ON ABJM BDS EXPONENTIATION

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In collaboration with **Marco S. Bianchi**

arXiv:1403.3398

N=4 SYM Exponentiation

Consider color ordered MHV 4p amplitude in $D = 4 - 2\epsilon$

Bern, Dixon, Smirnov hep-th/0505205

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We define the ratio $\mathcal{M}_4^{(l)} = \frac{A_4^{(l)}}{A_4^0}$ s.t. $\mathcal{M}_4 = 1 + \sum_{n=1}^{\infty} \lambda^l \, \mathcal{M}_4^{(l)}$

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1 loop $\mathcal{M}_4^{(1)}(\epsilon) = -\frac{(s/\mu^2)^{-\epsilon}}{\epsilon^2} - \frac{(t/\mu^2)^{-\epsilon}}{\epsilon^2} + \frac{1}{2}\log^2\left(\frac{s}{t}\right) + C + \mathcal{O}(\epsilon)$

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BDS Ansatz $\mathcal{M}_4 = \exp\left[\sum_{l=1}^{\infty} \lambda^l \left(f^{(l)}(\epsilon)\mathcal{M}_4^{(\mathbf{1})}(\mathbf{l}\,\epsilon) + c^{(l)}\right)\right]$

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ABJM Theory

- 3D Superconformal field Theory
- N=6 SUSY, 24 supercharges
- Dual to IIA Strings on $AdS_4 \times CP_3$

Aharony, Bergman, Jafferis, Maldacena arXiv:0806.1218

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- Two CS fields $U_k(N) \times U_{-k}(N)$
- Matter in the bifundamental:

$$\phi^I, \psi^I, \quad I = 1, 2, 3, 4$$

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ABJM scattering amplitudes

Bianchi, ML, Mauri, Penati, Santambrogio arXiv:1107.3139; Chen, Huang arXiv:1107.2710

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On shell:

$$\Phi(\eta) = \phi^4 + \eta^I \psi_I + \frac{1}{2}\epsilon_{IJK}\eta^I\eta^J\phi^K + \frac{1}{3!}\epsilon_{IJK}\eta^I\eta^J\eta^K\psi_4$$

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Unitarity Cuts ➡ Integrand of $\mathcal{M}_4^{(3)}(\epsilon)$

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Unitarity Cuts ➡ Integrand of $\mathcal{M}_4^{(3)}(\epsilon)$ ➡ **3** loop result

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Results

$$\mathcal{M}_4^{(3)}(\epsilon) = -\frac{\pi}{2}\left((s/\mu^2)^{-3\epsilon} + (t/\mu^2)^{-3\epsilon}\right)\left[\frac{1}{\epsilon}\left(\arctan^{-1}\sqrt{\frac{s}{s+t}} + \arctan^{-1}\sqrt{\frac{t}{s+t}}\right) + f(s/t)\right]$$

M. S. Bianchi, ML, arXiv:1403.3398

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- Evidence for exponentiation - DCI??

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- Need 4 loops to verify if recurrence is satisfied with 2 loop result