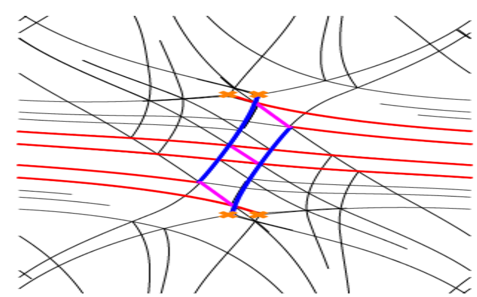


Singular Monopole Moduli Space

Yang-Mills-Higgs system for compact simple G

$$(A, X) \qquad \int_{\mathbb{R}^4} \mathrm{Tr}(F * F + DX * DX)$$

$$F = *DX$$

$$F = \gamma_m \mathrm{vol}(S^2) + \cdots \qquad X \to X_\infty - \frac{\gamma_m}{2r} + \cdots$$

$$\mathcal{M}(\gamma_m; X_\infty) \qquad \gamma_m \in \Lambda_{cr} \subset \mathfrak{t} \subset \mathfrak{g}$$

$$\vec{x} \to 0$$

$$F = P\mathrm{vol}(S^2) + \cdots \qquad X \to -\frac{P}{2r} + \cdots$$

$$\overline{\mathcal{M}}([P]; \gamma_m; X_\infty) \; [P] \in \Lambda(G)^*/W \subset \mathfrak{t}/W$$

Dimension Formula

Repeat computation of Callias; E. Weinberg to find:

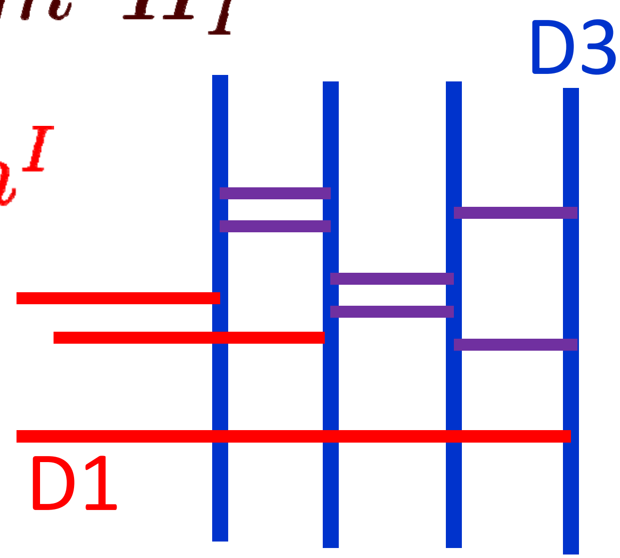
Regular X_∞ ⟹ Simple roots $\langle \alpha_I, X_\infty \rangle > 0$

P^+ : Weyl group image such that $\langle \alpha_I, P^+ \rangle \geq 0$

$$\tilde{\gamma}_m := \gamma_m + P^+ = \sum_I \tilde{m}^I H_I$$

$$\dim \overline{\mathcal{M}}([P]; \gamma_m; X_\infty) = 4 \sum_I \tilde{m}^I$$

$\overline{\mathcal{M}} \neq \emptyset$ ⟺ ? $\tilde{m}^I \geq 0$



Properties of $\overline{\mathcal{M}}$

$\overline{\mathcal{M}}$ Hyperkähler (with singular loci - monopole bubbling)

$\overline{\mathcal{M}}$ has an action of $so(3) \oplus \mathfrak{t}$

so(3): spatial rotations

$\mathfrak{t}$-action: global gauge transformations commuting with X_∞

$$v \in \mathfrak{t} \Longrightarrow G(v) \in \mathrm{VECT}(\overline{\mathcal{M}})$$

$\mathcal{N}$=2 Super-Yang-Mills

Second real adjoint scalar Y

Vacuum requires $[X_\infty, Y_\infty]=0$.

$$X_\infty + iY_\infty = \zeta^{-1} a \in \mathfrak{t} \otimes \mathbb{C}$$

Collective coordinate quantization ⟶

Dirac Operator: $\hat{D} := \gamma^\mu (D + G(Y_\infty))_\mu$

(Sethi, Stern, Zaslow; Gauntlett & Harvey ; Tong; Gauntlett, Kim, Park, Yi; Gauntlett, Kim, Lee, Yi; Bak, Lee, Yi; Bak, Lee, Lee, Yi; Stern & Yi)

Semiclassical Framed BPS states

Organize L^2-harmonic spinors by $\mathfrak{t}$-representation:

$$\mathrm{Ker}_{L^2}\hat{D} = \oplus_{\gamma_e}\mathrm{Ker}_{L^2}^{\gamma_e}\hat{D} \qquad \gamma_e \in \Lambda_{wt} \subset \mathfrak{t}^*$$

$$\mathcal{H}^{\mathrm{BPS}}(L_\zeta;\gamma_m,\gamma_e;a) := \underbrace{\mathrm{Ker}_{L^2}^{\gamma_e}\hat{D}}$$

$SU(2)_R$ acts on sections of $T\mathcal{M}$ rotating the 3 complex structures;

-1 acts on spinors as chirality.

$$SO(3)_{\mathrm{rot}} \times SU(2)_R$$

Some Physical Mathematics

Easy fact: There are no L^2 harmonic spinors for ordinary Dirac operator on a noncompact hyperkähler manifold.

$\exists$ Semiclassical chamber (Y_∞=0) where all populated magnetic charges are just simple roots

Other semiclassical chambers have nonsimple magnetic charges filled.

Nontrivial semi-classical wall-crossing

(Higher rank is different.)

Interesting math prediction for

$$\hat{D} = \gamma^\mu \left(D + G(Y_\infty)\right)_\mu$$

Jumping Index

The L^2-kernel of $\hat{D}$ jumps.

No exotics theorem ⇒

Harmonic spinors have definite chirality

⇒ L^2 index jumps! How?!

Along hyperplanes in Y-space zeromodes mix with continuum and $\hat{D}^+$ fails to be Fredholm.

(Similar picture proposed by M. Stern & P. Yi.)

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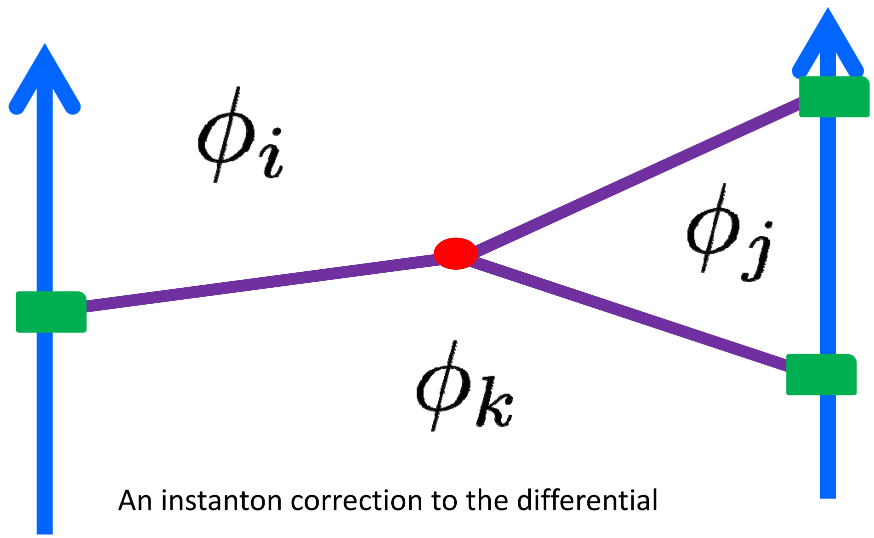

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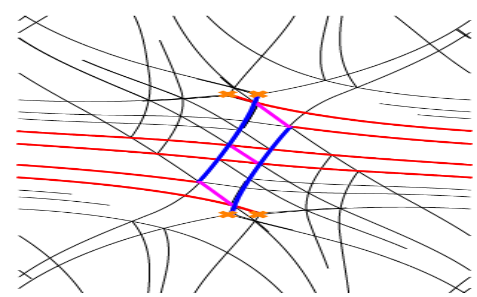

WILD WALL CROSSING

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Higher rank is different

SU(2), N_f = 0,1,2,3,4: All BPS states are in hypermulitplets or vectormultiplets and have Ω=1,-2

More generally: This holds for all theories S[A1]. Katz, Klemm, Mayr, Vafa,Warner; GMN; Bridgeland & Smith.

Already in the semiclassical limit in SU(3) you can see higher spin multiplets (Gauntlett, Kim, Park, Yi)

But there are still more suprises:

Sometimes Ω grows <u>expontentially</u> with charge.

Last year I conjectured that this couldn't happen -- for a good reason

Thermodynamic Argument

Consider d-dimensional CFT in box of spatial volume V :

$$E(T,V) = \alpha V T^d \quad S(T,V) = \beta V T^{d-1}$$

$$S(E,V) = \kappa V^{1/d} E^{(d-1)/d}$$

$$\Longrightarrow \quad \frac{\log |\Omega(\gamma)|}{|\gamma|} \to 0 \quad ?$$

However, explicit computations, both with spectral networks and wall – crossing independently show that there are regions of Coulomb branch and charges where growth is ***exponential***.

$$\exists u_{wild} \quad \exists \gamma_1, \gamma_2 \quad \langle \gamma_1, \gamma_2 \rangle = m > 2$$

$$\gamma_{a,b} = a\gamma_1 + b\gamma_2 \quad a, b \geq 1$$

$$\log |\Omega(n\gamma_{a,b}; u_{wild})| \sim nC_{a,b}(m)$$

$$n \to \infty \qquad C_{a,b}(m) > 0$$

How do you prove this?

Proof Using Spectral Networks

This is a technique whereby the study of soliton degeneracies on surface defects is used to produce four-dimensional BPS degeneracies. (GMN)

We get an exact result for $\Omega(n\gamma_{1,1}; u_{wild})$

$$P_m(z) = \prod_{n=1}^{\infty}(1-(-1)^{mn} z^n)^{n\Omega(n\gamma_{1,1})/m}$$

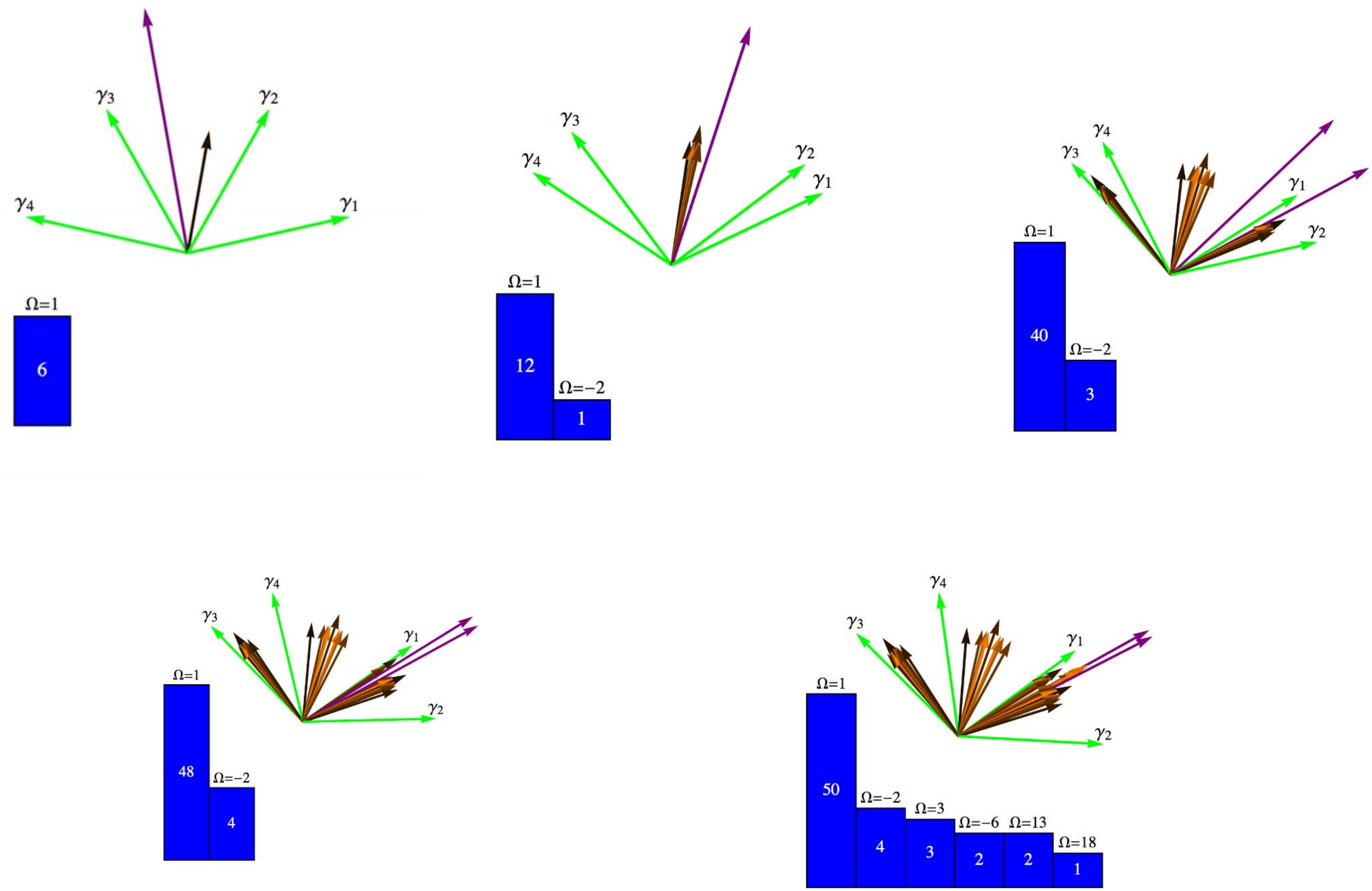
$$P_m = 1 + z(P_m)^{(m-1)^2}$$

(Same equation appears in Kontsevich & Soibelman; Gross & Pandharipande)

$$\Omega(n\gamma_{1,1}) = \frac{m}{(m-1)^2 n^2} \sum_{d|n} (-1)^{md+1} \mu\left(\frac{n}{d}\right) \binom{(m-1)^2 d}{d}$$

Spectral networks show similar generating functions always satisfy ***algebraic equations***. (Confirming M.K.)

Proof Using Wall-Crossing



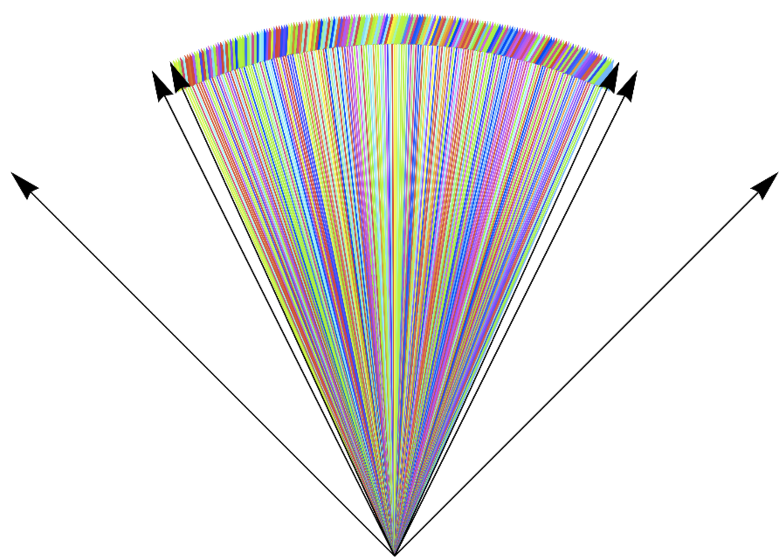
Kronecker m-Quiver

The critical wall-crossing has two hypermultiplets with charges $\langle \gamma_1, \gamma_2 \rangle = m > 2$

No nearby occupied rays ⇒

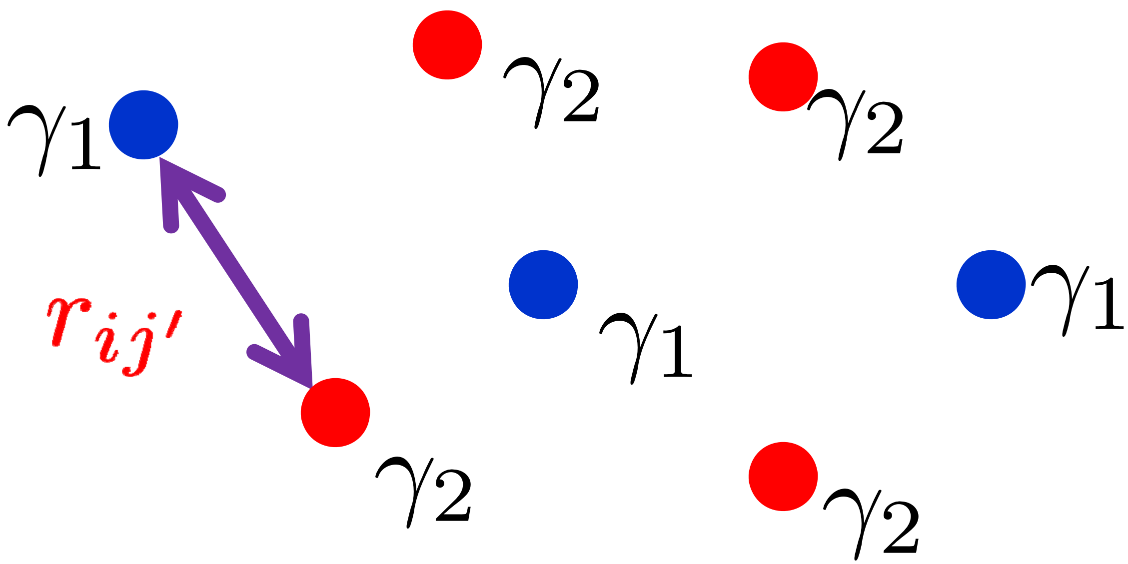
We get the wild spectrum of the Kronecker m-quiver:

Rays through occupied BPS charges:



BPS Giants

States from wall-crossing should have a semiclassical multi-centered picture:



Denef equations for $na\gamma_1 + nb\gamma_2$ ⟹

$$C|na\gamma_1 + nb\gamma_2| \leq \langle r_{ij'} \rangle$$

No contradiction with thermodynamic argument.

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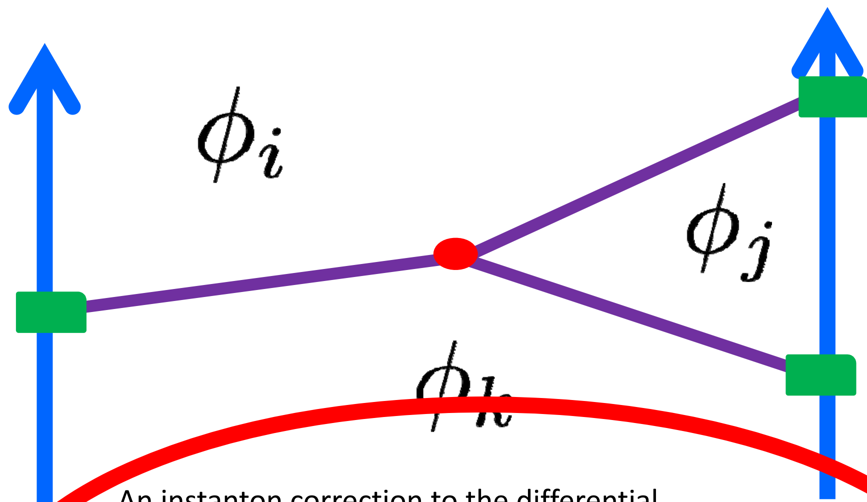

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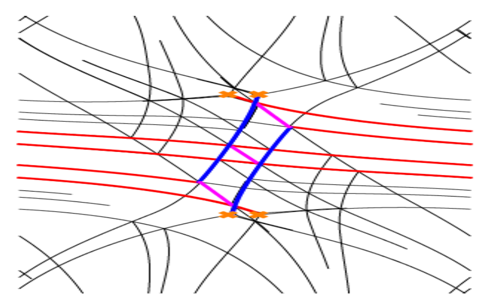


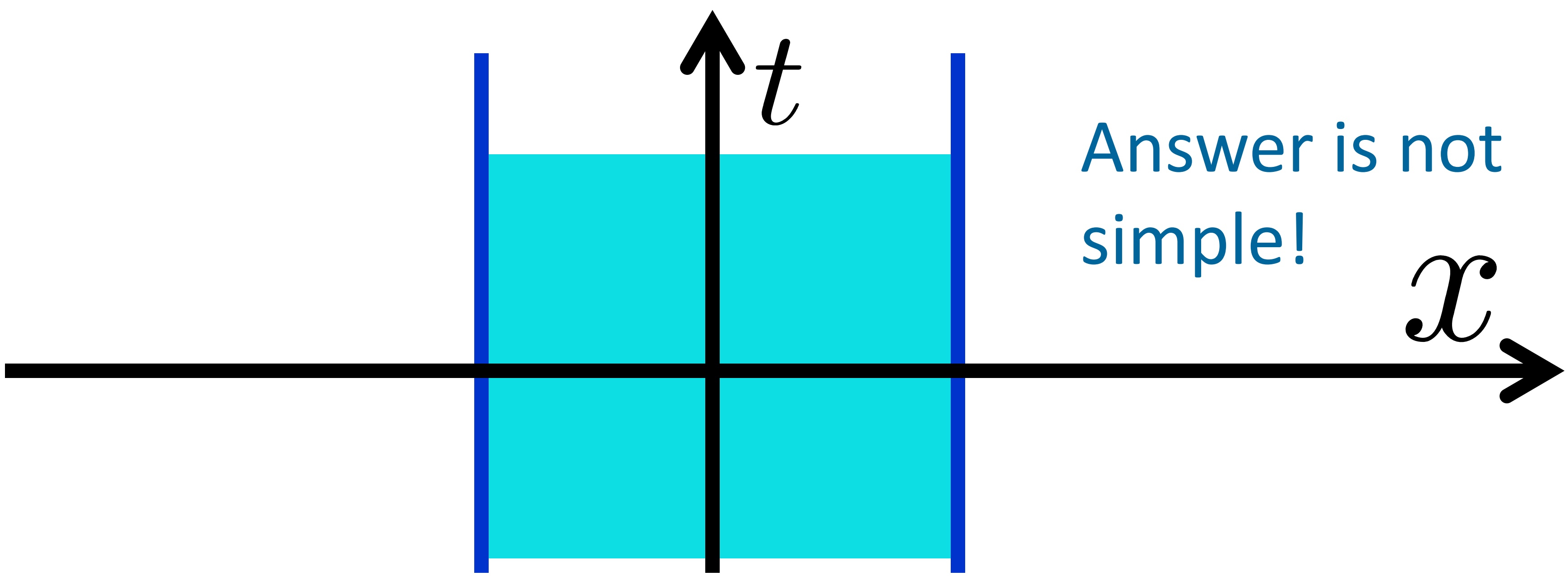
D=2, $\mathcal{N}$=2 Landau-Ginzburg Theory

X: Kähler manifold

W: X $\longrightarrow \mathbb{C}$ Superpotential (A holomorphic Morse function)

Simple question:

What is the space of BPS states on an interval ?



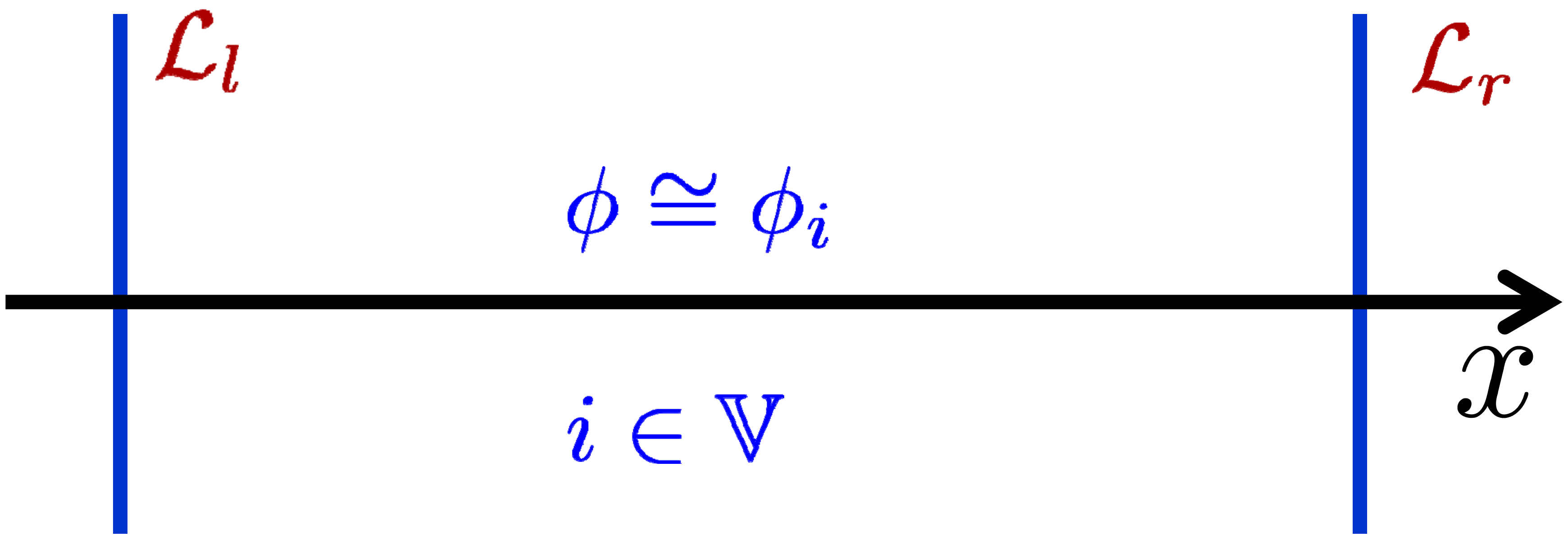
Answer is not simple!

Choose boundary conditions to preserve two susy's, parametrized by phase ζ,

For LG field ϕ we can choose any pair of Lagrangian subspaces (subject to some fine print)

The theory is massive:

Field in the middle of a large interval is close to a vacuum:



Does the Problem Factorize?

For the Witten index: Yes

$$\mu_{\mathcal{L}_l,i} = \mathrm{Tr}(-1)^F e^{-\beta H}$$

$$\boxed{\mu_{\mathcal{L}_l,\mathcal{L}_r} = \sum_{i\in\mathbb{V}} \mu_{\mathcal{L}_l,i} \cdot \mu_{i,\mathcal{L}_r}}$$

For the BPS states? No!

$$\mathcal{H}_{\mathcal{L}_l,\mathcal{L}_r} \neq \sum_{i\in\mathbb{V}} \mathcal{H}_{\mathcal{L}_l,i} \otimes \mathcal{H}_{i,\mathcal{L}_r}$$

BPS Solitons on half-line D:

Semiclassically:

Q_ζ -preserving BPS states must be solutions of differential equation

$$\frac{\partial \phi}{\partial x} = \zeta \frac{\partial \bar{W}}{\partial \bar{\phi}} \qquad X = \mathbb{C}$$

$$\phi|_\partial \in \mathcal{L}_l \qquad \begin{matrix} \phi \to \phi_i \\ x \to \infty \end{matrix}$$

Quantum?

Morse Theory & SQM à la Witten

View 1+1 Landau-Ginzburg model as

Supersymmetric Quantum Mechanics for target space

$$\phi \in \mathrm{Map}(D \to X)$$

Kähler form on X: $\omega = d(pdq)$

$$h = \int_D \left(\phi^*(pdq) + \mathrm{Re}(\zeta^{-1} W)\right)$$

$$\mathbb{M}_{\mathcal{L}_l,i} = \oplus_p \phi^p_{\mathcal{L}_l,i}(x)\mathbb{Z} \qquad d_{\mathcal{L}_l,i}$$

Factorizing the Complex

When the interval is much longer than the scale set by W the Morse complex is

$$\mathbb{M}_{\mathcal{L}_l,\mathcal{L}_r} = \oplus_{i\in\mathbb{V}}\mathbb{M}_{\mathcal{L}_l,i} \otimes \mathbb{M}_{i,\mathcal{L}_r}$$

But!

$$d_{\mathcal{L}_l,\mathcal{L}_r} \neq d_{\mathcal{L}_l,i} \otimes 1 + 1 \otimes d_{i,\mathcal{L}_r}$$

Why?

Instantons

$$h = \int_D \left(\phi^*(pdq) + \mathrm{Re}(\zeta^{-1}W)\right)$$

$$\frac{d}{d\tau}\phi = -\frac{\delta h}{\delta \phi}$$

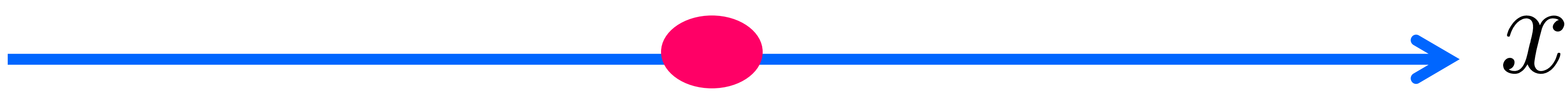
``ζ-instanton equation'':

$$\left(\frac{\partial}{\partial x} + \mathrm{i}\frac{\partial}{\partial \tau}\right)\phi = \zeta\frac{\partial \bar{W}}{\partial \bar{\phi}}$$

Stationary points are solitons: $$\frac{\partial \phi}{\partial x} = \zeta\frac{\partial \bar{W}}{\partial \bar{\phi}}$$

Now we will construct some special ζ-instantons using the solitons on $\mathbb{R}$

Solitons For D=$\mathbb{R}$



$\phi \cong \phi_i$ $\qquad\qquad$ $\phi \cong \phi_j$

For general ζ there is ***no*** solution

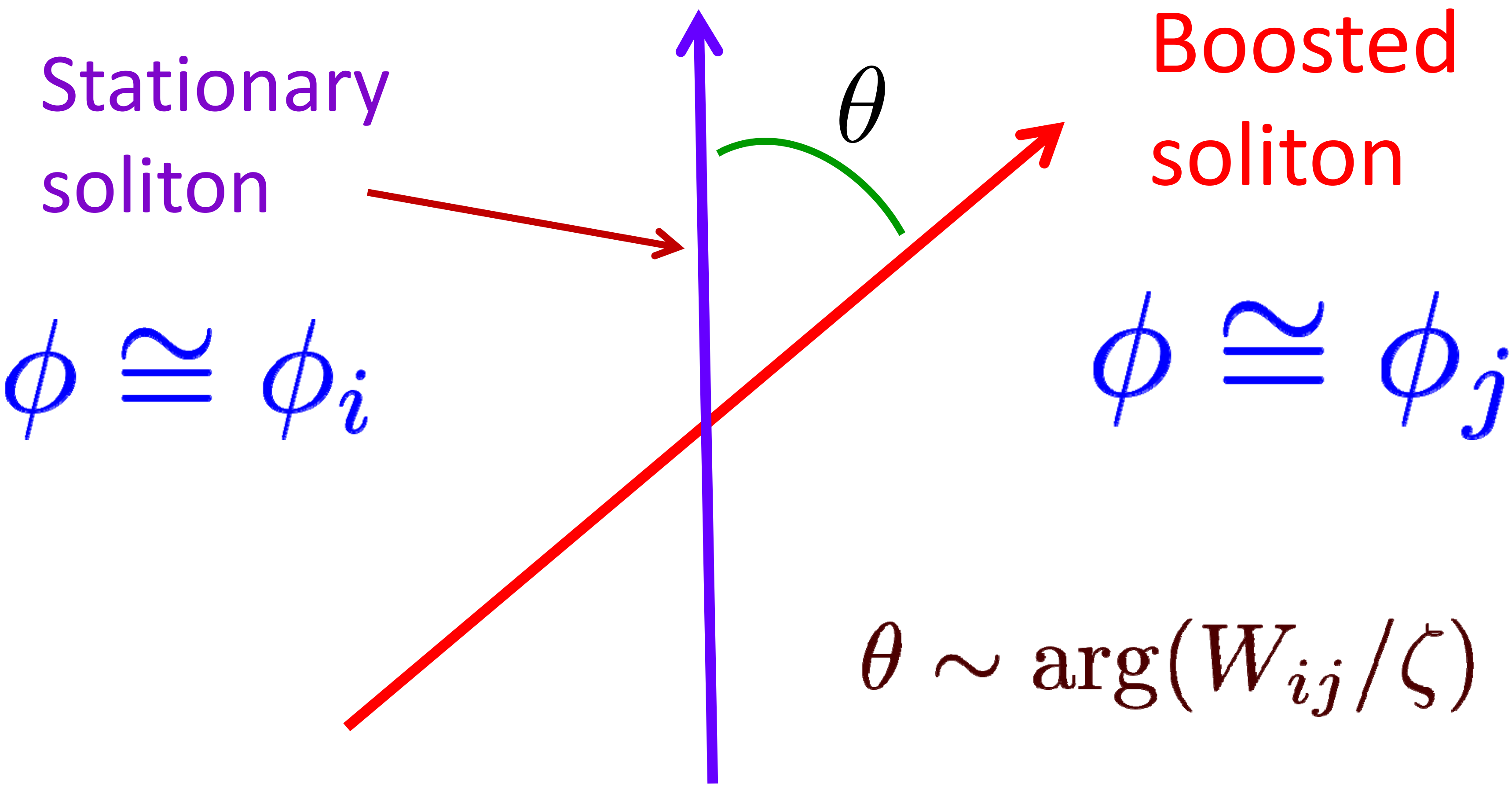
$$\frac{\partial \phi}{\partial x} = \zeta \frac{\partial \bar{W}}{\partial \bar{\phi}}$$

Solutions exist only for:

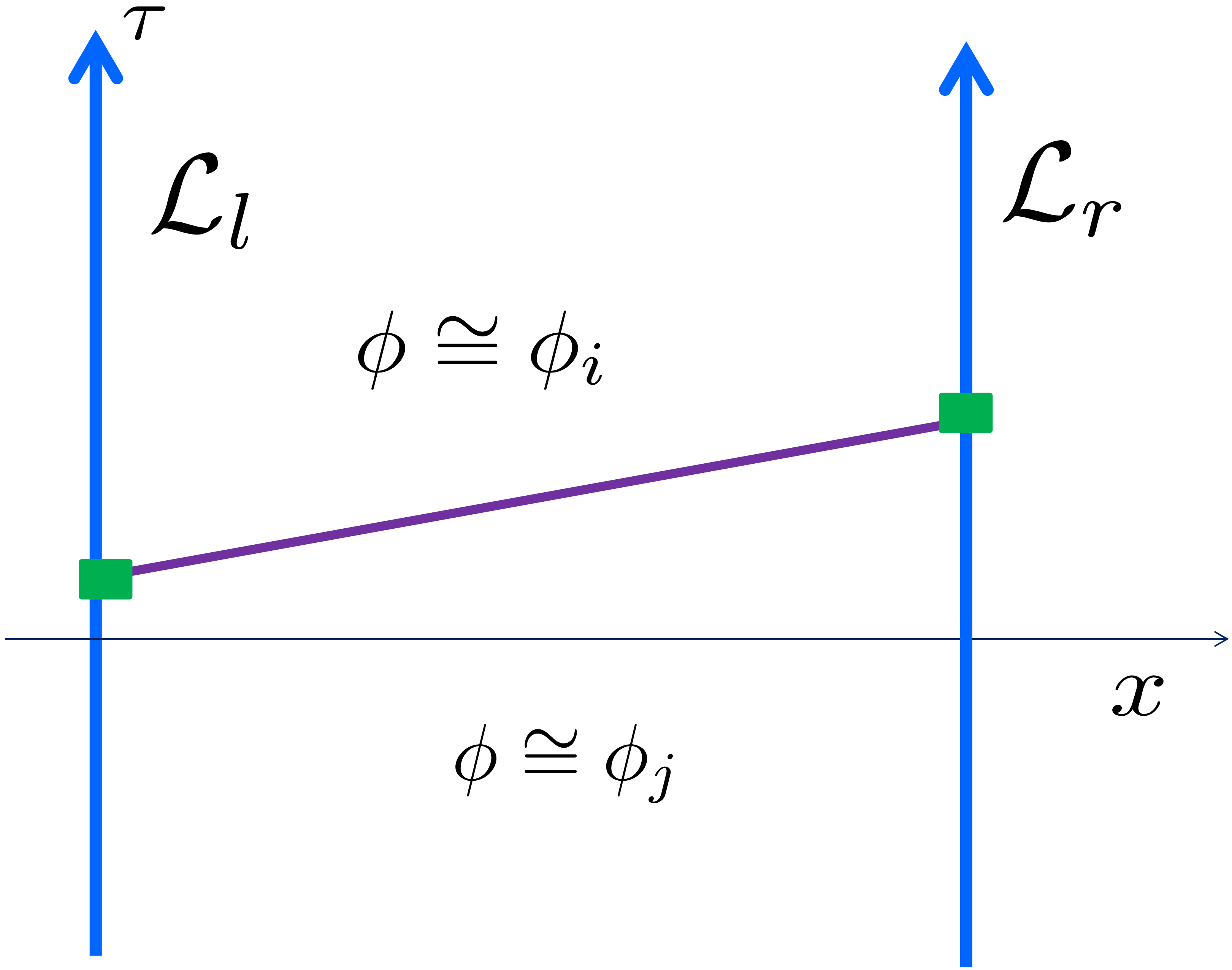
$$\zeta = \zeta_{ji} := \frac{W_j - W_i}{|W_j - W_i|} = \frac{W_{ji}}{|W_{ji}|}$$

The Boosted Soliton

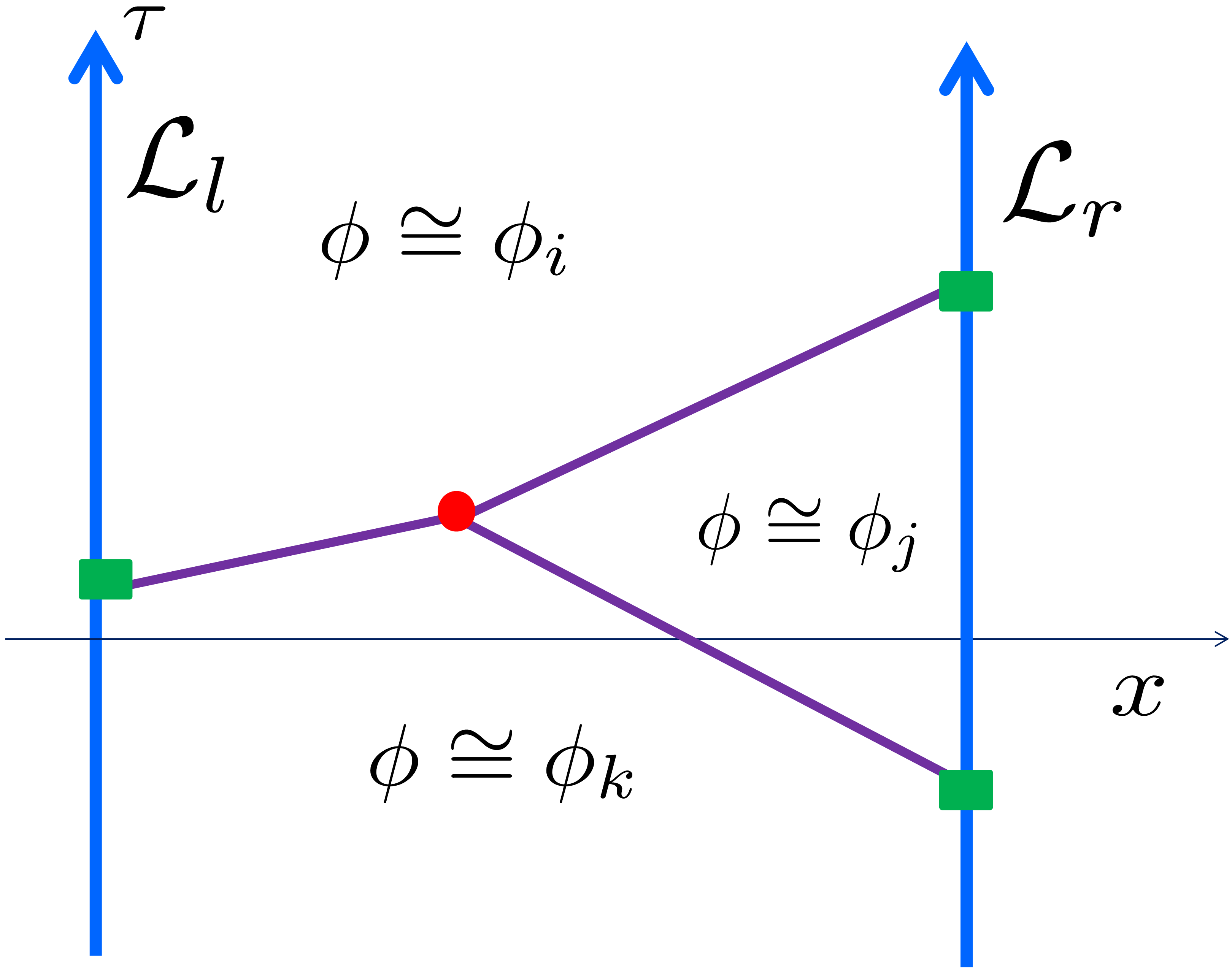
$$\phi_{ij}^{\text{inst}}(x,\tau) := \phi_{ij}^{\text{sol}}(\cos\theta x + \sin\theta\tau)$$



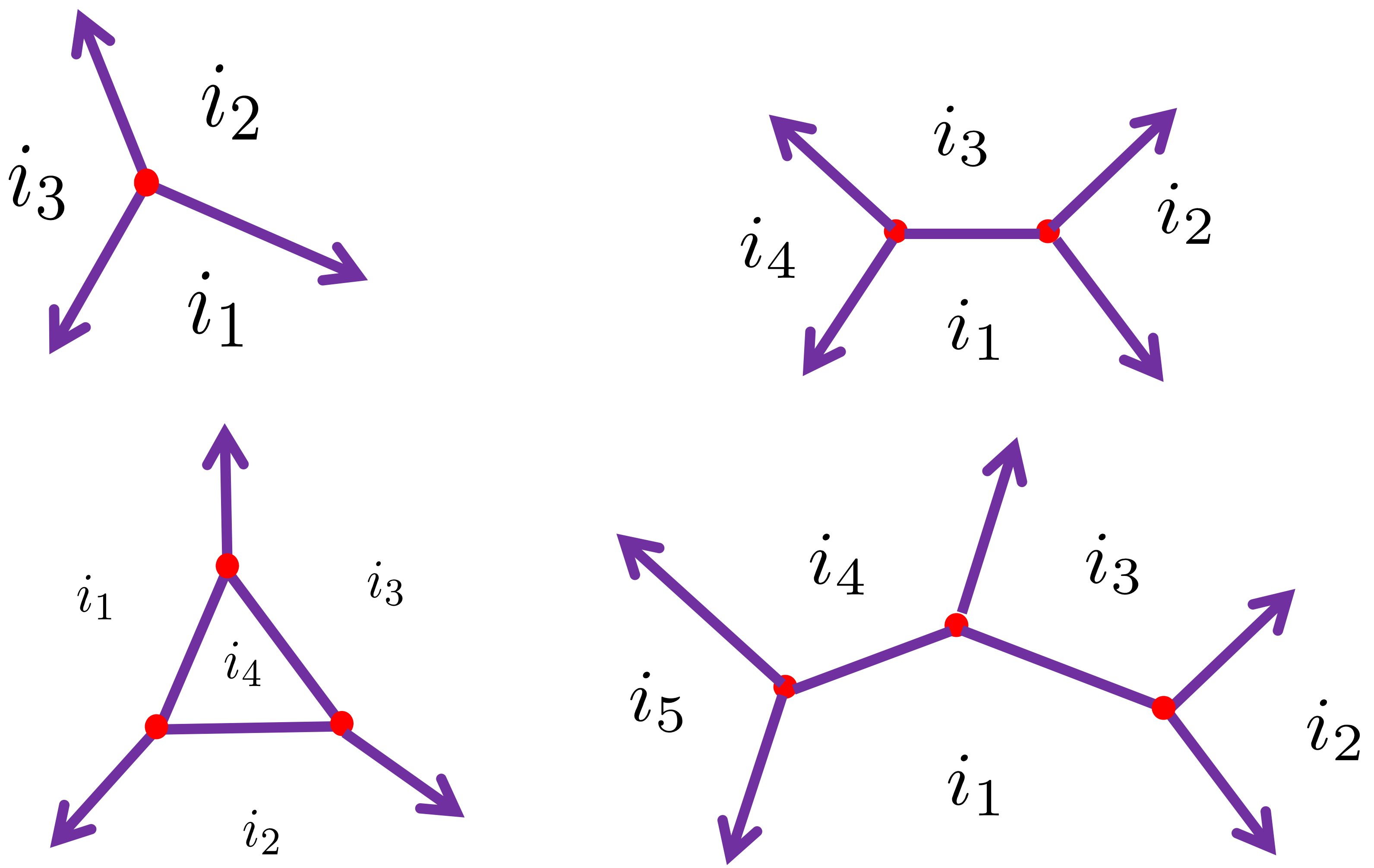
Instanton Correction to Naïve d

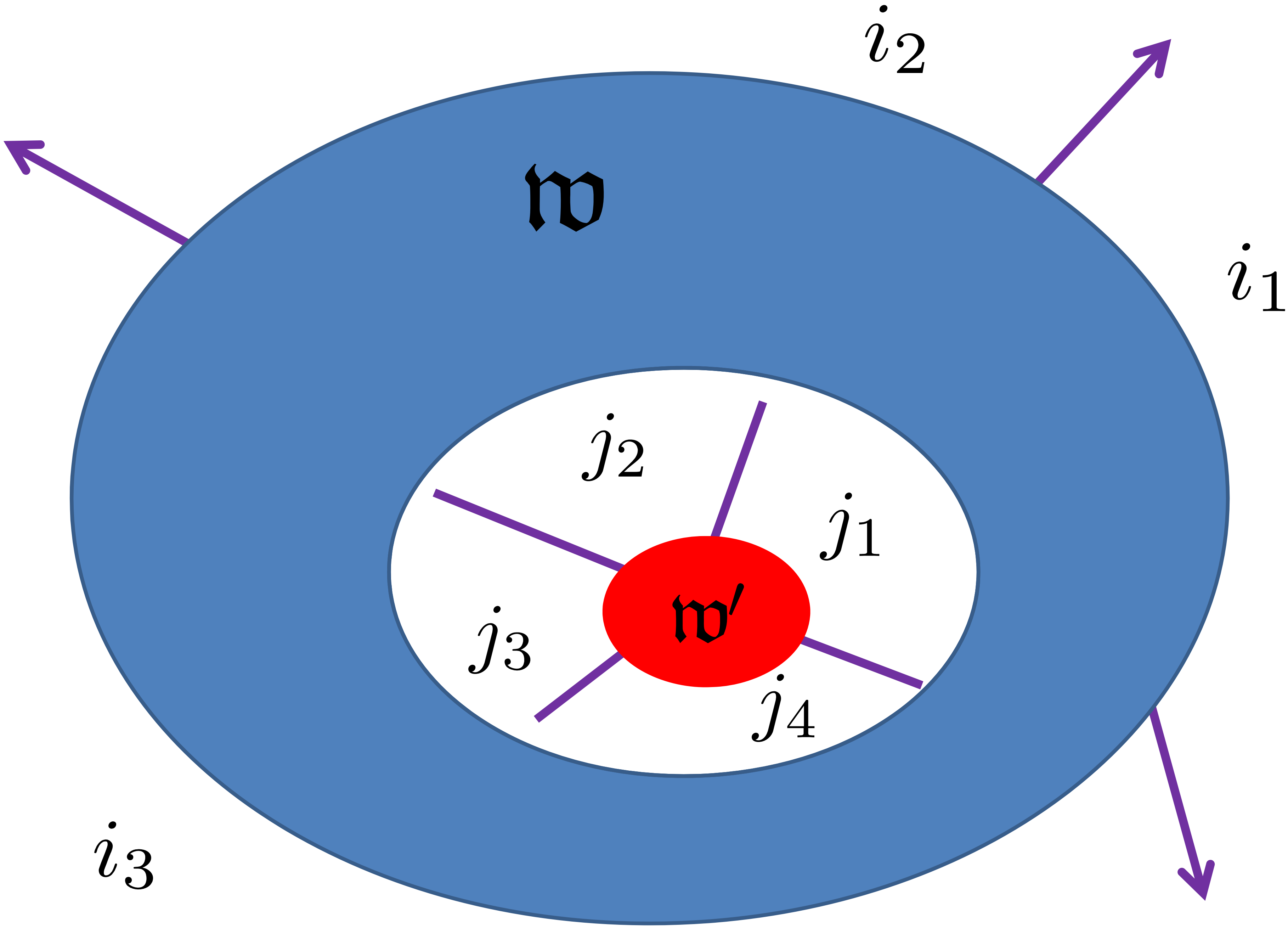


More Instanton Corrections to Naïve d



(Planar) Instanton Webs





i_2
$\mathfrak{w}$
i_1
j_2
j_1
$\mathfrak{w}'$
j_3
j_4
i_3

Algebraic Structures

This defines a convolution structure on the space of webs:

$$\mathfrak{w}_1 * \mathfrak{w}_2$$

Applied to planar instanton webs it describes a graded Lie algebra …

… together with higher multiplications defining an L_∞ structure.

Applied to half-plane instanton webs it defines an A_∞ category

D-brane categories

Branes are solutions of the Maurer-Cartan equations:

$$dA + A * A + A * A * A + \cdots = 0$$

This gives an ``infrared'' alternative to the (``ultraviolet'') Fukaya-Seidel construction of the category of D-branes in the LG model.

Choosing a brane at either end of the interval we thereby construct the full differential for computing BPS states.

Categorified Wall-Crossing

Applied to supersymmetric interfaces we obtain an A_∞ functor of D-brane categories.

This categorifies Cecotti-Vafa wall-crossing for solitons.

Similar ideas applied to surface defects in 4d should categorify the KSWCF.

and so on, and so forth …

And that's the way it is...

June 28, 2013

대단히

감사합니다