NON-PERTURBATIVE EFFECTS IN M-THEORY

Marcos Mariño
University of Geneva

[Hatsuda-M.M.-Moriyama-Okuyama, 1306.1734]
[Johan Kallen-M.M., 1308.6485]
[Alba Grassi-M.M.,1403.4276]

+ recent papers by Hatsuda, Honda, Matsumoto, Moriyama, Okuyama

Non-perturbative effects and AdS/CFT

The large *N* or ‘t Hooft expansion provides an asymptotic expansion of observables in gauge theories:

free energy

$$F(\lambda, N) = \sum_{g \geq 0} N^{2-2g} F_g(\lambda)$$

$$\lambda = g^2 N$$

‘t Hooft coupling

However, there might be exponentially small corrections which are *invisible* in the ‘t Hooft expansion

$$\sim \exp\left(-N S(\lambda)\right)$$

If the large *N* theory has a string dual, these corrections correspond to non-perturbative effects in the *string coupling constant*

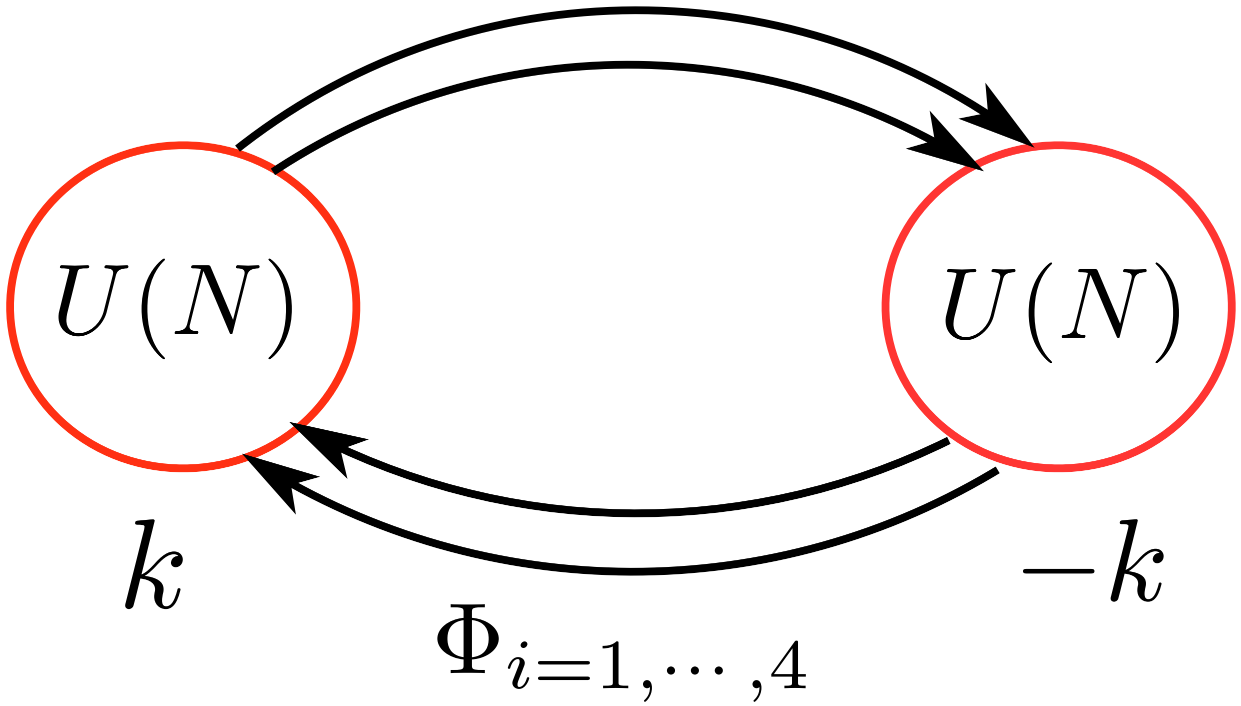
We also expect that, in type IIA string theory, these non-perturbative effects will appear naturally when we go to M-theory.

In this talk I will present some *exact results* for these effects in ABJM theory, which has a type IIA/M-theory dual.

In particular, I will describe an exact expression for the ABJM partition function on the three-sphere, in the M-theory regime, which *resums* the type IIA genus expansion and incorporates *membrane instanton effects*.

This clarifies, in a precise quantitative way, the relationship between type IIA and M-theory, and shows that a theory based only on fundamental strings is fundamentally *incomplete*

ABJM theory & localization



N=6, *3d* SCFT with 't Hooft parameter

$$\lambda = \frac{N}{k}$$

k= CS level

We will focus on the *free energy on the three-sphere*

$$F(N,k) = \log Z(N,k) = \sum_{g=0}^{\infty} F_g(\lambda) N^{2-2g}$$

By using localization techniques [Kapustin-Willett-Yaakov] showed that *Z(N,k)* reduces to a matrix integral:

$$Z(N,k) = \frac{1}{N!} \int \prod_{i=1}^{N} \frac{\mathrm{d}x_i}{8\pi k} \frac{1}{\cosh \frac{x_i}{2}} \prod_{i<j} \left(\tanh \left(\frac{x_i - x_j}{2k} \right) \right)^2$$

‘t Hooft expansion

The free energy can be studied in two different regimes:

N large, λ fixed → ‘t Hooft/genus expansion

[cf. my talk at Strings2011]

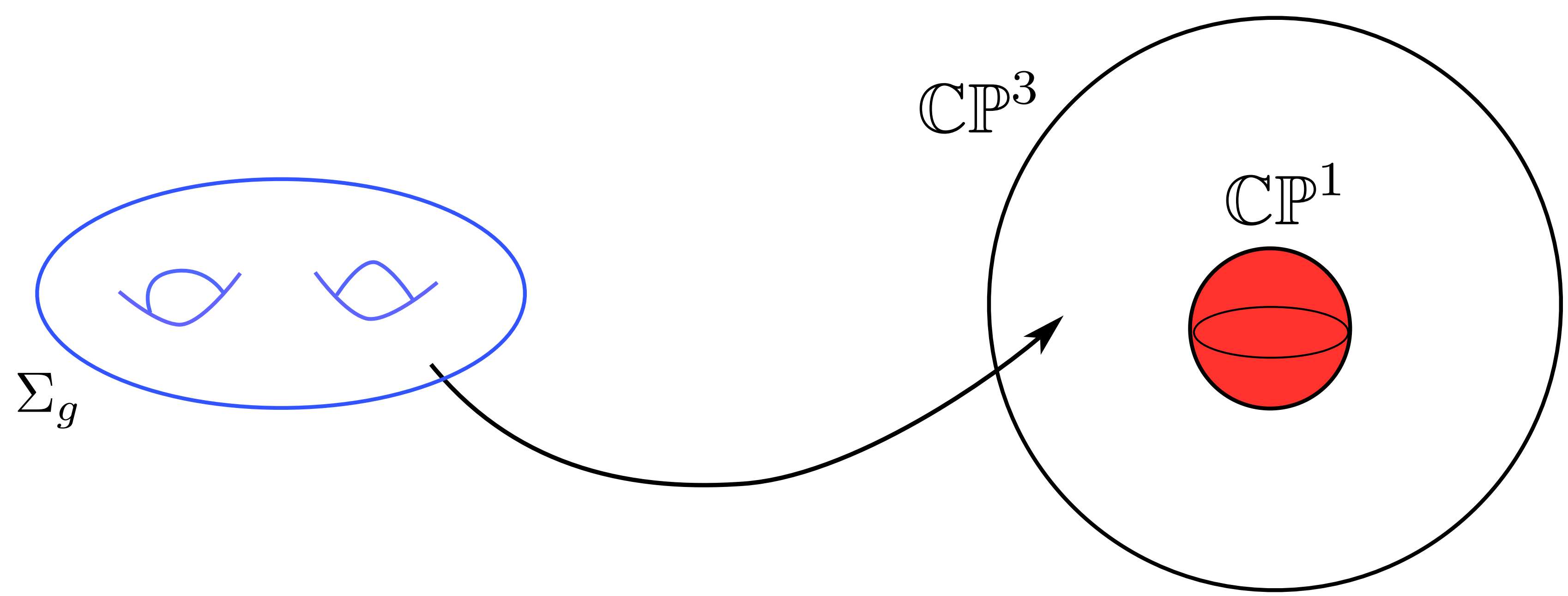
dual to type IIA on $\mathrm{AdS}_4 \times \mathbb{CP}^3$

$$(L/\ell_s)^4 \sim \lambda$$

$$g_{\rm st} \sim 1/k$$

The genus g free energies $F_g(\lambda)$ can be computed for arbitrary g with large *N* techniques. They are determined by *topological string theory* on a non-compact CY (local P1xP1)

They encode *worldsheet instanton* corrections in type IIA



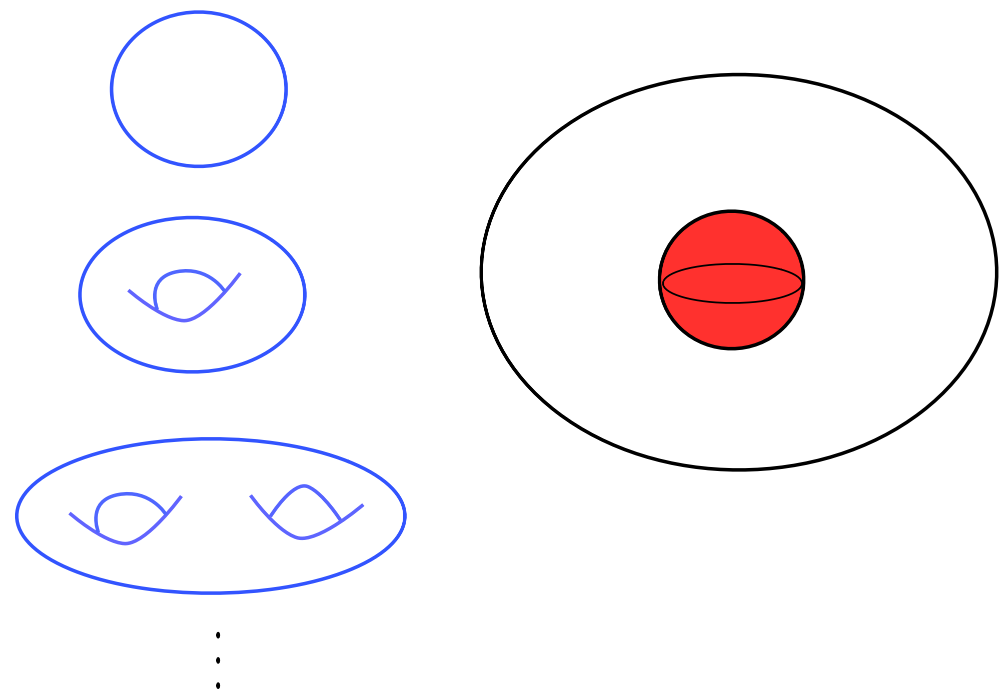
M-theory expansion

N large, k fixed	→	M-theory expansion/ "thermodynamic" limit

dual to M-theory on $\mathrm{AdS}_4 \times \mathbb{S}^7/\mathbb{Z}_k$

$$(L/\ell_p)^6 \sim kN$$

To go from type IIA to M-theory: *resum* the genus expansion at fixed, strong 't Hooft coupling -it can be done! (Gopakumar-Vafa resummation)



$$F_{\text{'t Hooft}}(N,k)=\sum_{\ell\geq 1} c_\ell(N,k)\mathrm{e}^{-2\pi\ell\sqrt{2N/k}}$$

they have *poles* for all rational *k*!

Beyond the ‘t Hooft expansion

This indicates that the ‘t Hooft/genus expansion is *incomplete*. We need a *new treatment of the matrix model* which gives the missing information in the ’t Hooft expansion.

In the *Fermi gas approach* [M.M.-Putrov], the matrix integral for $Z(N,k)$ is interpreted as the canonical partition function of an one-dimensional *ideal* Fermi gas of N particles

The energy levels are determined by the spectral problem (here *T=1*):

density matrix

$$\hat{\rho}|\psi_n\rangle = \mathrm{e}^{-E_n}|\psi_n\rangle \quad n = 1, 2, \cdots$$

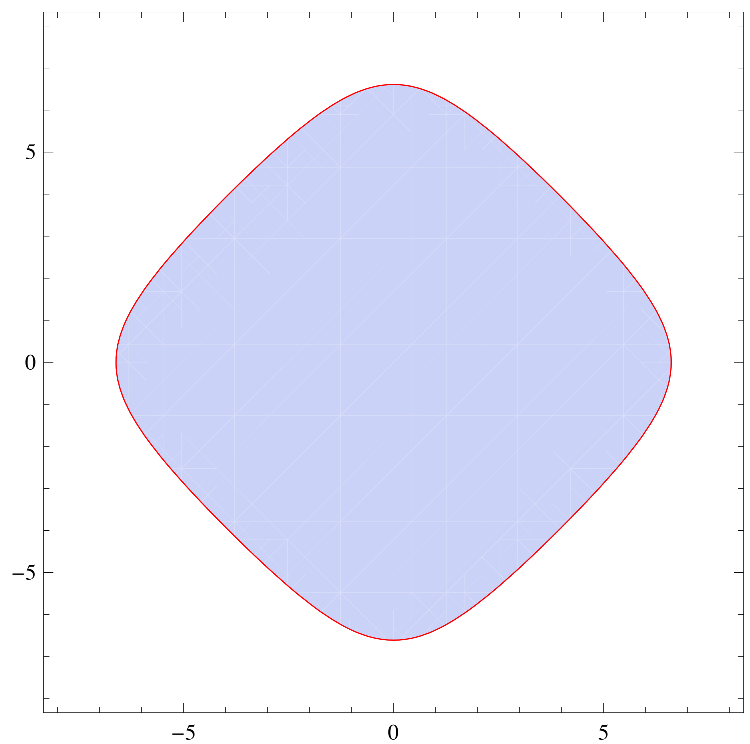
$$\rho(x_1, x_2) = \frac{1}{2\pi k} \frac{1}{\left(2\cosh\frac{x_1}{2}\right)^{1/2}} \frac{1}{\left(2\cosh\frac{x_2}{2}\right)^{1/2}} \frac{1}{2\cosh\left(\frac{x_1 - x_2}{2k}\right)}$$

k plays the role of *Planck's constant*: $\hbar = 2\pi k$

Semiclassical regime $\longleftrightarrow$ strong string coupling

This is an unconventional spectral problem (integral rather than differential equation). However, for large energies we have a gas of *N* fermions with Hamiltonian

$$H(q,p) \approx \frac{|p|}{2} + \frac{|q|}{2}$$



$H(q,p) = E$

Fermi surface has volume

$$\mathrm{vol}(E) \approx 8E^2, \;\; E \gg 1$$

$\longrightarrow N^{3/2}$ behavior of *F(N,k)*

To encode the information on the quantum ideal gas, we can use the *quantum-corrected phase-space volume*:

$$\mathrm{vol}(E;k)$$

spectrum:

$$\mathrm{vol}(E;k) = 4\pi^2 k \left(n + \frac{1}{2}\right)$$

thermodynamics:

$$\frac{\mathrm{d}}{\mathrm{d}E}\mathrm{vol}(E;k) \propto \rho(E;k)$$

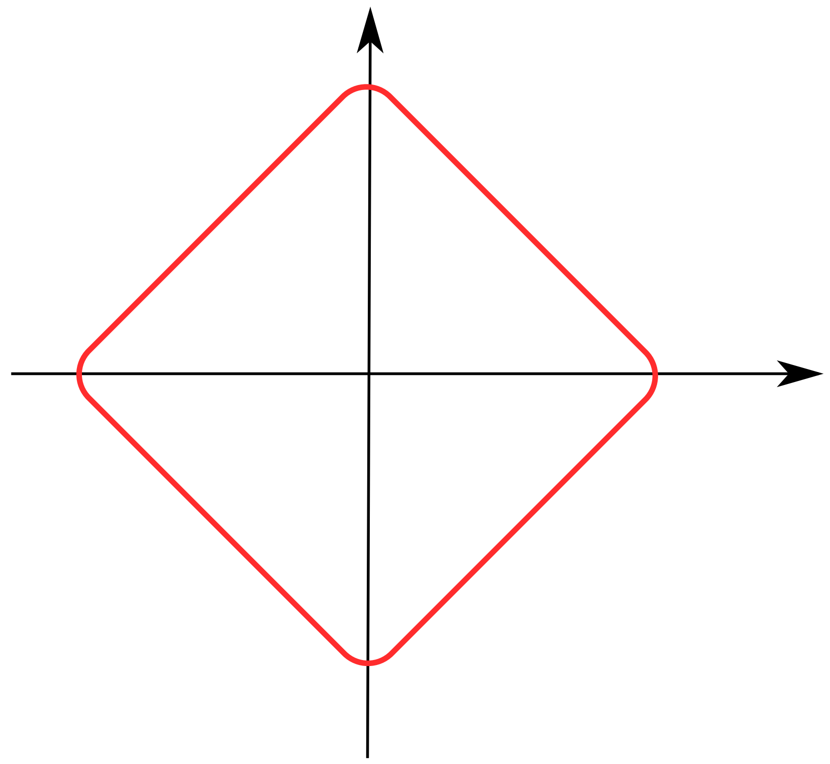
→ grand potential $J(\mu, k)$

→ $Z(N, k)$

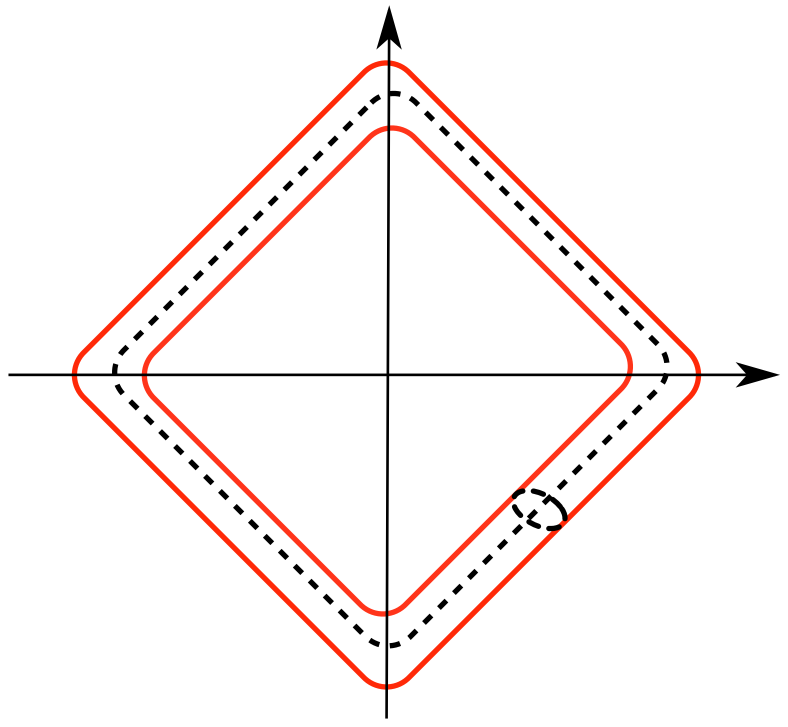
In this problem, large N corresponds to large E, *i.e.* to large quantum numbers

This suggests using the WKB expansion to calculate the quantum volume. We should work however at *large E* but *fixed* Planck constant. Non-perturbative effects in N in $F(N,k)$ come from *exponentially small corrections in E* to $vol(E;k)$

perturbative WKB:
real trajectories +
quantum fluctuations



non-perturbative WKB:
complex trajectories/
instantons
[cf. Balian-Parisi-Voros]



$$\mathrm{vol}_{\mathrm{np}}(E;k) = \sum_{\ell \geq 1} s_\ell(k)\, \mathrm{e}^{-4\ell E/k}$$

$$s_1(k) = 8\pi k \cot\left(\frac{2\pi}{k}\right)$$

1) It has *poles* for all rational values of *k*

2) $$\mathrm{e}^{-4E/k} \longrightarrow \mathrm{e}^{-\sqrt{N/k}} \sim \mathrm{e}^{-(L/\ell_s)^2}$$

worldsheet instantons $\longleftrightarrow$ (resummed) 't Hooft expansion

3) It is determined by the *conventional topological string on local P1xP1*

$$\mathrm{vol_p}(E;k) = 8E^2 + \sum_{\ell \geq 1} \left(E a_\ell(k) + b_\ell(k) \right) \mathrm{e}^{-2\ell E}$$

$$b_1(k) = \frac{2}{\pi} \cos^2\left(\frac{\pi k}{2}\right) \csc\left(\frac{\pi k}{2}\right)$$

1) It has *poles* for all rational values of *k*

2) $\mathrm{e}^{-2E} \longrightarrow \mathrm{e}^{-\sqrt{kN}} \sim \mathrm{e}^{-(L/\ell_p)^3}$

membrane instantons!

3) It is a "quantum period" of the complexified Fermi surface. It is determined by the *refined topological string on local P1xP1, in the NS limit*

The sum of perturbative and non-perturbative quantum volumes

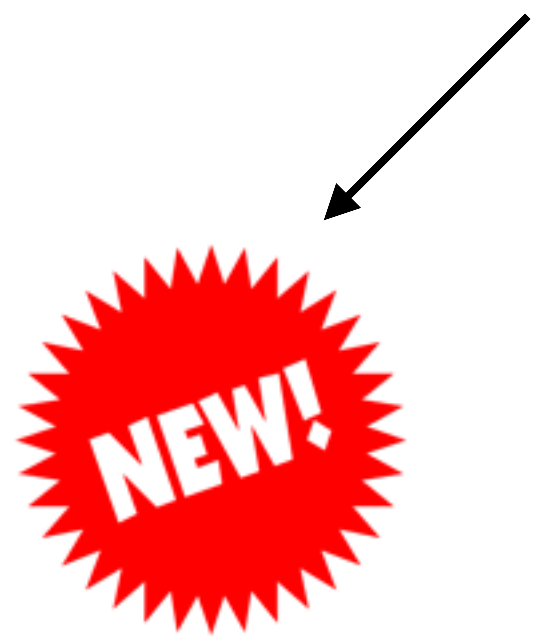
$$\mathrm{vol}(E;k) = \mathrm{vol}_{\mathrm{np}}(E;k) + \mathrm{vol}_{\mathrm{p}}(E;k)$$

has *no* poles and determines the spectrum through an *exact quantization condition* [cf. Zinn-Justin]

$$\frac{\mathrm{vol}(E;k=1)}{4\pi^2} = \frac{2}{\pi^2}E^2 - \frac{7}{24} + \frac{8}{\pi^2}\mathrm{e}^{-4E} + \frac{1}{\pi^2}\mathrm{e}^{-4E} - \frac{52}{\pi^2}E\mathrm{e}^{-8E} - \frac{1}{4\pi^2}\mathrm{e}^{-8E} + \frac{1472}{3\pi^2}E\mathrm{e}^{-12E} - \frac{152}{9\pi^2}\mathrm{e}^{-12E} + \mathcal{O}(E\mathrm{e}^{-16E})$$

From the total quantum volume, by standard StatMech, we obtain the *exact, singularity-free* ABJM free energy in the M-theory expansion. It includes the (resummed) ’t Hooft expansion *and* the contribution from membranes. Schematically,

$$F(N,k) = F_{\text{'t Hooft}}(N,k) + F_{\text{M2}}(N,k)$$



Consequences

(1) HMO mechanism [Hatsuda-Moriyama-Okuyama]): In the ABJM partition function, singularities in the contribution of fundamental strings cancel against the singularities in the contributions of membranes

(2): A large *N* dual based only on fundamental strings is inconsistent, since it leads to unphysical poles in the partition function

(3): The ‘t Hooft expansion is radically insufficient

(4): The genus expansion of the free energy is an asymptotic, divergent expansion. The M-theory expansion leads to a *convergent* expansion for N large and fixed k

(5): In the final answer for the ABJM free energy, topological string amplitudes and refined topological string amplitudes complete each other non-perturbatively. They are unified into a well-defined spectral problem

Generalizations

The above phenomena are *generic* for *N=3* Chern-Simons-matter (CSM) theories. In particular, two models have been studied in detail:

1) ABJ theory [Matsumoto-Moriyama, Honda-Okuyama, Kallen]

2) N_f matrix model: *N=4* SYM in 3d+adjoint hyper+ N_f fundamental hypers
[Mezei-Pufu,Grassi-M.M.,Hatsuda-Okuyama]

- In general $\mathcal{N}=3$ CSM theories, the M-theory expansion of the matrix model/partition function can be encoded in a spectral problem. It involves membrane instantons which are invisible in the 't Hooft expansion

- In some cases, the 't Hooft expansion can be resummed and has poles which should be cured by membrane instantons

The connection to topological string theory is an accident of ABJ(M) theory. However, the structures appearing in the analysis of general *N=3* CSM theories seem to be closely related to it [cf. Dabholkar-Drukker-Gomes]

Conclusions

I have presented an exact result for the ABJM partition function, in the M-theory regime, which goes beyond the 't Hooft expansion and incorporates stringy non-perturbative effects. This result shows very explicitly that fundamental strings need to be supplemented by membranes in order to avoid singularities

The resulting structures seem to pervade *N=3* CSM theories.

Two challenges:

1) Develop analytic tools to analyze the spectral problems and matrix models appearing in more general CSM theories

2) These exact results might give precious clues to answer the crucial question: what is M-theory?