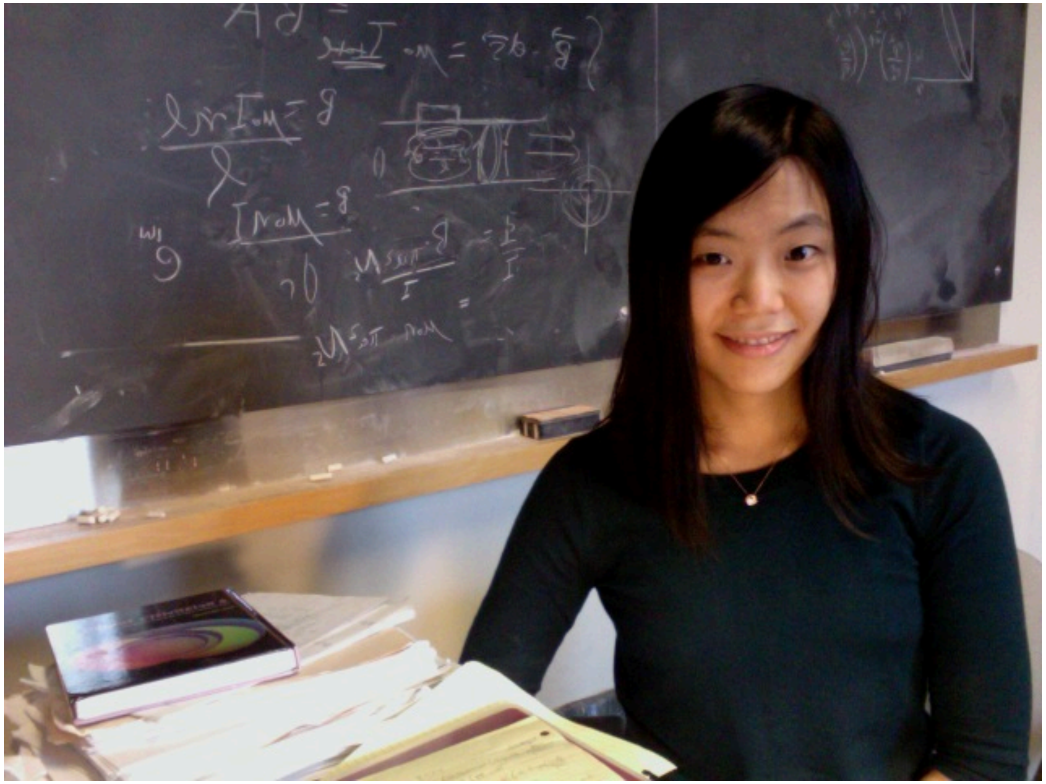
Entanglement Tsunami

Hong Liu



Josephine Suh

Mark Mezei

Based on

HL and Josephine Suh, 1305.7244, PRL 112, 011601 (2014)
HL and Josephine Suh, 1311.1200, PRD 89, 066012 (2014)
HL, Mezei, Suh, unpublished, Casini, unpublished
Casini, Hubeny, HL, Maxfield, Mezei, Suh, to appear

See also

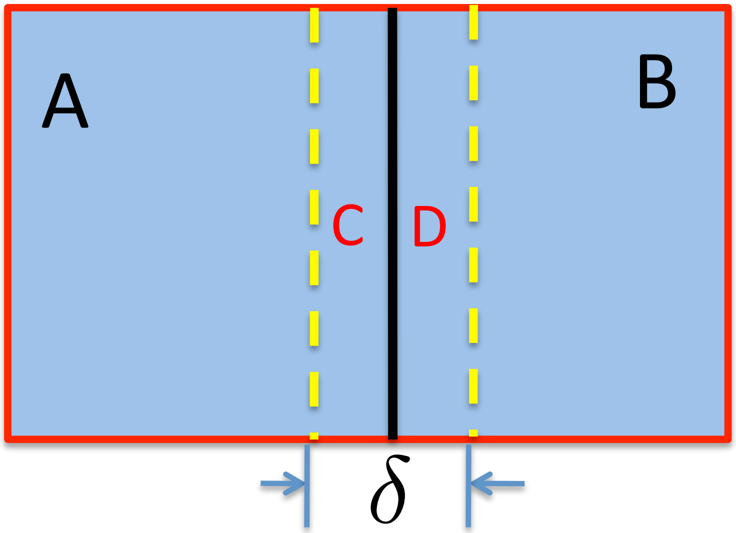
Hyungwon Kim and Huse arXiv:1306.4306

Hartman and Maldacena arXiv:1303.1080

Shenker and Stanford arXiv: 1306.0622, 1312.3296

Hubeny and Maxfield arXiv: 1312.6887

Entanglement generation



$$\psi(t=0) = \psi_A \otimes \psi_B$$

$$\psi(t) = e^{-iHt}\psi(0)$$

$$H = H_A + H_B + H_{AB}$$

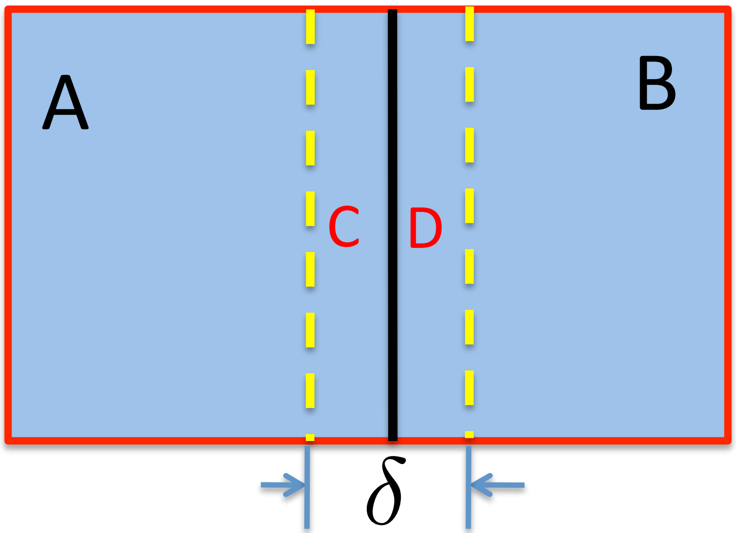
How fast can entanglement be generated?

In most physical systems: Local Hamiltonian

$$H_{AB} = H_{CD}$$

δ : UV cutoff

Small incremental entangling conjecture/ theorem



$$H = H_A + H_B + H_{CD}$$

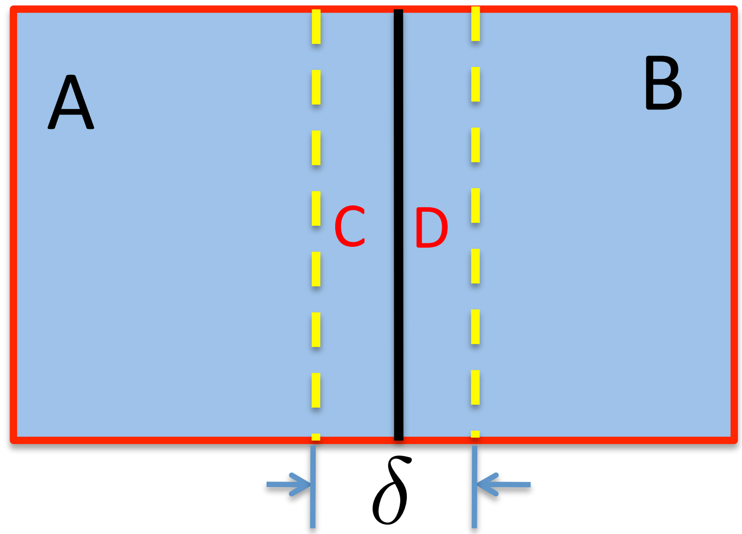
S_A : entanglement entropy of A

Dur, Vidal et al, Bravyi, Kitaev
Bennett et al, Van Acoleyen, Marien, Verstraete

For spin systems:

$$\frac{dS_A}{dt} \leq c||H|| \log d, \qquad d = \min(d_C, d_D)$$

d_C : dimension of Hilbert space of C



For more general quantum systems, e.g. a QFT

$$\frac{dS_A}{dt} < ???$$

In this talks we will describe some hints.

A simple setup: global quenches

1. Start with a QFT in the ground state.

2. At t=0 in a very short time interval inject a uniform energy density

   - initial state homogeneous, isotropic, entanglement properties as vacuum

3. The system evolves to (thermal) equilibrium

$S_A(t)$?

Also a question of interest for thermalization.

$$\Delta S_A(t) = S_A(t) - S_A(t=0)$$

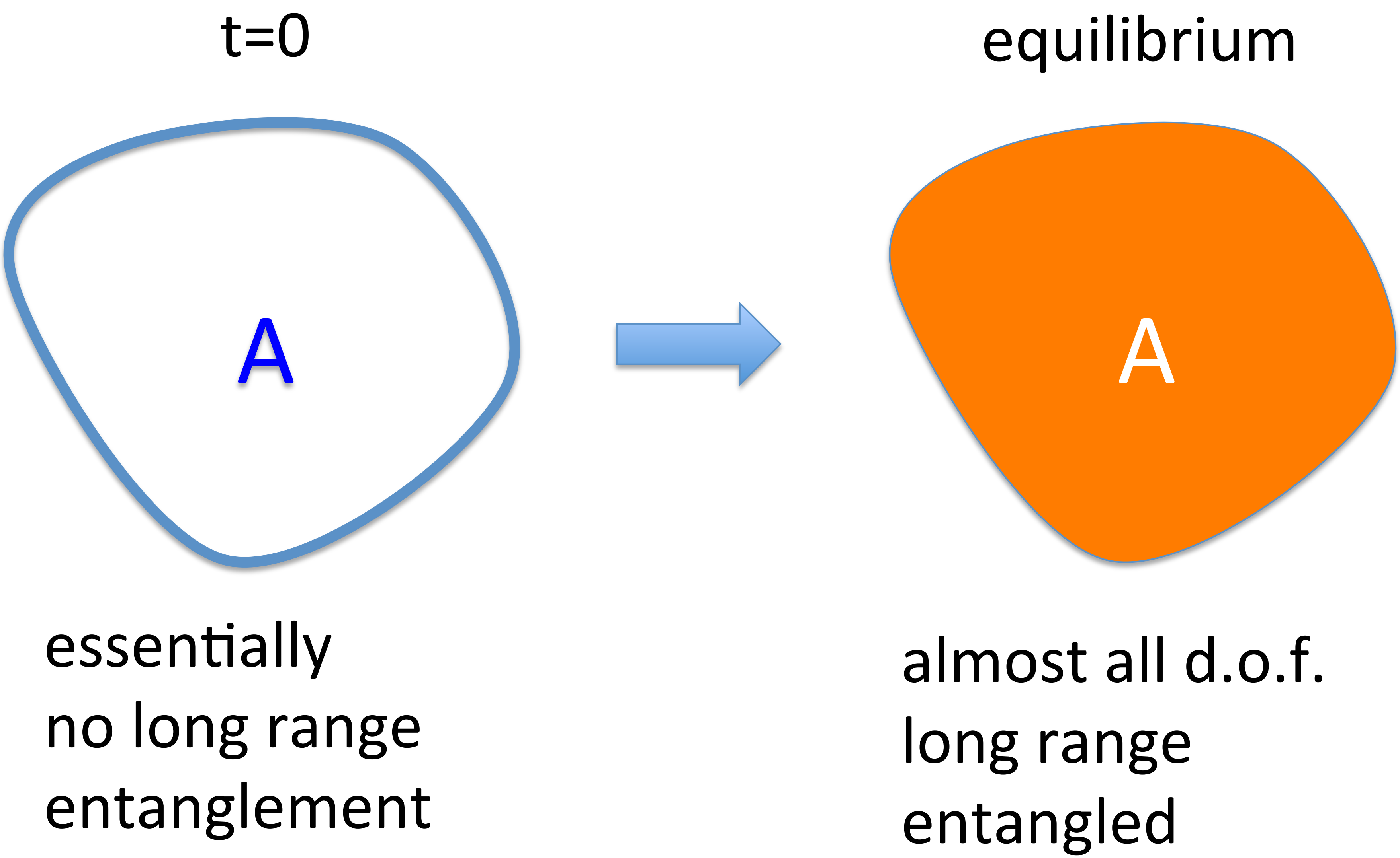
$$\Delta S_A(0) = 0$$

In equilibrium, system behaves macroscopically as a thermal state, with entanglement entropy disguised as thermal entropy:

$$\Delta S_A^{\rm eq} = s_{\rm eq} V_A$$ V_A : volume of region A

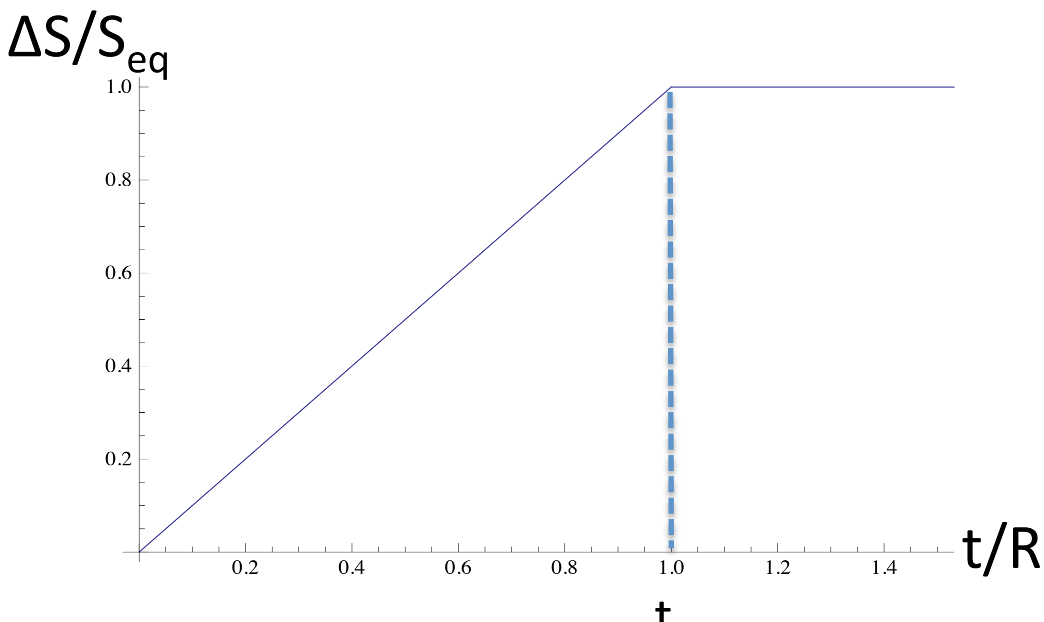
$$s_{\rm eq} : \text{ equilibrium entropy density}$$

Essentially all d.o.f. inside A becomes long ranged entangled with those outside A.


t=0
equilibrium
A
A
essentially
no long range
entanglement
almost all d.o.f.
long range
entangled

Previous results in (1+1)-d CFTs

Calabrese and Cardy

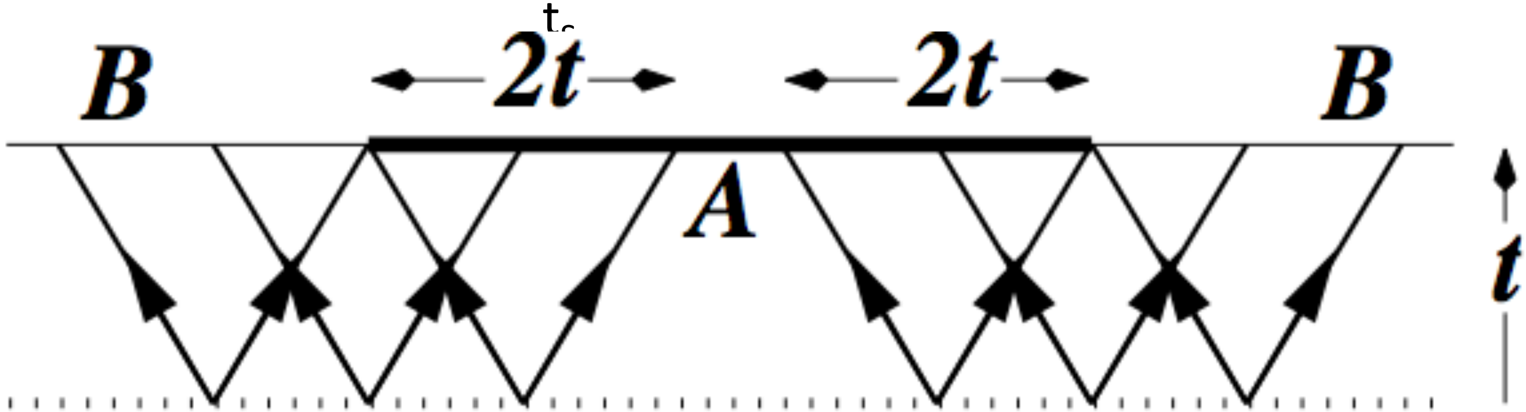


1. Linear growth with time

   (not too early and too late)

2. Slope = 1

3. Can be reproduced by free particles



Special techniques in one spatial dimension do not apply to higher dimensions:

- Equilibration processes: complicated non-equilibrium many-body dynamics, generally out of theoretical control.

- Entanglement entropy is notoriously difficult to calculate even for simple regions in the vacuum of a free theory, not to mention for general regions in interacting theories far from equilibrium.

String theory to the rescue!

Earlier work:

Hubeny, Rangamani, Takayanagi: arXiv:0705.0016

Abajo-Arrastia, Aparicio and Lopez, arXiv:1006.4090

Albash and Johnson, arXiv:1008.3027

Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Muller, Schafer, Shigemori, Staessens arXiv:1012.4753, arXiv:1103.2683

Aparicio and Lopez, arXiv:1109.3571

Caceres and A. Kundu, arXiv:1205.2354

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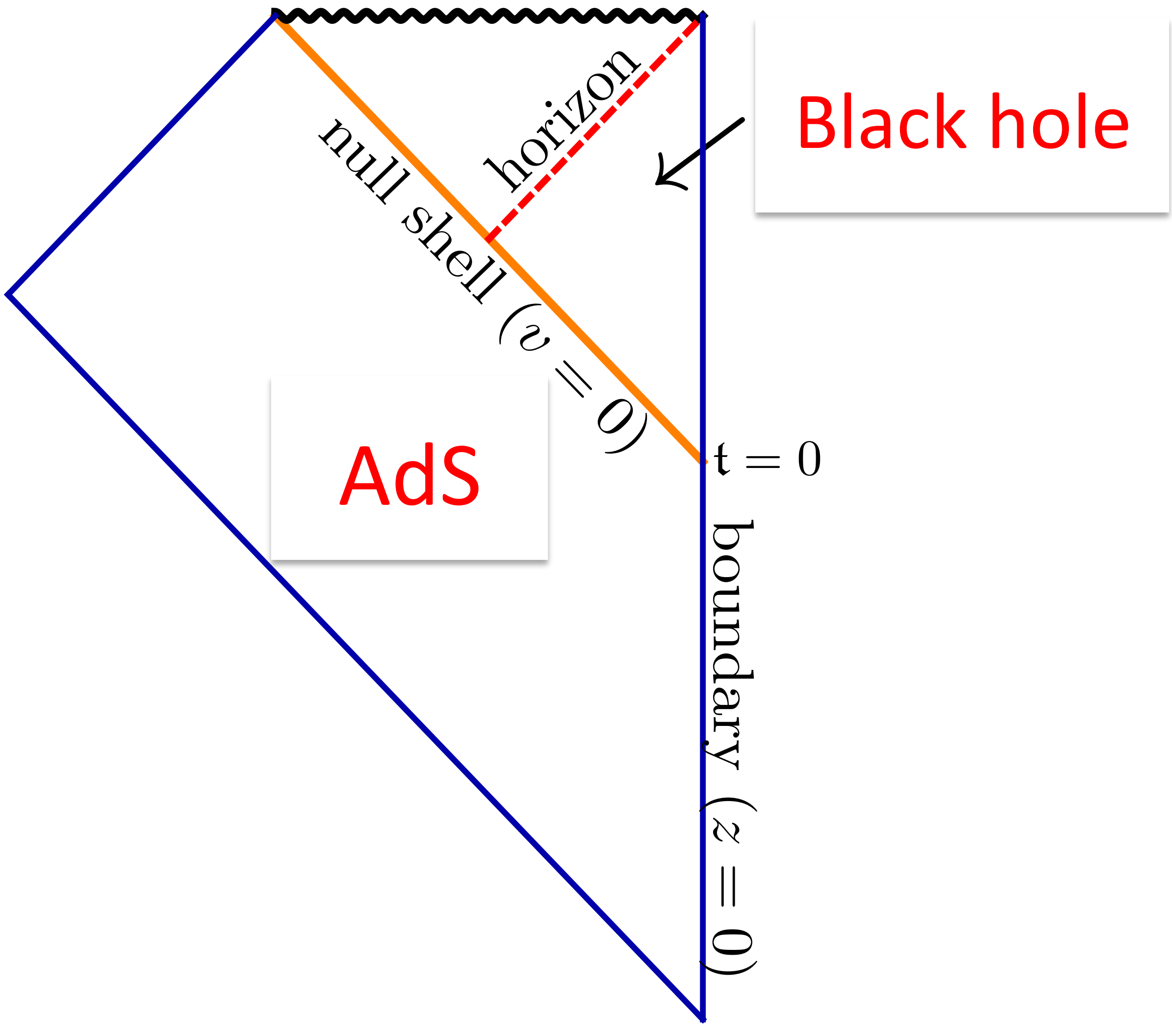
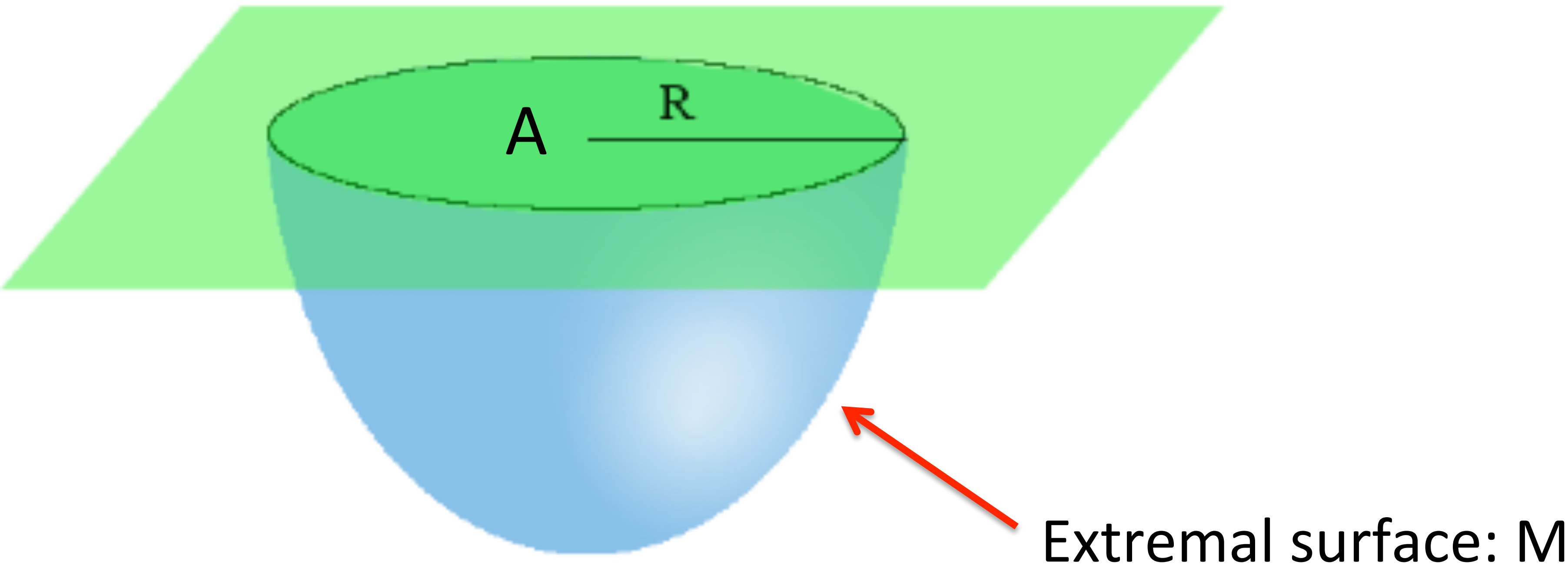
Holographic description of quench



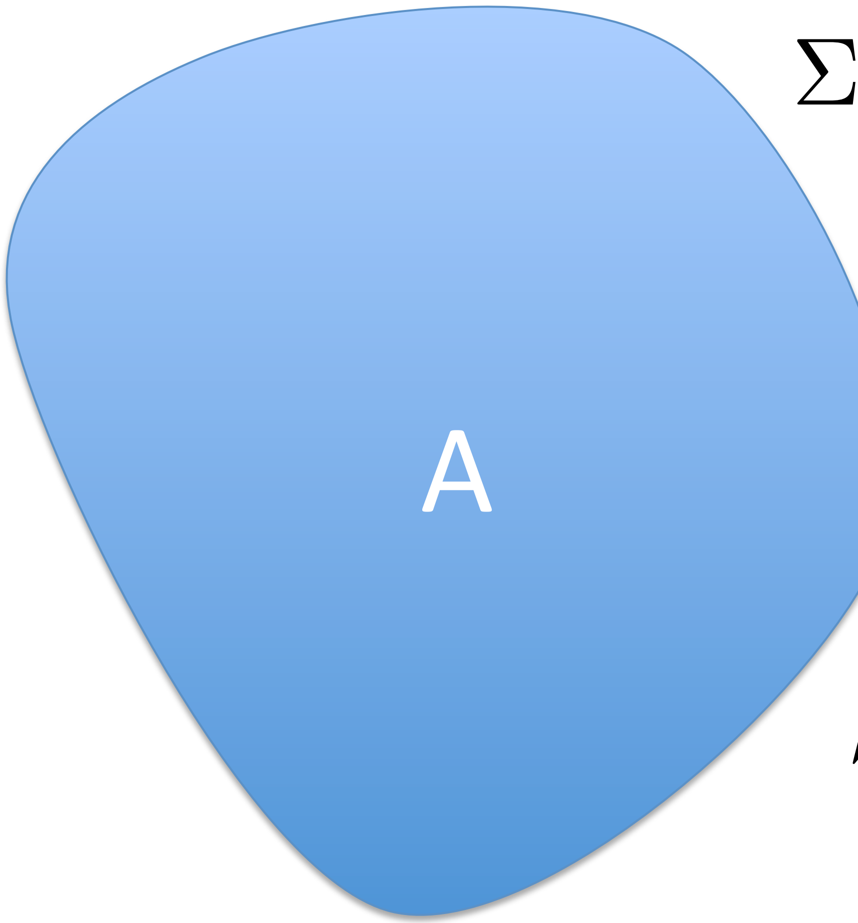
quench: thin shell collapse to form a black hole.

Holographic Entanglement entropy

Ryu, Takayanagi
Hubeny, Rangamani, Takayanagi



$$S_A = \frac{\text{Area of M}}{4\mathrm{G_N}}$$



Area: A_Σ

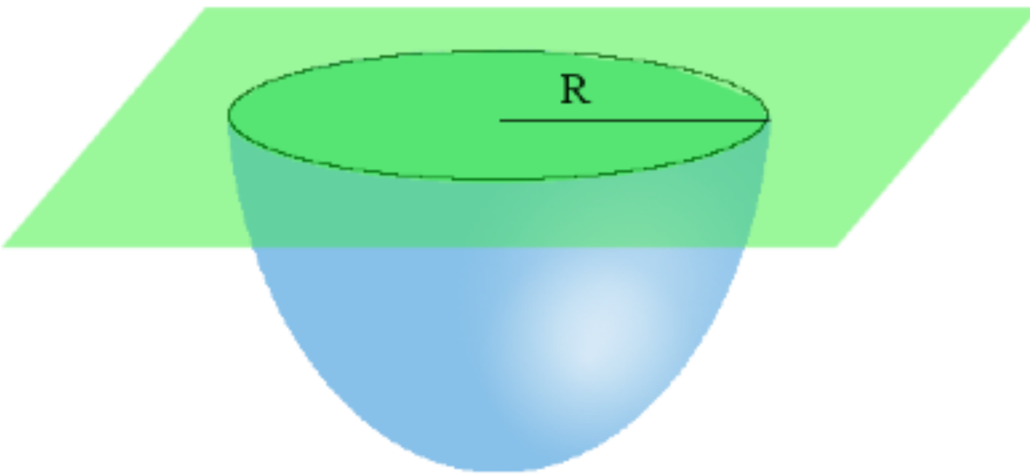
Volume: V_A

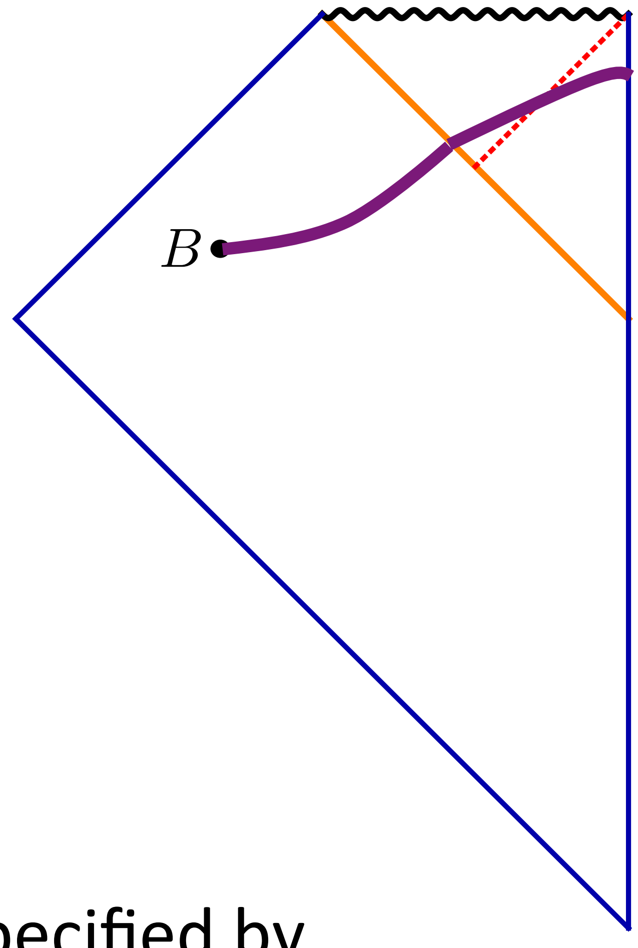
$S_A(t)?$

R: characteristic size of the region

Interested in long-distance physics: $R \to \infty$

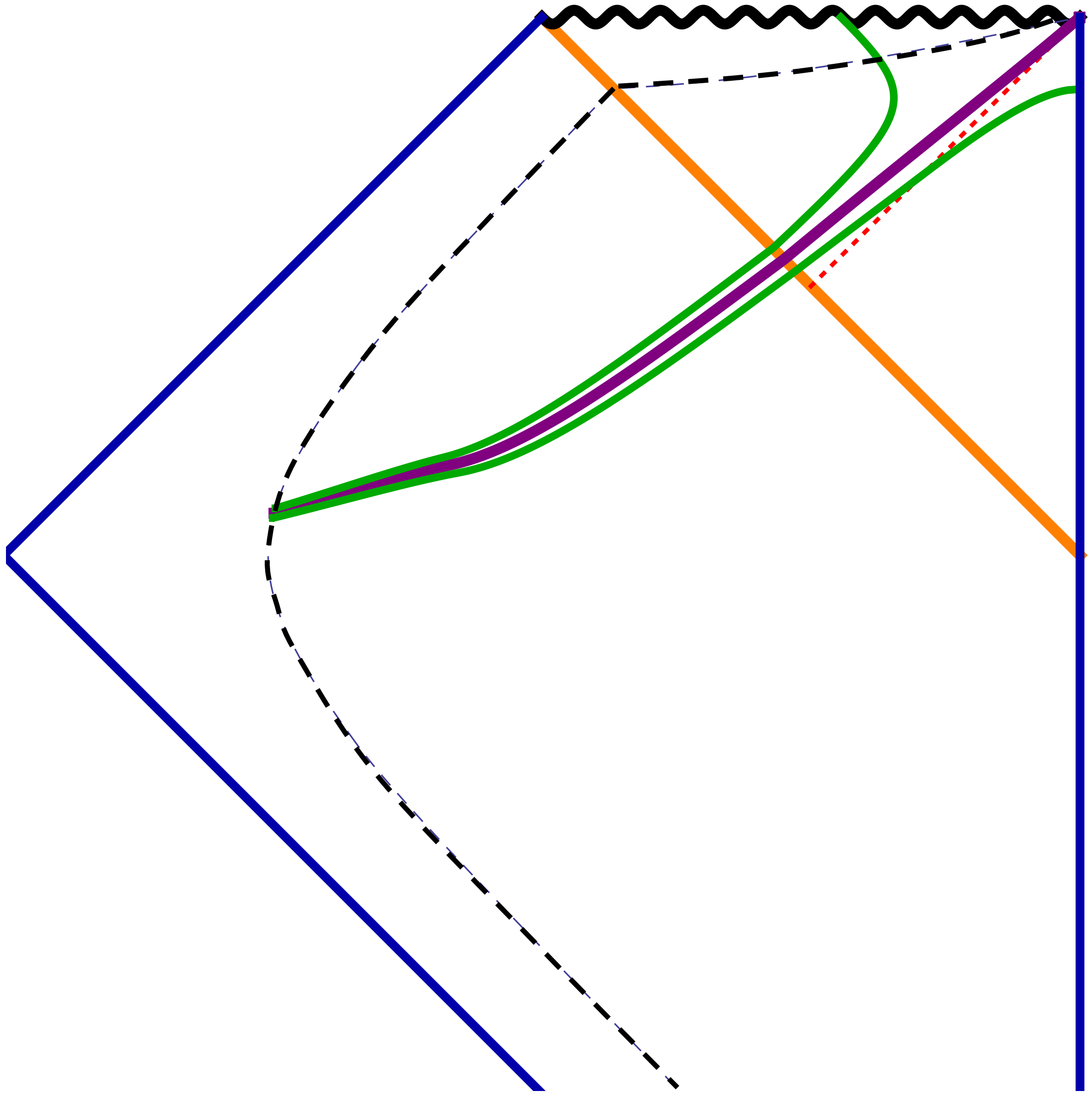
Gravity description





Each extremal surface can also be specified by the location of (and boundary conditions at) the tip.

Large size and critical extremal surfaces

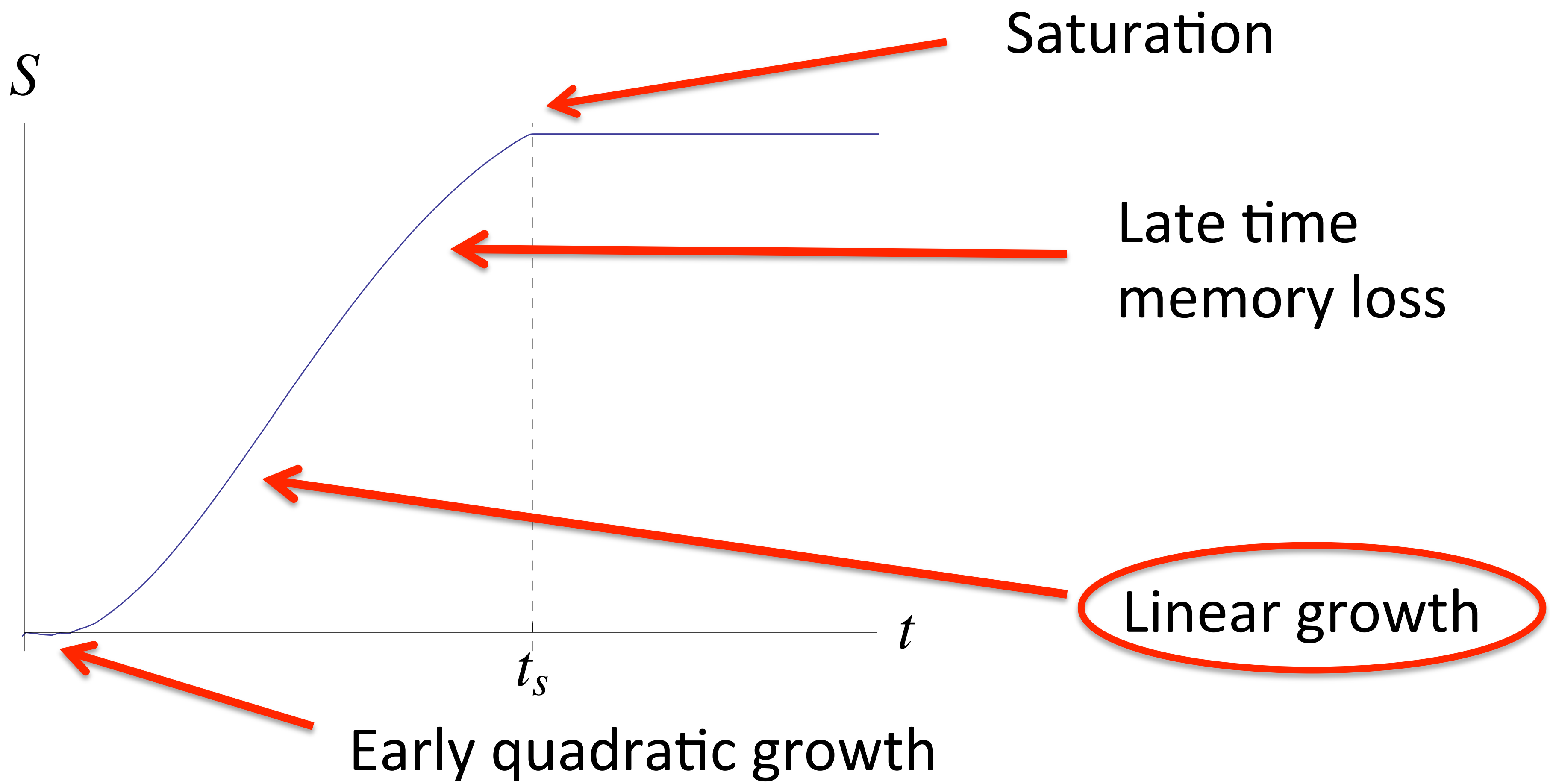


In general a rather complicated problem to determine time evolution of extremal surfaces

Critical extremal surfaces determine large R, large time behavior

Four scaling regimes in general dimensions

In the large size R limit: $$R \gg 1/T$$



Linear growth

For $R \gg t \gg 1/T$

See also
Hartman, Maldacena

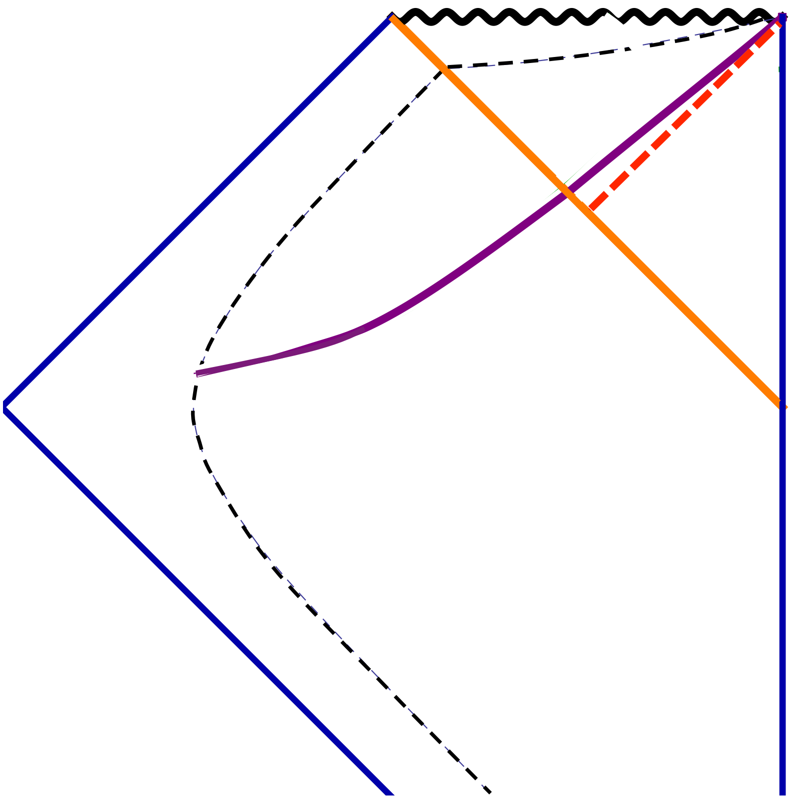
$$\Delta S_A(t) = v_E \, s_{\rm eq} \, A_\Sigma t + \cdots$$

$s_{\rm eq}$: Equilibrium entropy density

independent of shape, holographic theories under consideration, the nature of equilibrium state, also likely thermalization processes

v_E: dimensionless number characterizing final eq state.

Critical extremal surface for linear growth



The critical extremal surface runs along a constant radial slice inside the horizon

$$ds^2 = \frac{L^2}{z^2}\left(-h dt^2 + \frac{1}{f}dz^2 + d\vec{x}^2\right)$$

$$z_m \;:\; \text{minimum of} \;\; \frac{h(z)}{z^{2D}}$$

D: # of spatial dimensions

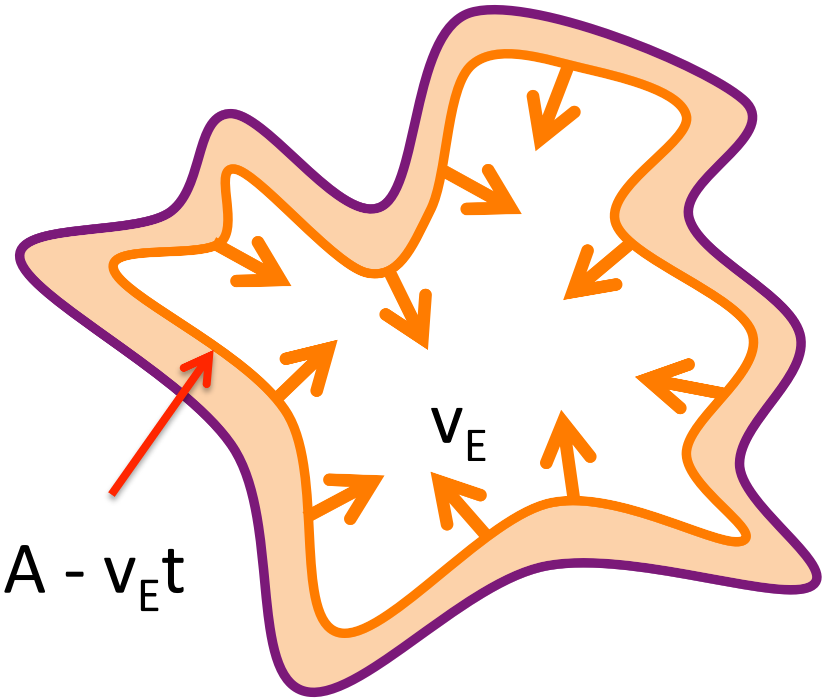
$$v_E = (z_h/z_m)^D \sqrt{-h(z_m)}$$

z_h : horizon size

Determined by equilibrium state

Entanglement Tsunami

$$\Delta S_A(t) = v_E\, s_{\rm eq}\, A_\Sigma\, t = s_{\rm eq}\, \left(V_A - V_{A-v_E t}\right)$$



suggests a picture of tsunami wave of entanglement, with a sharp wave front.

d.o.f. in the region covered by the wave is now entangled with those outside A

natural with evolution from a local Hamiltonian

Tsunami velocity

$$\Delta S_A(t) = v_E \, s_{\rm eq} \, A_\Sigma t + \cdots$$

Neutral system (AdS Schwarzschild):

$$v_E^{(\rm S)} = \frac{(\eta-1)^{\frac{1}{2}(\eta-1)}}{\eta^{\frac{1}{2}\eta}} = \begin{cases} 1 & D=1 \\ \frac{\sqrt{3}}{2^{\frac{4}{3}}} = 0.687 & D=2 \\ \frac{\sqrt{2}}{3^{\frac{3}{4}}} = 0.620 & D=3 \\ \frac{1}{2} & D=\infty \end{cases}$$

$$\eta \equiv \frac{2D}{D+1}$$

Turning on chemical potential reduces v_E.

Upper bound on v_E ?

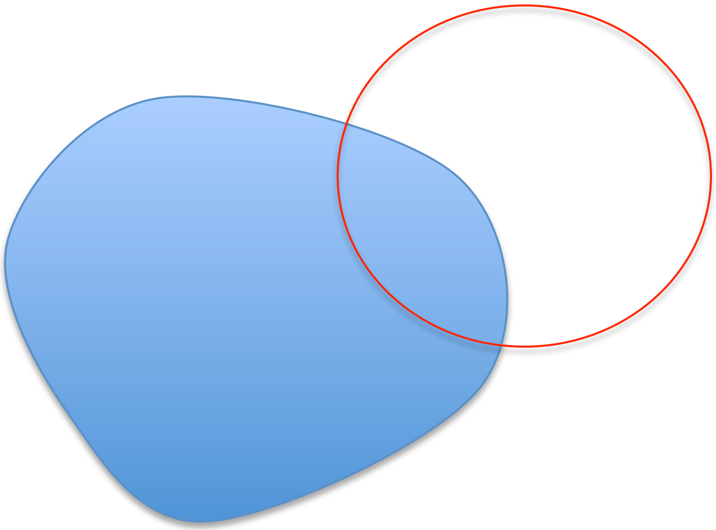
v_E should be constrained by causality.

In all gravity examples: $$v_E = (z_h/z_m)^D \sqrt{-h(z_m)}$$

$$v_E \le v_E^{(\mathrm{S})} = \frac{(\eta - 1)^{\frac{1}{2}(\eta-1)}}{\eta^{\frac{1}{2}\eta}} \qquad \eta \equiv \frac{2D}{D+1}$$

Null energy condition important

Comparing with free particle streaming



Assume:

- At t=0, there is a uniform density of "photons" with only local entanglement correlations.
- Entanglement spreads when photons propagate.

Leading to shape independent linear growth,

$$\Delta S_\Sigma(t) = v_E s_{\rm eq} A_\Sigma t + \cdots$$

For D=1:

$$v_{\text{streaming}} = v_{\text{CFT}} = v_{\text{gravity}} = 1$$

HL, Mezei, Suh

Casini

$$D \geq 2$$

$$v_{\text{streaming}} = \frac{\Gamma(\frac{D}{2})}{\sqrt{\pi}\Gamma(\frac{D+1}{2})} < \quad v_E^{(S)} \quad < 1$$

In strongly coupled systems, entanglement tsunami propagates faster than those from free particles traveling at speed of light !

$$D \to \infty : \; v_E^{(S)} \to \frac{1}{2}, \; v_{\text{streaming}} \to \sqrt{\frac{2}{\pi(D+1)}} \to 0$$

Bound on entanglement growth?

For any non-equilibrium processes:

$$\mathfrak{R}_A(t) \equiv \frac{1}{s_{\rm eq} A_\Sigma} \frac{dS_A}{dt}$$

dimensionless, can be compared among region A of different shapes, sizes, and systems of different number of d.o.f.

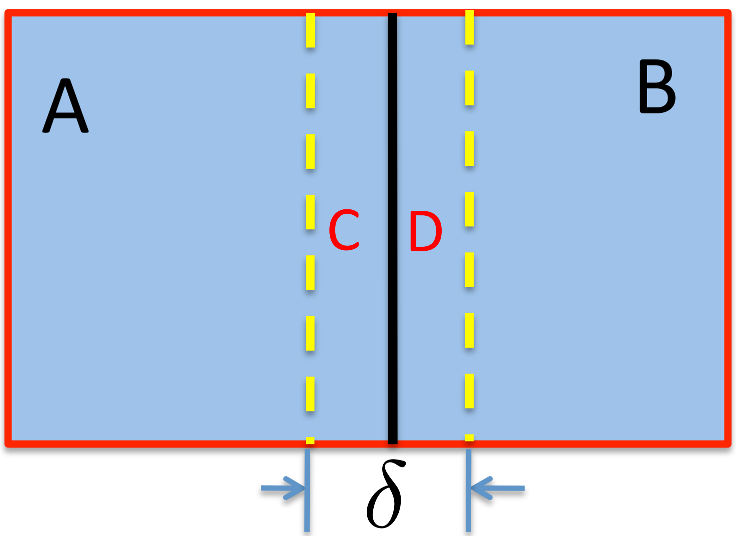
Indications from gravity: after local equilibration (t >>1/T)

$$\mathfrak{R}_A(t) \leq v_E^{(S)}$$

Comparing with small incremental entangling conjecture/theorem:

$$\frac{dS_A}{dt} \leq v_E^{(S)}\, s_{\text{eq}} A_\Sigma$$

$$\frac{dS_A}{dt} \leq c||H|| \log d, \qquad d = \min(d_C, d_D)$$



Future directions

- More examples:

    Both holographic and field theoretical

- More physical intuition on $v_E^{(S)}$
- Direct probe of entanglement tsunami
- Continuum limit of small incremental theorem
- Implications for black hole physics

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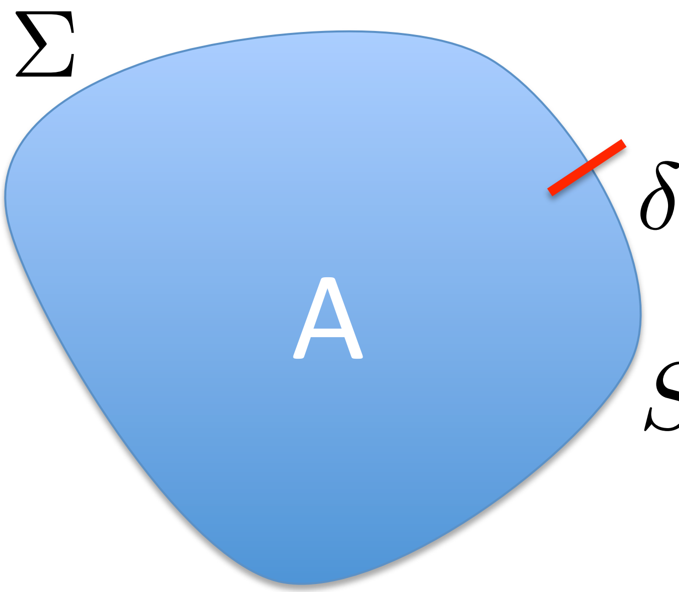
Thank You

Entanglement in the vacuum



$$S_A = S_A^{\text{short}} + S_A^{\text{long}}$$

S_A^{short} : short-range entanglement near Σ, cutoff dependent

S_A^{long} : long range entanglement, insensitive to UV physics near Σ

Vacuum: $S_A^{\text{long}} = \text{const or } \log R$ R: characteristic size of A

Long ranged entangled d.o.f. are measure zero.

Entanglement in equilibrium state

The system behaves macroscopically as a thermal state,
with entanglement entropy disguised as thermal entropy:

$$S_A^{\mathrm{long,eq}} = s_{\mathrm{eq}} V_A$$

$$s_{\mathrm{eq}} : \text{ equilibrium entropy density}$$

V_A : volume of region A

Essentially all d.o.f. inside A becomes long ranged entangled with those outside A.