The Tadpole Problem

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Work in collaboration with

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Iosif Bena, Callum Brodie

arXiv: 2010.10519

arXiv: 2103.03250

arXiv: 2107.xxxx

Strings, June 2021

Introduction

- Realistic string compactifications should have no massless scalar fields (moduli)

- However, the landscape of Calabi-Yau flux compactifications is populated mostly from CY with lots of moduli

  10^{272000} vacua from CY_4 with $h^{3,1} = 303148$

Ashok, Denef, Douglas 03

Taylor, Wang 15

- These are also the corners that can give rise to interesting phenomenology

  $\longrightarrow$ Possibility of uplifting anti-de Sitter vacua with small c.c.

- Moduli need to be stabilized

  $\longrightarrow$ Can be done with fluxes

Gukov, Vafa, Witten 99

Dasgupta, Rajesh, Sethi 99

Giddings, Kachru, Polchinski 01

- Fluxes induce charges

- How large is the charge induced by fluxes needed to stabilize a given number of moduli?

- Can fluxes that stabilize a large number of moduli have $\mathcal{O}(1)$ induced charge?

  → Common lore: yes

  → We argue: no

- Furthermore: we believe there is a relation between the induced charge and the number of moduli stabilized

The tadpole conjecture

Bena, Blåbäck, M.G., Lüst 20

- For a large number N of moduli

$$\boxed{Q_{\text{flux s.t. all mod stabilized}} > \alpha \, N} \quad \text{at a generic point in mod space}$$

with $\alpha > \frac{1}{3}$

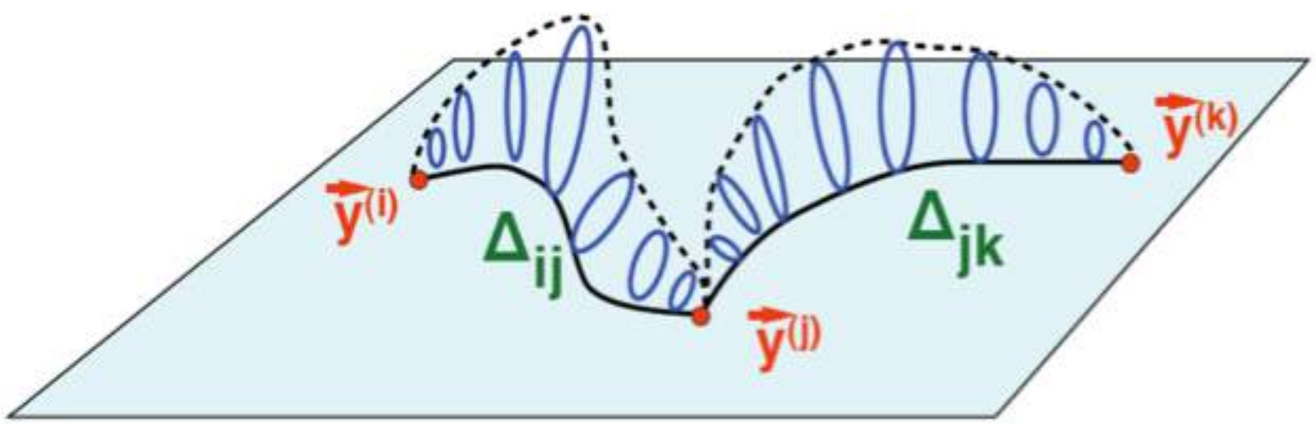
Motivation

- Black-Hole Microstate *Bubbling Geometries*

  -Same mass and charge as a D1-D5-P black hole, but no horizon

  -AdS-CFT: dual to states of D1-D5 CFT. This CFT counts the black-hole entropy Strominger, Vafa 96

2-cycles + magnetic fluxes



Black Hole charge dissolved in fluxes

Two supporting arguments for the Tadpole Conjecture:

"Experimental" observation: Bena, Wang, Warner 06

Charge contributions from cycles add up

Otherwise closed time-like curves

BH entropy $\supset$ number of ways to put susy fluxes on cycles within a given charge

If possible to get this charge with positive and negative individual contributions $\Rightarrow$ $S_{\text{fluxes}} > S_{BH}$

Here: IIB flux Compactifications on Calabi-Yau

$M_{10} = M_4 \times_w CY_3$

- $h^{2,1}$ complex structure moduli (volumes of 3-cycles) $\sim \mathcal{O}(100)$

- Add 3-form fluxes

$$\int_{\alpha_I} F_3 = \underbrace{M^I}_{\in \mathbb{Z}} \qquad \int_{\alpha_I} H_3 = K^I \qquad Q_{\text{flux}} = \int F_3 \wedge H_3 = M^I K_I > 0$$

basis of 3-cycles

$I = 1, ..., 2h^{2,1} + 2$

at minimum

$H_3 = \star F_3$

- Potential for complex structure moduli (and dilaton)

Dasgupta, Rajesh, Sethi 99

Giddings, Kachru, Polchinski 01

$$S \sim \int F_3 \wedge \star F_3 + e^{-2\phi} H_3 \wedge \star H_3$$

depends on complex structure moduli

- Minimum at $e^{-\phi} H_3 = \star F_3$ fixes complex structure moduli in terms of M, K

- Fluxes induce D3-charge. In a compact space total charge should be zero

Tadpole cancelation condition

- Sum charges should be zero

Positive charge

- Fluxes: $Q_{\rm flux} = M^I K_I$

- D3-branes

Negative charge

- O3-planes

(maximum charge from O3-planes 100($-\frac{1}{4}$))

Carta, Moritz, Westphal 20

- D7-branes and O7-planes

wrapped on curved 4-cycles

have moduli associated
stabilized by world-volume flux

- Unified description in F-theory

F-theory on CY_4

$h^{2,1}$ complex structure moduli
D7-brane moduli
} $h^{3,1}$ complex structure moduli of CY_4

3-form fluxes H_3, F_3
2-form fluxes F_2 on D7
} 4-form flux G_4

Tadpole cancelation condition

moduli stabilized by fluxes

$$\boxed{Q_{\text{flux}} = \frac{1}{2}\int \overbrace{G_4 \wedge G_4}^{N^I d_{IJ} N^J}} \leq \boxed{\frac{\chi(CY_4)}{24}} = \frac{1}{4}(h^{3,1} + h^{1,1} - h^{2,1} + 8) \boxed{\sim \frac{1}{4}N}$$

at minimum
$\star G_4 = G_4$
> 0

all the negative
3-charge
from D7/O7

for large $h^{3,1}$

Tadpole conjecture

$$\boxed{\frac{1}{2}\int G_4 \wedge G_4 \Big|_{\text{all moduli are stabilized}}} > \boxed{\frac{1}{3}N}$$

If true, cannot stabilize
a large number of moduli!!

Supporting arguments

Tadpole conjecture $\alpha > \frac{1}{3}$

Description	N	Q_{flux}	$\alpha = Q_{\text{flux}}/N$	Ref
IIB at highly symm pt in mod space	$h^{2,1} = 128$	48	0.38	Giryavets, Kachru, Tripathy, Trivedi 03
	$h^{2,1} = 272$	124	0.46	Demirtas, Kim, Mc Allister, Morritz 19
F-theory on sextic CY	$h^{3,1} = 426$	775/4	0.45	Braun, Valandro 20
F-theory on $\mathbb{CP}^3$ base	$n_7 = 3728$	1638	0.44	Collinucci, Denef Esole 08
F-theory on any weak-Fano base	$n_7 = 58c_1^3(B) + 16$	$\frac{7}{16}(58c_1^3(B) + 15)$	0.44	Bena, Brodie, M.G. 21
M-theory on K3xK3	57	25	0.44	Bena, Blåbäck, M.G., Lust 20

M-theory on K3 x K3

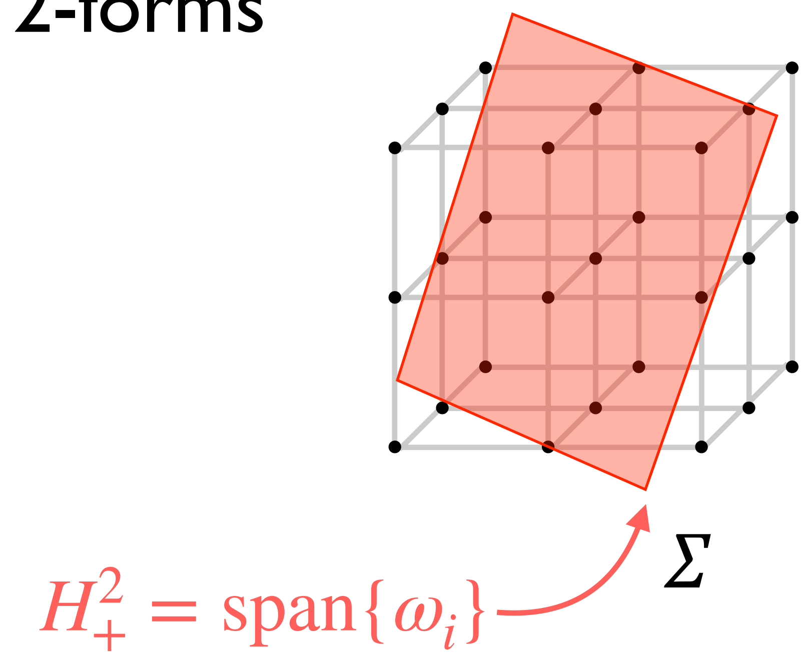
Dasgupta, Rajesh, Sethi 99, Aspinwall Kallosh 05,…

- Fixing moduli on K3: choosing 3-plane Σ of self-dual 2-forms

$$H^2(K3,\mathbb{Z}) = (-E_8)\oplus(-E_8)\oplus U\oplus U\oplus U$$

lattice of signature (3,19)

$$\Omega = \omega_1 + i\omega_2 \qquad J \sim \omega_3$$

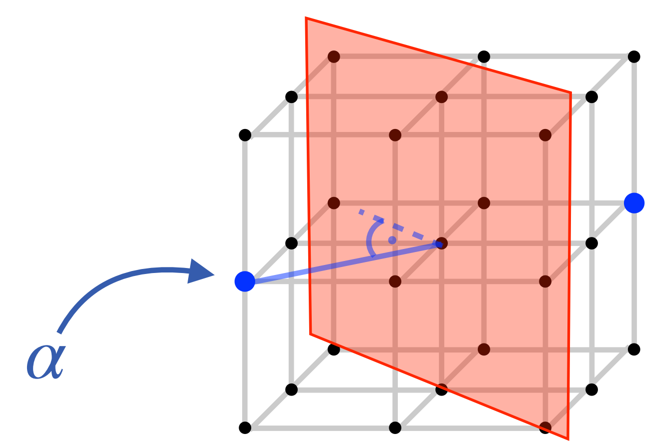


- We require smooth compactification (no orbifold singularity)

orbifold singularity if ∃

root $\alpha \in H^2(K3,\mathbb{Z})$ such that $\alpha \perp \Sigma$

$(\alpha,\alpha) = -2$



- K3 x K3' with 4-form flux

$G_4 \in H^2(K3,\mathbb{Z}) \times H^2(K3',\mathbb{Z})$

basis of $H^2(K3,\mathbb{Z})$ $\quad I = 1, ..., 22$

$$G_4 = N^{I\tilde{J}} \alpha_I \wedge \alpha'_{\tilde{J}}$$

22x22 integer matrix

- Gives a potential for all K3 moduli (except volumes)

- Moduli stabilization can be turned into algebraic problem

Braun, Hebecker, Ludeling, Valandro 08

-Define a map $M \,:\, H^2(K3) \to H^2(K3)$

$d_{IJ} = \int_{K3} \alpha_I \wedge \alpha_J$

$$M^I{}_J = N^{I\tilde{K}} d_{\tilde{K}\tilde{L}} N^{M\tilde{L}} d_{MJ}$$

- All moduli are stabilized at regular points iff

$$M^I{}_J = N^{I\tilde{K}} d_{\tilde{K}\tilde{L}} N^{M\tilde{L}} d_{MJ}$$

(i) M is diagonalizable with non-negative eigenvalues

$$\{\underbrace{a_1^2, a_2^2, a_3^2}_{\substack{\text{eigenvectors with}\\ \text{positive norm}\\ \Sigma}}, \underbrace{b_1^2, ..., b_{19}^2}_{\substack{\text{eigenvectors with}\\ \text{negative norm}}}\}$$

(ii) All $a \neq b$ (otherwise can rotate the 3-plane $\Sigma \Rightarrow$ unstabilized moduli)

(iii) No root in the lattice $\perp \ \Sigma$

- Goal: find N satisfying all three requirements and minimizing the flux charge

$$Q_{\text{flux}} = \frac{1}{2}\int G_4 \wedge G_4 = \frac{1}{2}\text{tr}(M)$$

- Used evolutionary algorithm

Evolutionary algorithm

- Optimization inspired by biological evolution (population, mutation, selection)

- Random initial population: P={ $N \in \mathbb{R}^{\overset{22 \times 22}{484}}$ } (rounded to $\mathbb{Z}$)

- For each N, mutate some entries using other elements of population

- From original and mutated, select the one that minimizes a fitness function

weights (determined empirically)

$$f = \sum_{k=1}^{3} w^k p_k(N) + w^Q Q_{\text{flux}}(N)$$

penalty if (i)-(iii) is violated

penalty for large flux charge

(i) M is diagonalizable $\{a_1^2, a_2^2, a_3^2, b_1^2, \ldots, b_{19}^2\}$

(ii) $a \neq b$

(iii) No root $\perp\ \Sigma$

Note: condition (iii) No root in the lattice $\perp\ \Sigma$ is NP hard problem!

- Perform local search (brute force) around minima

Results

Bena, Blåbäck, M.G., Lüst 20

- 100,989 matrices with $Q_{\text{flux}} = 25$

- No matrix $Q_{\text{flux}} \leq 24$

- Tadpole cancelation condition cannot be satisfied

$$Q_{\text{flux}} \leq \frac{\chi(K3\times K3)}{24} = 24$$

- Cannot stabilize moduli at generic point !

- Tadpole conjecture constant

$$\alpha = \frac{min(Q_{\text{flux}})}{\text{moduli}} = \frac{25}{57} \approx 0.44 > \frac{1}{3}$$

- This behavior confirmed by looking at smaller dimensional lattices

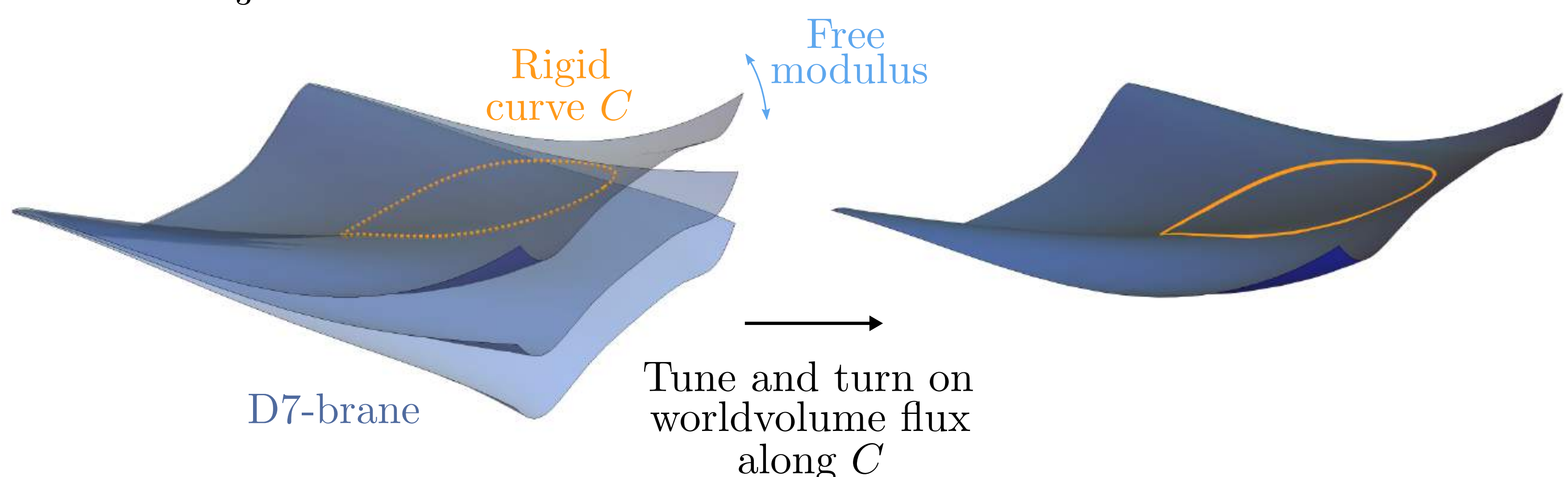
Bena, Blåbäck, M.G., Lüst 21

D7 moduli stabilization

- F-theory on CY 4-fold fibered over a base B₃ in Sen limit

  • D7-brane moduli $$n_7 = 58 \int_{B_3} c_1(B_3)^3 + 16$$

- Stabilized by F_2



Tadpole cancelation condition

$$\boxed{Q_{\text{flux}} = \frac{1}{2}\int_S F_2 \wedge F_2} \leq \boxed{15 \int_{B_3} c_1(B_3)^3 + 12} \sim \frac{15}{58} n_7 \simeq \boxed{0.26\, n_7} \quad \text{for large } n_7$$

negative
3-charge from D7/O7

Tadpole conjecture

$$\boxed{\frac{1}{2}\int_S F \wedge F \,\Big|_{\substack{\text{all D7-moduli}\\\text{are stabilized}}}} > \boxed{\frac{1}{3} n_7}$$

If true, cannot stabilize
a large number of moduli

D7-moduli stabilization

I.Bena, C. Brodie, M.G. 21

- We verified tadpole conjecture for any Weak Fano base !

- Moduli stabilized by flux $F \leftrightarrow C$ complex curve

$C \cdot (-K) = 4d$ for $\mathbb{CP}^3$

$$n_{\text{stab.moduli}} \leq 8\tilde{d} + 1 \qquad\qquad Q_{\text{flux}} \geq \frac{7}{2}\tilde{d} + 1 - g$$

$$\left(n_7 = 58 \int c_1(B_3)^3 + 16\right)$$

- For large n_7 and fixed genus, we recover $\alpha \geq \frac{7}{16} \simeq 0.44 > 0.26$ allowed by tadpole cancelation condition

$> \frac{1}{3}$ Tadpole conjecture

- Moduli cannot be stabilized within tadpole, Tadpole conjecture satisfied

- This reduces to the result for B3= $\mathbb{CP}^3$, genus 0

Collinucci, Denef, Esole 08

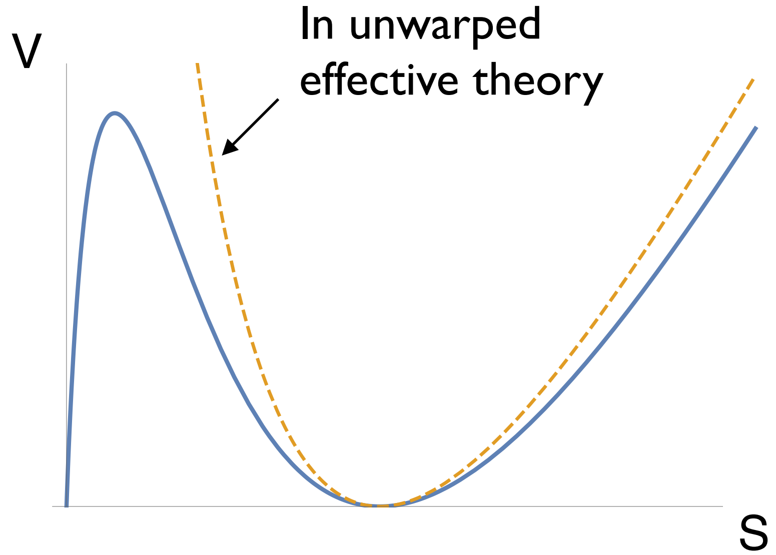
$$n_{\text{stab.moduli}} = 32d + 1 = 3728 \qquad Q_{\text{flux}} \geq 14d + 1 \geq 1640 \qquad |Q_{\text{neg}}| = 972 \qquad \alpha \geq \frac{14}{32} \simeq 0.44$$

Implications for de Sitter with anti-brane uplift

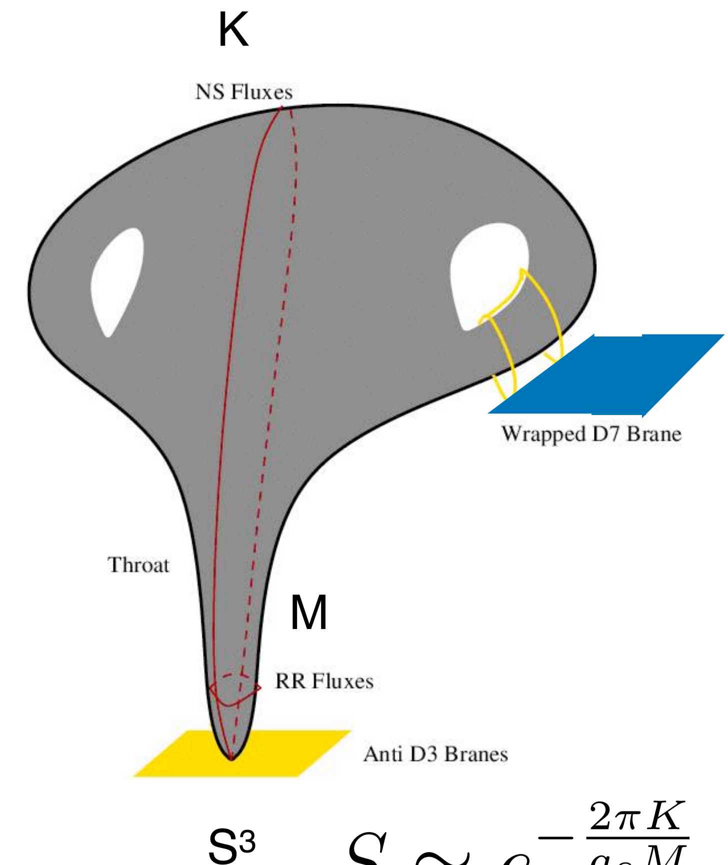
Bena, Dudas, M.G., Lust 18

Moduli stabilization using warped effective field theory for conifold modulus

Douglas, Torroba 08



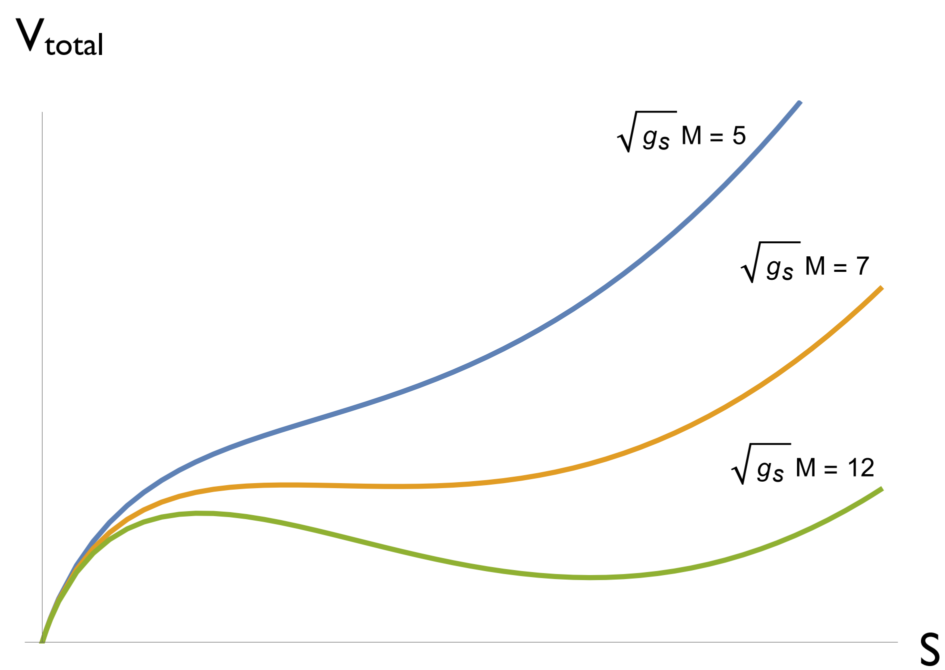
$$ds^2 = e^{2A} ds_4^2 + e^{-2A} ds_{\rm CY}^2$$



$$S \simeq e^{-\frac{2\pi K}{g_s M}}$$

- Add $\overline{\text{D3}}$ wants to collapse the S^3!

Full flux + $\overline{\text{D3}}$ warped potential for size of S^3



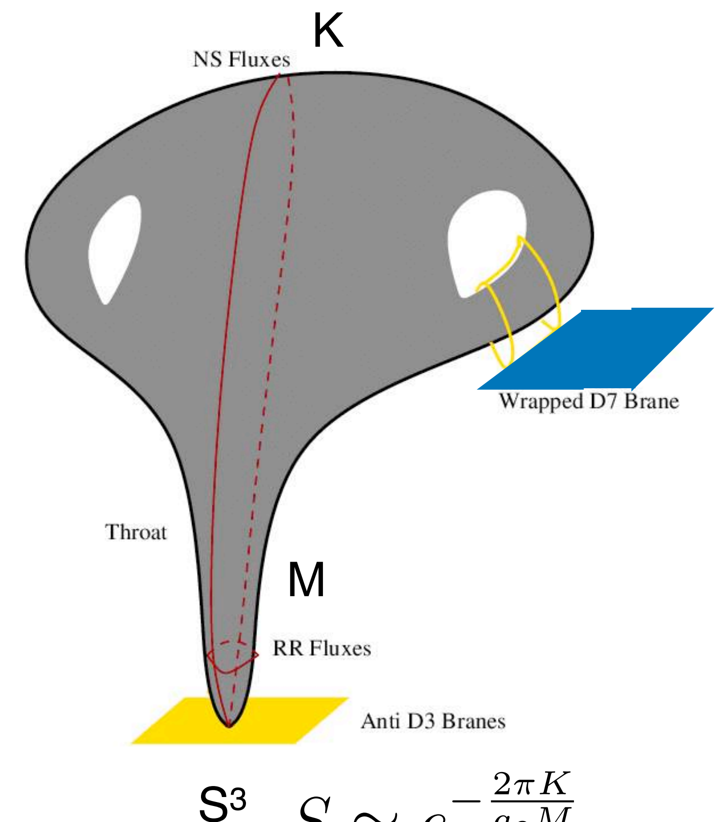
Need $\sqrt{g_s} M \geq 6.7$ to avoid collapse

similar bound obtained from demanding existence of KS black-hole Bena, Buchel, Lust 19

- But then hierarchy:

$$e^A|_{\text{bottom}} = \frac{\Lambda_{IR}}{\Lambda_{UV}} = \exp\left(-\frac{2\pi}{3}\frac{KM}{g_s M^2}\right) > e^{-\frac{2}{3}\pi\frac{Q_{\text{flux}}^{\text{throat}}}{(6.7)^2}} \sim \mathcal{O}(10^{-2})$$

needs $\quad Q_{\text{flux}}^{\text{throat}} \gtrsim \mathcal{O}(100)$



$$S \simeq e^{-\frac{2\pi K}{g_s M}}$$

- Requires a large tadpole charge ⇒ large number of moduli

- Large number of moduli need to be stabilized with extra fluxes

- Cannot be done if tadpole conjecture is true

- No anti-brane uplift, no dS vacua à la KKLT

Conclusions

- Tadpole conjecture: for large number N of moduli

$$Q_{\text{flux}}\ _{\text{s.t. all mod stabilized}} > \alpha\ N \qquad \alpha > \frac{1}{3}$$

- Conjecture supported by several examples, evolutionary algorithm for K3xK3 and analytic computation for D7-moduli

- If true, cannot stabilize a large number of moduli in F-theory (or in IIB limit)

  10^{272000} vacua not phenomenologically relevant

- If true, no anti-brane uplift in long warped throats, no dS vacua à la KKLT

- Analytic proof?

  - in K3 x K3
  - in particular regions in moduli space

STAY TUNED!