Deconfinement Transition As Black Hole Formation By The Condensation Of QCD Strings

Jonathan Maltz[1]
Masanori Hanada[2] and Leonard Susskind[3]

[1]Kavli IPMU - University of Tokyo
[2]Yukawa Institute for Theoretical Physics - University of Kyoto [3]Stanford Institute for Theoretical Physics - Stanford University

Strings 2014 - Princeton University
arXiv: 1405.1732 to be published in PRL

In gauge/gravity duality the deconfinement transition of a gauge theory is dual To the formation of a Black Hole in the gravity bulk [Witten -1998]

We want to describe an intuitive way of understanding this Duality without referring to a sophisticated duality dictionary

Our initial motivation was to study a simple Matrix Model for a Black Hole by looking at the deconfinement transition of 4d $\mathcal{N} = 4$ SYM on an $\mathcal{S}^3$ and the Hawking-Page Transition of the Black hole in the corresponding AdS bulk [Hawking,Page - 1983]

Such a black can be modeled as a long and winding string [Susskind,Teitelboim - 1993; Horowitz,Polchinski - 1997]

Since we do not assume the dual gravity description, our argument is applicable to a generic Gauge theories

We do this by paying attention to the behavior of the stringy degrees of freedom of a gauge theory (the Wilson Lines) as the gauge theory undergoes a deconfinement transition. This was achieved through a Monte-Carlo Lattice gauge theory simulation of the transition

In gauge/gravity duality the deconfinement transition of a gauge theory is dual To the formation of a Black Hole in the gravity bulk [Witten -1998]

We want to describe an intuitive way of understanding this Duality without referring to a sophisticated duality dictionary

Our initial motivation was to study a simple Matrix Model for a Black Hole by looking at the deconfinement transition of 4d $\mathcal{N} = 4$ SYM on an $\mathcal{S}^3$ and the Hawking-Page Transition of the Black hole in the corresponding AdS bulk [Hawking,Page - 1983]

Such a black can be modeled as a long and winding string [Susskind,Teitelboim - 1993; Horowitz,Polchinski - 1997]

Since we do not assume the dual gravity description, our argument is applicable to a generic Gauge theories

We do this by paying attention to the behavior of the stringy degrees of freedom of a gauge theory (the Wilson lines) as the gauge theory undergoes a deconfinement transition. This was achieved through a Monte-Carlo Lattice gauge theory simulation of the transition

In gauge/gravity duality the deconfinement transition of a gauge theory is dual To the formation of a Black Hole in the gravity bulk [Witten -1998]

We want to describe an intuitive way of understanding this Duality without referring to a sophisticated duality dictionary

Our initial motivation was to study a simple Matrix Model for a Black Hole by looking at the deconfinement transition of 4d $\mathcal{N} = 4$ SYM on an $\mathcal{S}^3$ and the Hawking-Page Transition of the Black hole in the corresponding AdS bulk [Hawking,Page - 1983]

Such a black can be modeled as a long and winding string [Susskind,Teitelboim - 1993; Horowitz,Polchinski - 1997]

Since we do not assume the dual gravity description, our argument is applicable to a generic Gauge theories

We do this by paying attention to the behavior of the stringy [illegible] gauge theory undergoes a deconfinement transition. This was achieved through a Monte-Carlo Lattice gauge theory simulation of the transition

In gauge/gravity duality the deconfinement transition of a gauge theory is dual To the formation of a Black Hole in the gravity bulk [Witten -1998]

We want to describe an intuitive way of understanding this Duality without referring to a sophisticated duality dictionary

Our initial motivation was to study a simple Matrix Model for a Black Hole by looking at the deconfinement transition of 4d $\mathcal{N} = 4$ SYM on an $\mathcal{S}^3$ and the Hawking-Page Transition of the Black hole in the corresponding AdS bulk [Hawking,Page - 1983]

Such a black can be modeled as a long and winding string [Susskind,Teitelboim - 1993; Horowitz,Polchinski - 1997]

Since we do not assume the dual gravity description, our argument is applicable to a generic Gauge theories

We do this by paying attention to the behavior of the stringy degrees of freedom of a gauge theory (the Wilson Lines) as the gauge theory undergoes a deconfinement transition. This was achieved through a Monte-Carlo Lattice gauge theory simulation of the transition

In gauge/gravity duality the deconfinement transition of a gauge theory is dual To the formation of a Black Hole in the gravity bulk [Witten -1998]

We want to describe an intuitive way of understanding this Duality without referring to a sophisticated duality dictionary

Our initial motivation was to study a simple Matrix Model for a Black Hole by looking at the deconfinement transition of 4d $\mathcal{N} = 4$ SYM on an $\mathcal{S}^3$ and the Hawking-Page Transition of the Black hole in the corresponding AdS bulk [Hawking,Page - 1983]

Such a black can be modeled as a long and winding string [Susskind,Teitelboim - 1993; Horowitz,Polchinski - 1997]

Since we do not assume the dual gravity description, our argument is applicable to a generic Gauge theories

We do this by paying attention to the behavior of the stringy degrees of freedom of a gauge theory (the Wilson Lines) as the gauge theory undergoes a deconfinement transition. This was achieved through a Monte-Carlo Lattice gauge theory simulation of the transition

In gauge/gravity duality the deconfinement transition of a gauge theory is dual To the formation of a Black Hole in the gravity bulk [Witten -1998]

We want to describe an intuitive way of understanding this Duality without referring to a sophisticated duality dictionary

Our initial motivation was to study a simple Matrix Model for a Black Hole by looking at the deconfinement transition of 4d $\mathcal{N}=4$ SYM on an $\mathcal{S}^3$ and the Hawking-Page Transition of the Black hole in the corresponding AdS bulk [Hawking,Page - 1983]

Such a black can be modeled as a long and winding string [Susskind,Teitelboim - 1993; Horowitz,Polchinski - 1997]

Since we do not assume the dual gravity description, our argument is applicable to a generic Gauge theories

We do this by paying attention to the behavior of the stringy degrees of freedom of a gauge theory (the Wilson Lines) as the gauge theory undergoes a deconfinement transition. This was achieved through a Monte-Carlo Lattice gauge theory simulation of the transition

As concrete example consider $(D+1)$ pure $U(N)$ YM Theory on a discrete lattice

$$H = K + V \qquad\qquad K = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E^{\alpha}_{\mu,\vec{x}} \right)^2$$

$$V = \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}) \right).$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu} \delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}},$$

$$[E_{\mu,\vec{x}}, E_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U^{\dagger}_{\nu,\vec{y}}] = 0. \quad E^{\alpha}_{\mu,\vec{x}} |0\rangle$$

$$W_{C_1} W_{C_2} \cdots W_{C_k} |0\rangle \quad W_C = Tr \left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} \cdots U_{\rho,\vec{x}-\hat{\rho}} \right)$$

$$E = K = \frac{\lambda}{2} L_{total}(T). \quad S = L_{total} \log(2D-1).$$

$$F = L_{total}(T) \left(\frac{\lambda}{2} - T \log(2D-1) \right). \quad T_c = \lambda / (2 \log(2D-1)).$$

As concrete example consider $(D+1)$ pure $U(N)$ YM Theory on a discrete lattice

$$H = K + V \qquad K = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E^{\alpha}_{\mu,\vec{x}} \right)^2$$

$$V = \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}) \right).$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu} \delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}},$$

$$[E_{\mu,\vec{x}}, E_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U^{\dagger}_{\nu,\vec{y}}] = 0. \quad E^{\alpha}_{\mu,\vec{x}} |0\rangle$$

$$W_{C_1} W_{C_2} \cdots W_{C_k} |0\rangle \quad W_C = Tr\left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} \cdots U_{\rho,\vec{x}-\hat{\rho}} \right)$$

$$E = K = \frac{\lambda}{2} L_{total}(T). \quad S = L_{total} \log(2D-1).$$

$$F = L_{total}(T) \left(\frac{\lambda}{2} - T \log(2D-1) \right). \quad T_c = \lambda/(2 \log(2D-1)).$$

As concrete example consider $(D+1)$ pure $U(N)$ YM Theory on a discrete lattice

$$H = K + V \qquad\qquad K = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E^{\alpha}_{\mu,\vec{x}} \right)^2$$

$$V = \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}) \right).$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu} \delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}},$$

$$[E_{\mu,\vec{x}}, E_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U^{\dagger}_{\nu,\vec{y}}] = 0. \quad E^{\alpha}_{\mu,\vec{x}} |0\rangle$$

$$W_{C_1} W_{C_2} \cdots W_{C_k} |0\rangle \quad W_C = Tr \left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} \cdots U_{\rho,\vec{x}-\hat{\rho}} \right)$$

$$E = K = \frac{\lambda}{2} L_{total}(T). \qquad S = L_{total} \log(2D-1).$$

$$F = L_{total}(T) \left(\frac{\lambda}{2} - T \log(2D-1) \right). \qquad T_c = \lambda/(2 \log(2D-1)).$$

As concrete example consider $(D+1)$ pure $U(N)$ YM Theory on a discrete lattice

$$H = K + V \qquad K = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E^{\alpha}_{\mu,\vec{x}} \right)^2$$

$$V = \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}) \right).$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu} \delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}},$$

$$[E_{\mu,\vec{x}}, E_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U^{\dagger}_{\nu,\vec{y}}] = 0. \quad E^{\alpha}_{\mu,\vec{x}}|0\rangle$$

$$W_{C_1} W_{C_2} \cdots W_{C_k} |0\rangle \quad W_C = Tr \left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} \cdots U_{\rho,\vec{x}-\hat{\rho}} \right)$$

$$E = K = \tfrac{\lambda}{2} L_{total}(T). \qquad S = L_{total} \log(2D-1).$$

$$F = L_{total}(T) \left(\tfrac{\lambda}{2} - T \log(2D-1) \right). \qquad T_c = \lambda/(2\log(2D-1)).$$

Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space

In order to describe a black hole 0-brane let us consider two lattice models

First the dimensionally reduced D-matrix model This is the Eguchi-Kawai model with with continuous time direction

At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory Is then know to be equivalent to the D+1 dim. YM at large N. In the sense that translationally Invariant observables are reproduced from the former at leading order

At weak coupling this model is Equivalent to the bosonic part of The BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity In the ̀t Hooft large N limit.

For $D \leq 2$ this theory exhibits a deconfinement transition, characterized by the non-vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order N^2 and a typical state contains a long winding string such as $Tr(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$

Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space

In order to describe a black hole 0-brane let us consider two lattice models

First the dimensionally reduced D-matrix model This is the Eguchi-Kawai model with with continuous time direction

At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory Is then know to be equivalent to the D+1 dim. YM at large N. In the sense that translationally Invariant observables are reproduced from the former at leading order

At weak coupling this model is Equivalent to the bosonic part of The BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity In the t̀ Hooft large N limit.

For $D \leq 2$ this theory exhibits a deconfinement transition, characterized by the non-vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order N^2 and a typical state contains a long winding string such as $Tr(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$

Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space

In order to describe a black hole 0-brane let us consider two lattice models

First the dimensionally reduced D-matrix model This is the Eguchi-Kawai model with with continuous time direction

At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory Is then know to be equivalent to the D+1 dim. YM at large N. In the sense that translationally Invariant observables are reproduced from the former at leading order

At weak coupling this model is Equivalent to the bosonic part of The BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity In the t̀ Hooft large N limit.

For $D \leq 2$ this theory exhibits a deconfinement transition, characterized by the non-vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order N^2 and a typical state contains a long winding string such as $Tr(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$

Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space

In order to describe a black hole 0-brane let us consider two lattice models

First the dimensionally reduced D-matrix model This is the Eguchi-Kawai model with with continuous time direction

At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory Is then know to be equivalent to the D+1 dim. YM at large N. In the sense that translationally Invariant observables are reproduced from the former at leading order

At weak coupling this model is Equivalent to the bosonic part of The BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity In the ̀t Hooft large N limit.

For $D \leq 2$ this theory exhibits a deconfinement transition, characterized by the non vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order N^2 and a typical state contains a long winding string such as $Tr(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$

Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space

In order to describe a black hole 0-brane let us consider two lattice models

First the dimensionally reduced D-matrix model This is the Eguchi-Kawai model with with continuous time direction

At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory Is then know to be equivalent to the D+1 dim. YM at large N. In the sense that translationally Invariant observables are reproduced from the former at leading order

At weak coupling this model is Equivalent to the bosonic part of The BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity In the ì Hooft large N limit.

For $D \leq 2$ this theory exhibits a deconfinement transition, characterized by the non-vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order N^2 and a typical state contains a long winding string such as $Tr(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$

Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space

In order to describe a black hole 0-brane let us consider two lattice models

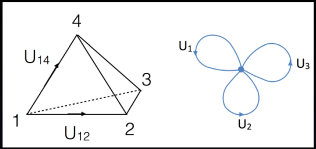
First the dimensionally reduced D-matrix model This is the Eguchi-Kawai model with with continuous time direction

At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory Is then know to be equivalent to the D+1 dim. YM at large N. In the sense that translationally Invariant observables are reproduced from the former at leading order

At weak coupling this model is Equivalent to the bosonic part of The BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity In the ȉ Hooft large N limit.

For $D \leq 2$ this theory exhibits a deconfinement transition, characterized by the non-vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order N^2 and a typical state contains a long winding string such as $Tr(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$

The second Model is the tetrahedron Lattice, here the entropy and temperature scale as
$S = L_total \log 2$ and $T_c = \lambda/(2 \log 2)$ This system also possesses a deconfinement transition with a long string described as
$Tr(U_{12}U_{23}U_{31}U_{14}U_{42}\ldots)$



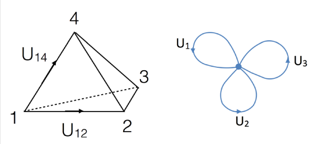
$$S_{lattice} = -\frac{N}{2a\lambda}\sum_{\mu,t}\mathrm{Tr}\left(V_t U_{\mu,t+a} V^\dagger_{\mu,t} U_{\mu,t} + c.c.\right) + \frac{aN}{\lambda}\sum_{\mu\neq\nu,t}\left(N - \mathrm{Tr}(U_{\mu,t}U_{\nu,t}U^\dagger_{\mu,t}U^\dagger_{\nu,t})\right)$$

$$S_{tet} = -\frac{N}{2a\lambda}\sum_t\sum_{m<n}\left(\mathrm{Tr}(V_{m,t}U_{mn,t+a}V^\dagger_{n,t}U_{nm,t}) + c.c.\right) - \frac{aN}{\lambda}\sum_t\sum_{l<m<n}\left((N - \mathrm{Tr}(U_{lm,t}U_{mn,t}U_{nl,t})) + c.c.\right).$$

$P_{tet} = \frac{1}{4N}\sum_{m=1}^{4}\mathrm{Tr}(V_{m,t=a}V_{m,t=2a}\cdots V_{m,t=n_t a})$

$P = \frac{1}{N}\mathrm{Tr}(V_{t=a}V_{t=2a}\cdots V_{t=n_t a})$.

We use the absolute value of P in order to eliminate the $U(1)$

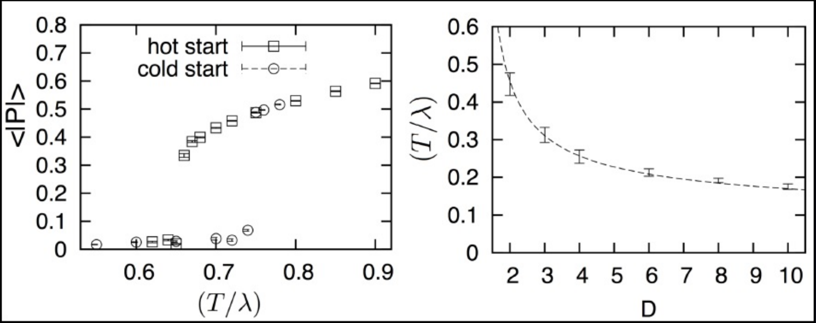


$$S_{lattice} = -\frac{N}{2a\lambda}\sum_{\mu,t}\mathrm{Tr}\left(V_t U_{\mu,t+a} V^\dagger_{\mu,t} U_{\mu,t} + c.c.\right) + \frac{aN}{\lambda}\sum_{\mu\neq\nu,t}\left(N - \mathrm{Tr}(U_{\mu,t}U_{\nu,t}U^\dagger_{\mu,t}U^\dagger_{\nu,t})\right)$$

$$S_{tet} = -\frac{N}{2a\lambda}\sum_t\sum_{m<n}\left(\mathrm{Tr}(V_{m,t}U_{mn,t+a}V^\dagger_{n,t}U_{nm,t}) + c.c.\right) - \frac{aN}{\lambda}\sum_t\sum_{l<m<n}\left((N - \mathrm{Tr}(U_{lm,t}U_{mn,t}U_{nl,t})) + c.c.\right).$$

$$P_{tet} = \tfrac{1}{4N}\textstyle\sum_{m=1}^{4}\mathrm{Tr}(V_{m,t=a}V_{m,t=2a}\cdots V_{m,t=n_t a})$$
$$P = \tfrac{1}{N}\mathrm{Tr}(V_{t=a}V_{t=2a}\cdots V_{t=n_t a}).$$

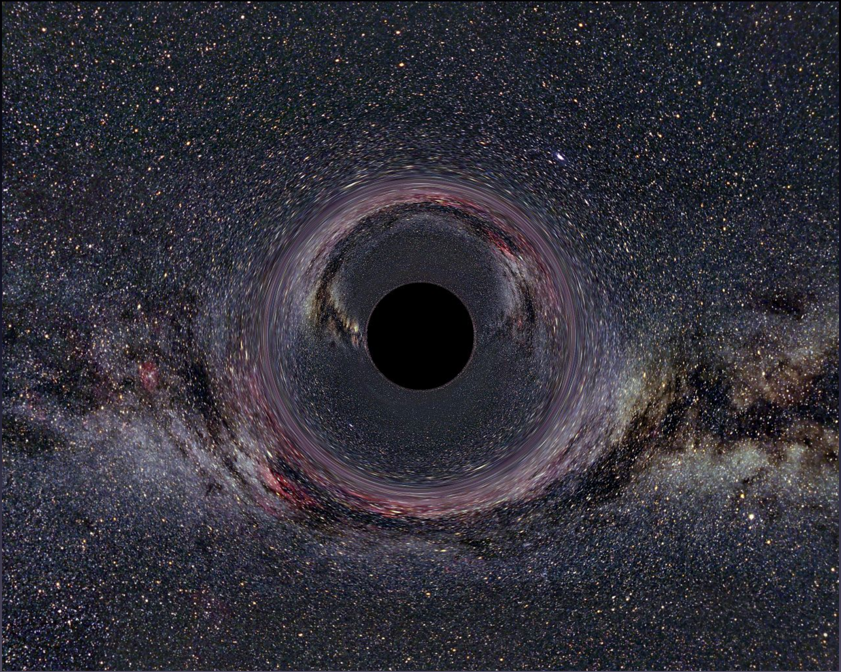
We use the absolute value of P in order to eliminate the $U(1)$



The expectation value of the tetrahedron
With N = 64 nt =12
There strong hysteresis about the
Theoretically predicted critical temperature
$(T_c/\lambda) = 1/(2\log 2) \simeq 0.721$

The range for hyteresis of the EK model
with N =64 nt =12
for various dimensions
The dashed curve is the critical temperature
$(T_c/\lambda) = 1/(2\log(2D-1))$

The End



- [Maldacena,1997] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
- E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
- O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, Adv. Theor. Math. Phys. **8**, 603 (2004).
- L. Susskind, arXiv:1311.7379 [hep-th].
- L. Susskind, arXiv:1402.5674 [hep-th].
- S. W. Hawking and D. N. Page, Commun. Math. Phys. **87**, 577 (1983).
- G. 't Hooft, Nucl. Phys. B **72**, 461 (1974).
- L. Susskind, In *Teitelboim, C. (ed.): The black hole* 118-131.
  E. Halyo, A. Rajaraman and L. Susskind, Phys. Lett. B **392**, 319 (1997).
- G. T. Horowitz and J. Polchinski, Phys. Rev. D **55**, 6189 (1997).
- J. B. Kogut and L. Susskind, Phys. Rev. D **11**, 395 (1975).

- A. Patel, Nucl. Phys. B **243**, 411 (1984).
  T. Kalaydzhyan and E. Shuryak, arXiv:1402.7363 [hep-ph].
- T. Eguchi and H. Kawai, Phys. Rev. Lett. **48**, 1063 (1982).
- T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997).
  B. de Wit, J. Hoppe and H. Nicolai, Nucl. Phys. B **305**, 545 (1988).
- N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998).
- George Fishman, Monte Carlo: Concepts, Algorithms,and Applications, Springer Series in ORFE, 9780387945279, 1996.
- Y. Sekino and L. Susskind, JHEP **0810**, 065 (2008).
- U. W. Heinz, AIP Conf. Proc. **739**, 163 (2005).