Three-point correlators from string theory amplitudes

Joseph Minahan

Uppsala University

arXiv:1206.3129
Till Bargheer, Raul Pereira, JAM: arXiv:1311.7461;
Raul Pereira, JAM: arXiv:1407.xxxx

Strings 2014 in Princeton; 27 June

Introduction

Spectrum of local operators in $\mathcal{N} = 4$ SYM effectively solved in the planar limit. Determined by:

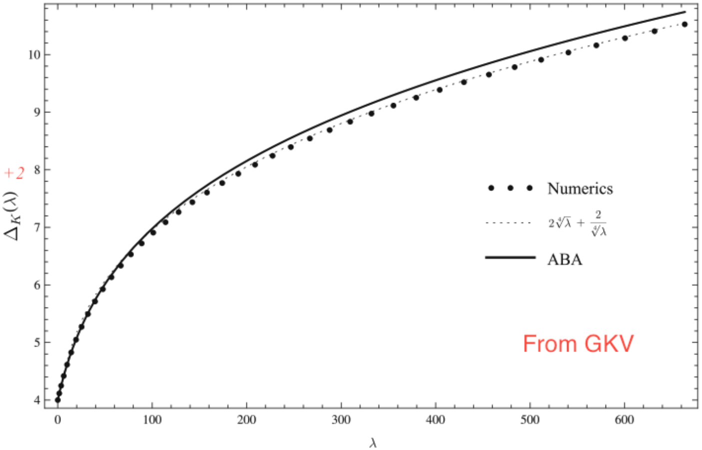
- Integrability: Asymptotic Bethe ansatz Staudacher (2004), Beisert-Staudacher(2005), Beisert (2005), Janik (2006), Eden-Staudacher (2006), Beisert-Hernandez-Lopez (2006), Beisert-Eden-Staudacher (2006) . . .
- Finite size complications (winding effects). Handled by TBA, Y-system, Hirota, FiNLIE, Q-functions Ambjorn-Janik-Kristjansen (2005), Bajnok-Janik (2008), Gromov-Kazakov-Vieira (2009,2009), G-K-Kozac-V (2009), Arutyunov-Frolov (2008,2009), Bombardelli-Fioravanti-Tateo (2009), Frolov (2010), Gromov-Kazakov-Leurent-Volin (2011, 2013, 2014) . . .

Introduction (cont)

A key example: *Konishi operator:* $\mathcal{O}_K = \mathrm{tr}(\phi^I \phi^I)$;

Primary: $[K^\mu, \mathcal{O}_K(0)] = 0$; $SO(6)$ singlet

$$\langle \mathcal{O}_K(x)\mathcal{O}_K(y)\rangle = \frac{\mathcal{Z}}{|x-y|^{2\Delta_K(\lambda)}} \qquad \lambda = g_{YM}^2 N$$



Introduction (cont)

Besides the spectrum, to really solve the theory we need the three-point correlators.

Correlator for three local operators:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{\mathcal{C}_{123}}{|x_{12}|^{2\alpha_3}|x_{23}|^{2\alpha_1}|x_{31}|^{2\alpha_2}}$$

$\alpha_1 = \frac{1}{2}(\Delta_2 + \Delta_3 - \Delta_1)$ $\alpha_2 = \frac{1}{2}(\Delta_3 + \Delta_1 - \Delta_2)$ $\alpha_3 = \frac{1}{2}(\Delta_1 + \Delta_2 - \Delta_3)$

$\mathcal{C}_{123} \sim N^{-1}$ for $N \gg 1$.

Introduction (cont)

$\mathcal{C}_{123}$ is protected for 3 chiral primaries.

- Chiral primary $\mathcal{O}_C(x)$: $[Q, \mathcal{O}_C(0)] = 0$ for half the Q's
- The gravity duals are K-K modes in the $AdS_5 \times S^5$ type IIB supergravity
- Supergravity calculation shows that $\mathcal{C}_{123}$ at large λ is the same as the zero-coupling result Lee, Minwalla, Rangamani and Seiberg (1998)

Introduction (cont)

Nonchiral primaries are not dual to sugra states but to massive string states.

- "Heavy" operators: Dual to long classical strings that stretch across the $AdS_5 \times S^5$.
- Semiclassical string calculation for 3-point correlators

  Janik-Surowka-Wereszczynski (2010)

  - Two heavy, one light Zarembo (2010), Costa-Monteiro-Santos-Zoakos (2010), Roiban-Tseytlin (2010) ...
  - Three heavy Janik-Wereszczynski (2011), Buchbinder-Tseytlin (2011), Klose-McLoughlin (2011), Kazama-Komatsu (2011-13), ...
- The Konishi operator is neither semi-classical nor light – it is dual to a short string state.
- Can one compute the 3-point correlators involving at least one Konishi operator for $\lambda \gg 1$?

Introduction (cont)

- General idea: Since Konishi is short it doesn't see the curvature of $AdS_5 \times S^5$ $(R = 1)$ $\Longrightarrow$ use the flat-space limit.
- Flat-space for the spectrum:
  Gubser-Klebanov-Polyakov (1998) String size $\sim \sqrt{\alpha'} = \lambda^{-1/4} \ll 1$

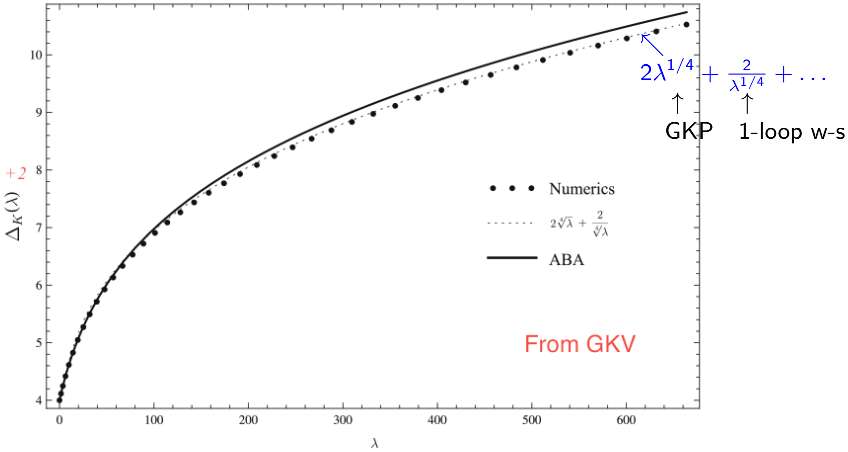
Flat-space closed strings: $m^2 = 4n/\alpha' = 4n\,\lambda^{1/2}$

AdS/CFT dictionary: $m^2 = \Delta^2 - d\Delta \approx \Delta^2$

$\Delta \approx 2\sqrt{n}\,\lambda^{1/4}$ $\qquad$ $n = 1$ for Konishi

Introduction (cont)

Back to the Gromov-Kazakov-Vieira plot:



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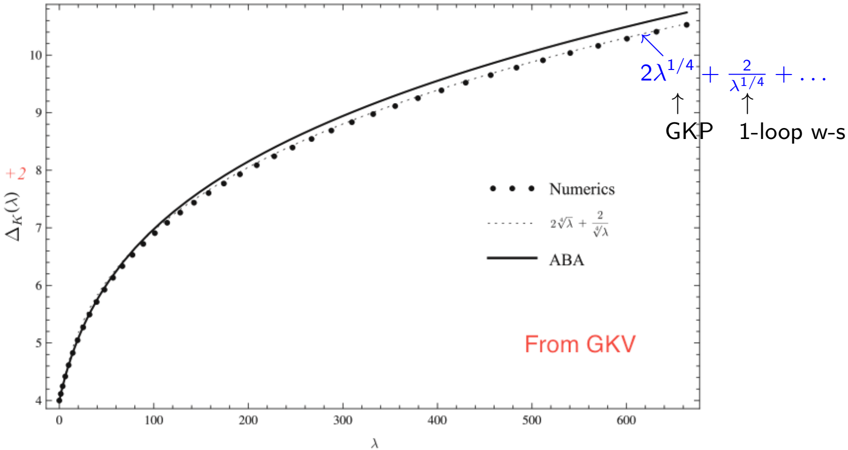
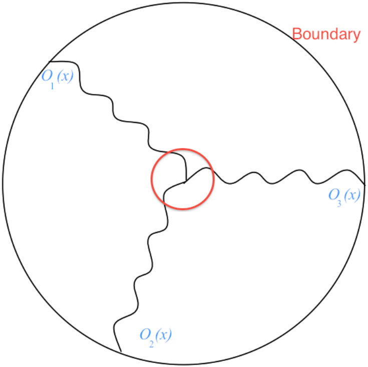


We would like a similar goal for 3-point correlators

3-point correlators – Witten diagrams

3-point correlators in supergravity Witten (1998)



Freedman-Mathur-Matusis-Rastelli (1998):

- Boundary to bulk propagators meet at an intersection point.
- Integrate over the intersection point.
- Multiply by sugra coupling $\mathcal{G}_{123}$

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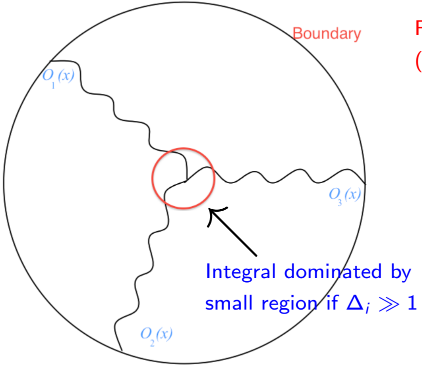

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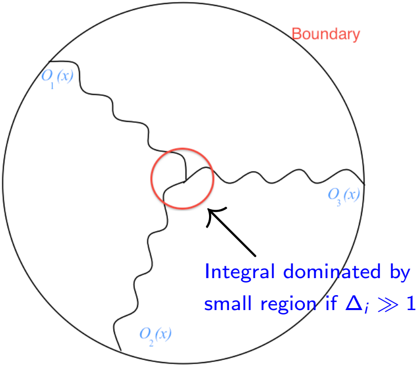

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- Treat as strings in the intersection region.

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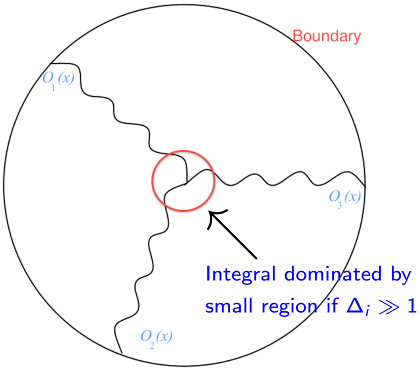

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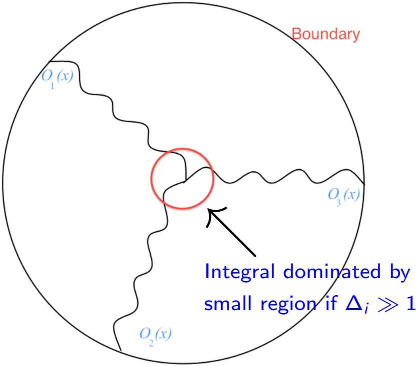

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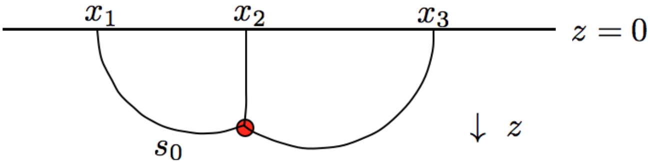
- Which vertex operators?

3-point correlators-particle path integrals in AdS

Three incoming particles meet at a joining point: $x^\mu(s_0) = x^\mu$, $z(s_0) = z$.

$\mathcal{Z}_{123} \equiv$



circular geodesics

$$S_{\rm cl}(x^\mu, z) = -\sum_{i=1}^{3} \Delta_i \log\left(\frac{z\,\epsilon}{z^2 + (x - x_i)^2}\right)$$

Saddle point: Conservation of momentum: (See also Klose & McLoughlin (2011))

$$\sum_{i=1}^{3} \Pi_{\mu,i} = 0 \qquad \sum_{z,i} \Pi_{z,i} = 0 \qquad \Pi_i \cdot \Pi_i = -\Delta_i^2$$

$$\mathcal{Z}_{123} \approx \frac{\pi^{\frac{2-d}{4}}}{4} \frac{(\Delta_1\Delta_2\Delta_3)^{d/4}}{(\alpha_1\alpha_2\alpha_3\Sigma^{d+1})^{1/2}} \frac{\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\alpha_3^{\alpha_3}\Sigma^{\Sigma}}{\Delta_1^{\Delta_1}\Delta_2^{\Delta_2}\Delta_3^{\Delta_3}} \frac{1}{|x_{12}|^{2\alpha_3}|x_{23}|^{2\alpha_1}|x_{31}|^{2\alpha_2}}\,\mathcal{G}_{123}$$

$$\Sigma = \tfrac{1}{2}(\Delta_1 + \Delta_2 + \Delta_3)$$

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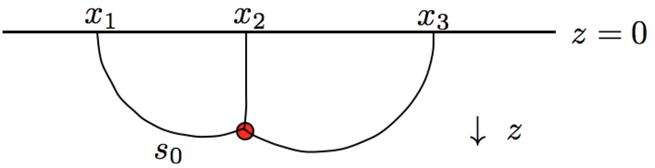

$$\mathcal{C}_{123} = \frac{\pi^{\frac{2-d}{4}}}{4} \frac{(\Delta_1\Delta_2\Delta_3)^{d/4}}{(\alpha_1\alpha_2\alpha_3\Sigma^{d+1})^{1/2}} \frac{\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\alpha_3^{\alpha_3}\Sigma^\Sigma}{\Delta_1^{\Delta_1}\Delta_2^{\Delta_2}\Delta_3^{\Delta_3}} \mathcal{G}_{123}$$

$$\approx \frac{2^{3/2}\,\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)\Gamma(\Sigma - d/2)}{\pi^{d/4}[\Gamma(\Delta_1)\Gamma(\Delta_2)\Gamma(\Delta_3)\Gamma(\Delta_1 + \frac{2-d}{2})\Gamma(\Delta_2 + \frac{2-d}{2})\Gamma(\Delta_3 + \frac{2-d}{2})]^{1/2}} \mathcal{G}_{123}$$

Freedman-Mathur-Matusis-Rastelli (1998)

$$\mathcal{G}_{123} = \mathcal{V}_{123}\,\langle \psi_{J_1}\psi_{J_2}\psi_{J_3}\rangle$$

$$\mathcal{C}_{123} = \mathcal{V}_{123} \times \ (AdS_5 \times S^5 \text{ Overlaps})$$

String vertex operators: Strategy

- Let $\Delta_i \gg 1$.
- States are wave-packets with wavelength $\sim \Delta^{-1}$, spread $\sim \Delta^{-1/2}$
- $\Rightarrow$ Treat as plane-waves in the intersection region Polchinski (1999)
- Momentum: $k_{Mi} = (\Pi_{\mu i}, \Pi_{zi}; \vec{J}_i)$, $M = 0 \ldots 9$
- Flat-space factors of $(2\pi)^{10}\delta^{10}(k_1+k_2+k_3)$ replaced with $AdS_5 \times S^5$ overlaps.
- Use level 1 (0) flat-space vertex operators for Konishi (chiral primaries).
  - $k^2 = -\Delta^2 + J^2 = -4n/\alpha' = -4n\sqrt{\lambda}$
  - $\vec{J}$ can be set to 0 for level 1, but not for level 0.
- The coupling factor uses the string result

$$\mathcal{V}_{123} = \frac{8\pi}{g_c^2 \alpha'} \langle V(k_1) V(k_2) V(k_3) \rangle$$

Polchinski, *String Theory, Vol 1, 2.*

$g_c = \pi^{3/2} N^{-1}$ in AdS/CFT dictionary

Which vertex operators?

$\mathcal{N}=4$ superconformal algebra in manifest $SO(2,4)$ form:

$$M_{\mu\nu}\,,\quad M_{-1\mu}\equiv\frac{1}{\sqrt{2}}(P_\mu-K_\mu)\qquad M_{4\mu}\equiv\frac{1}{\sqrt{2}}(P_\mu+K_\mu)\qquad M_{-14}\equiv -D\,,$$

$$Q^1_{\dot a a}\equiv(Q_{\alpha a},\tilde S_{\dot\alpha a})\,,\quad Q^{2\dot a a}\equiv(\epsilon^{\alpha\beta}S^a_\beta,\epsilon^{\dot\alpha\dot\beta}\tilde Q^a_{\dot\beta})\quad \alpha,\dot\alpha=1,2;\ \dot a=1\dots4$$

$$\{Q^1_{\dot a\,a},Q^{2\,\dot b\,b}\}=\frac{1}{2}\delta_a{}^b M_{mn}\gamma^{mn}{}_{\dot a}{}^{\dot b}-\frac{i}{2}\delta_{\dot a}{}^{\dot b}R_{IJ}\gamma^{IJ}{}_a{}^b\,,\qquad m,n=-1,\dots4$$

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Define:

$$Q^L_A\equiv Q^1_{\dot{a}a}+\gamma^{-1}_{\dot{b}\dot{a}}\gamma^6_{ba}Q^{2\dot{b}b}\,,\ \ Q^R_A\equiv i(Q^1_{\dot{a}a}-\gamma^{-1}_{\dot{b}\dot{a}}\gamma^6_{ba}Q^{2\dot{b}b})\,,\ \ P_m\equiv M_{-1,m}\,,\ \ P_J\equiv R_{J6}$$

$$\Rightarrow \qquad \{Q^{L,R}_A,Q^{L,R}_B\}=2\Gamma^M_{AB}P_M+\ldots \quad \{Q^L_A,Q^R_B\}=0$$

10d $\mathcal{N}=2$ Super-Poincaré algebra $\qquad A,B=1\ldots16,\ \ M=0\ldots9$

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10d $\mathcal{N}=2$ Super-Poincaré algebra $\qquad A,B=1\ldots 16,\quad M=0\ldots 9$

Primary Operator: $K^\mu\mathcal{O}(0)=0\ \Rightarrow\ S^b_\alpha\mathcal{O}(0)=\widetilde S_{\dot\alpha\,b}\mathcal{O}(0)=0$

Flat-space: $Q^L=\pm i\,Q^R$ (sign depends on component)

String vertex operators

$\Rightarrow$ Mixing of NS-NS and R-R modes:

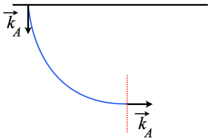
$$Q^L(|NS\rangle \otimes |NS\rangle + |R\rangle \otimes |R\rangle) = |R\rangle \otimes |NS\rangle + |NS\rangle \otimes R\rangle$$

$$Q^R(|NS\rangle \otimes |NS\rangle + |R\rangle \otimes |R\rangle) = |NS\rangle \otimes |R\rangle + |R\rangle \otimes NS\rangle$$

Setting $Q^L = \pm i\, Q^R$ requires a mixture of both sets of fields

String vertex operators

Choosing components:



- Boundary: $k = (\vec{0}, i\Delta; \vec{J}) \Rightarrow Q^L_{\alpha a} = +i\, Q^R_{\alpha a}, \quad Q^{L\,a}_{\dot\alpha} = -i\, Q^{R\,a}_{\dot\alpha}$
  $\alpha, \dot\alpha$ are 4-d space-time spinors. a are $\perp$ $SO(6)$ spinor indices
- Bulk: $k = (\vec{k}_A; \vec{J}) \Rightarrow Q^L_{\alpha' a'} = +i\, Q^R_{\alpha' a'}, \; Q^{L\,a'}_{\dot\alpha'} = -i\, Q^{R\,a'}_{\alpha}$
  $\alpha', \dot\alpha'$ are spinors in 4-d space $\perp$ to k_A. a' are $\perp$ $SO(6)$ spinor indices

String vertex operators: Massless example

- First consider a "twisted" version: set $Q_L = i\, Q_R$ for all spin comps. Then untwist by rotating the righthand part of the state: $T = exp(i\pi(M_{0'1'} + M_{2'3'})_R)$
- Chiral primaries:$\Leftrightarrow$ Massless vertex ops. Friedan-Martinec-Shenker (1985)
  - $k^M = (\vec{\Delta}; \vec{J}) \quad \Rightarrow \quad k^2 = 0$
  - NS-NS : $W_1 = g_c \varepsilon_{MN} \psi^M \tilde{\psi}^N e^{-\phi-\tilde{\phi}} e^{ik\cdot X}, \quad k^M \varepsilon_{MN} = 0$
  - R-R : $W_2 = g_c \left(\frac{\alpha'}{2}\right)^{1/2} t^{AB} \Theta_A \tilde{\Theta}_B e^{-\frac{1}{2}\phi - \frac{1}{2}\tilde{\phi}} e^{ik\cdot X}, \quad t\, \not{k} = 0$
- Only solution to $Q_L = i\, Q_R$: $\varepsilon^T_{MN} = \eta_{MN} - \frac{k_M \bar{k}_N}{k\cdot\bar{k}}, \qquad t^{AB}_T = (\mathcal{C}\not{k})^{AB}$
  - Mix of dilaton and axion; *descendant* of the chiral primary (LMRS)

String vertex operators: Massless example

- Untwist: dilaton $\to$ graviton, axion $\to$ self-dual tensor
- Normalized vertex: $W(k) = -\frac{1}{4}(W_1(k) + \frac{1}{\sqrt{2}} W_2(k))$
- Amplitude: $\mathcal{V}_{123} = \frac{8\pi}{g_c^2 \alpha'} \langle W(k_1) W(k_2) W(k_3) \rangle = 8\pi g_c \frac{\alpha_1 \alpha_2 \alpha_3 \Sigma^5}{J_1^2 J_2^2 J_3^2}$
- Using AdS/CFT dictionary and overlap integrals:

$$\mathcal{C}_{123} \approx \frac{1}{N} (J_1 J_2 J_3)^{1/2} \frac{\alpha_1^{\alpha_1} \alpha_2^{\alpha_2} \alpha_3^{\alpha_3} \Sigma^{\Sigma}}{J_1^{J_1} J_2^{J_2} J_3^{J_3}}$$

✓ agrees with LMRS

String vertex ops: Level one (Untwisted)

- Relevant vert. ops. can be found in 1980's literature
  - (FMS (1985); Kostelecky-Lechtenfeld-Lerche-Samuel-Watamura (1987); Koh, Troost, van Proeyen (1987))
- $Q_L = i\,Q_R$ requires two types of NS-NS and one R-R

$$
\begin{aligned}
V_{1T}(k) &= g_c \left(\tfrac{2}{\alpha'}\right) \sigma_{MN;\tilde{M}\tilde{N}}\, \psi^M(z)\partial X^N \tilde{\psi}^{\tilde{M}}(\bar{z})\bar{\partial} X^{\tilde{N}}\, e^{-\phi-\tilde{\phi}} e^{ik\cdot X}\,, \\
V_{2T}(k) &= g_c\, \alpha_{MNL;\tilde{M}\tilde{N}\tilde{L}}\, \psi^M(z)\psi^N(z)\psi^L(z)\, \psi^{\tilde{M}}(\bar{z})\psi^{\tilde{N}}(\bar{z})\psi^{\tilde{L}}(\bar{z})\, e^{-\phi-\tilde{\phi}} e^{ik\cdot X}\,. \\
V_{3T}(k) &= g_c \left(\tfrac{2}{\alpha'}\right)^{1/2} \left[i\,\bar{\partial} X^M \tilde{\Theta} - \left(\tfrac{\alpha'}{16}\right) \tilde{\psi}^M \not{k} \tilde{\not{\psi}} \tilde{\Theta} \right] e^{-\tilde{\phi}/2} \\
&\quad \times C\not{k}(\hat{\eta}_{MN} - \tfrac{1}{9}\hat{\Gamma}_M \hat{\Gamma}_N) \left[i\,\partial X^N \Theta - \left(\tfrac{\alpha'}{16}\right) \psi^N \not{k} \not{\psi} \Theta \right] e^{-\phi/2} e^{ikX}
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_{MN;\tilde{M}\tilde{N}} &= \tfrac{1}{2}(\hat{\eta}_{M\tilde{M}}\hat{\eta}_{N\tilde{N}} + \hat{\eta}_{M\tilde{N}}\hat{\eta}_{N\tilde{M}}) - \tfrac{1}{9}\,\hat{\eta}_{MN}\hat{\eta}_{\tilde{M}\tilde{N}} \\
\alpha_{MNL;\tilde{M}\tilde{N}\tilde{L}} &= \tfrac{1}{3!}(\hat{\eta}_{M\tilde{M}}\hat{\eta}_{N\tilde{N}}\hat{\eta}_{L\tilde{L}} - \text{ perms})
\end{aligned}
$$

$$
\hat{\eta}_{MN} \equiv \eta_{MN} - \frac{k_M k_N}{k^2} \qquad \hat{\Gamma}^M = \Gamma^M - \not{k} k^M / k^2
$$

Results for three Konishi operators

- Untwist
- $\Rightarrow$ Normalized vertex: $V(k) = \frac{1}{16}\left(V_1(k) + V_2(k) + \frac{1}{\sqrt{2}} V_3(k)\right)$
- Compute $\langle V(k_1)V(k_2)V(k_3)\rangle$
  - $k_i = (\Delta; 0)$, $\Delta = 2\lambda^{1/4} - 2 + \dots$
  - Various combinations:
    $\langle V_1(k_1)V_1(k_2)V_1(k_3)\rangle$, $\langle V_1(k_1)V_1(k_2)V_2(k_3)\rangle$,
    $\langle V_1(k_1)V_3(k_2)V_3(k_3)\rangle$, $\langle V_2(k_1)V_3(k_2)V_3(k_3)\rangle$, *etc.*
- Nasty combinatorics: $\langle V_2(k_1)V_3(k_2)V_3(k_3)\rangle$ is especially horrific.

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- Nasty combinatorics: $\langle V_2(k_1)V_3(k_2)V_3(k_3)\rangle$ is especially horrific.
- But big simplification: $\langle V(k_1)V(k_2)V(k_3)\rangle = g_c^3\frac{3^8}{2^9}$
- $\Rightarrow \mathcal{C}_{123} \approx \frac{1}{N}(4\cdot 3^5\pi)^{1/2}\lambda^{1/4}\left(\frac{3}{4}\right)^{3\lambda^{1/4}}$
  - Explicit λ dependence
  - Suppression for large λ

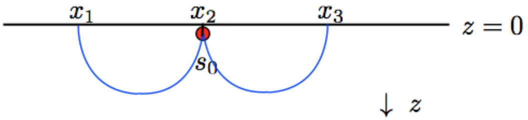
Two chiral primaries and a Konishi

- Two chiral primaries with R-charge $+J$ and $-J$

$$\mathcal{C}_{123} \approx \frac{1}{N} \frac{\sqrt{\pi}}{4\sqrt{\lambda}} 2^{-\Delta} J^{2(1-J)} (J - \tfrac{1}{2}\Delta)^{J-\Delta/2-1/2} (J + \tfrac{1}{2}\Delta)^{J+\Delta/2+3/2}$$

- Extremal limit: Intersection point approaches the boundary

Singular as $J \to_+ \Delta/2$



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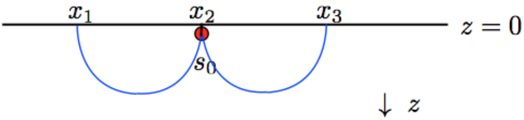
Singular as $J \to_+ \Delta/2$



- Analyze more closely: Use exact FMMR result

$$\begin{aligned}\mathcal{C}_{123} &= \frac{\sqrt{(\Delta_1-1)(\Delta_2-1)(\Delta_3-1)}}{2^{5/2}\pi} \frac{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)\Gamma(\Sigma-2)}{\Gamma(\Delta_1)\Gamma(\Delta_2)\Gamma(\Delta_3)} \mathcal{G}_{123} \\ &\approx \frac{16}{N}\lambda^{3/8} \frac{1}{2J-\Delta} \qquad \text{as } \alpha_3 = 2J-\Delta \to 0\end{aligned}$$

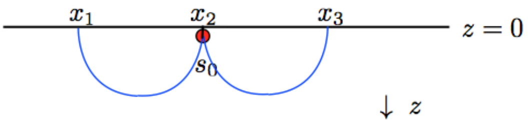
Two chiral primaries and a Konishi

- Two chiral primaries with R-charge $+J$ and $-J$

$$\mathcal{C}_{123} \approx \frac{1}{N} \frac{\sqrt{\pi}}{4\sqrt{\lambda}} 2^{-\Delta} J^{2(1-J)} (J - \tfrac{1}{2}\Delta)^{J-\Delta/2-1/2} (J + \tfrac{1}{2}\Delta)^{J+\Delta/2+3/2}$$

- Extremal limit: Intersection point approaches the boundary

  Singular as $J \to_+ \Delta/2$

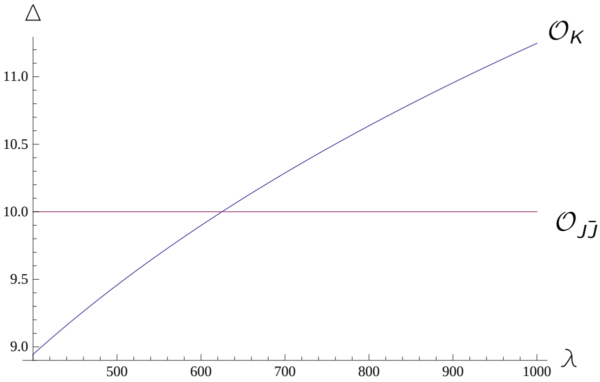


- Analyze more closely: Use exact FMMR result

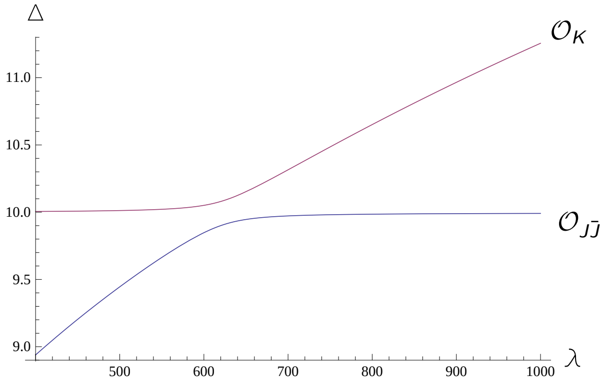
$$\begin{aligned}\mathcal{C}_{123} &= \frac{\sqrt{(\Delta_1-1)(\Delta_2-1)(\Delta_3-1)}}{2^{5/2}\pi} \frac{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)\Gamma(\Sigma-2)}{\Gamma(\Delta_1)\Gamma(\Delta_2)\Gamma(\Delta_3)} \mathcal{G}_{123} \\ &\approx \frac{16}{N}\lambda^{3/8}\frac{1}{2J-\Delta} \qquad \text{as } \alpha_3 = 2J - \Delta \to 0\end{aligned}$$

- No pole for 3 chiral primaries LMRS, D'Hoker-FMMR (1999)
- Pole indicates mixing of $\mathcal{O}_\Delta$ with double trace op. $\mathcal{O}_{J\bar{J}} =: \mathcal{O}_J \mathcal{O}_{\bar{J}}:$

Splitting at the crossover

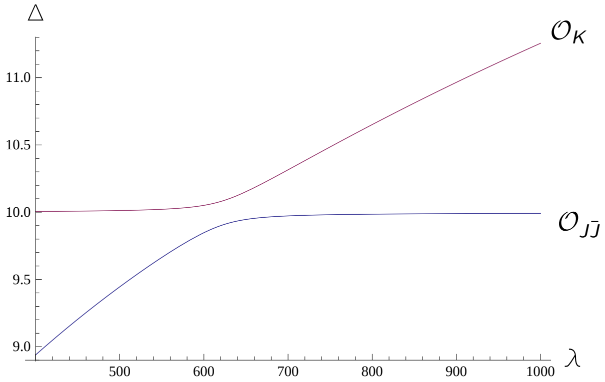


Splitting at the crossover



Splitting: $\delta\Delta = \frac{16}{N}\sqrt{M}\,\lambda^{3/8}$

Splitting at the crossover



Splitting: $\delta\Delta = \frac{16}{N}\sqrt{M}\,\lambda^{3/8}$

Interesting to compare with recent bootstrap results

Beem-Rastelli-van Rees (*to appear*)

Discussion

- There has been much progress on 3-point correlators at low loop orders using integrability: Escobedo-Gromov-Sever-Vieira (2010,2011), Gromov-Sever-Vieira (2011), Georgiou (2011), Bissi-Harmaark-Orselli (2011), Gromov-Vieira (2011,2012), Kostov (2012, 2012), Serban (2012), Grignani-Zayakin (2012), Plefka-Wiegant (2012), Bissi-Grignani-Zayakin (2012), Foda-Jiang-Kostov-Serban (2013), Jiang-Kostov-Loebbert-Serban (2014), Caetano-Fleury (2014)
- We can also do 3-point correlators containing an operator with nonzero spin. Compares favorably with recent results using Mellin amplitudes on Regge trajectories Costa-Goncalves-Penedones (2012)
- Many possible generalizations:
  - More massive operators at $n = 2$ or higher.
  - Can study four-point correlators and duality of operator products.
- The simplifications suggest an underlying symmetry playing an important role.
- Perhaps these results can help lead us to the exact vertex operators in $AdS_5 \times S^5$.