Aspects of 6d SCFTs & LSTs

Kimyeong Lee, KIAS
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Stefano Bolognesi, Hirotaka Hayasi, Hee-Cheol Kim, Joonho Kim, Seok Kim, Sung-Soo Kim, Eunkyung Koh, Sungjay Lee, Jaemo Park, Masato Taki, Cumrun Vafa, Futoshi Yagi, Ho-Ung Yee

Outline

- 6d (2,0) SCFTs
- 6d (1,0) SCFTs
- 5d reduction of 6d (1,0) SCFTs
- T-duality of 6d LSTs

6d (2,0) SCFTs

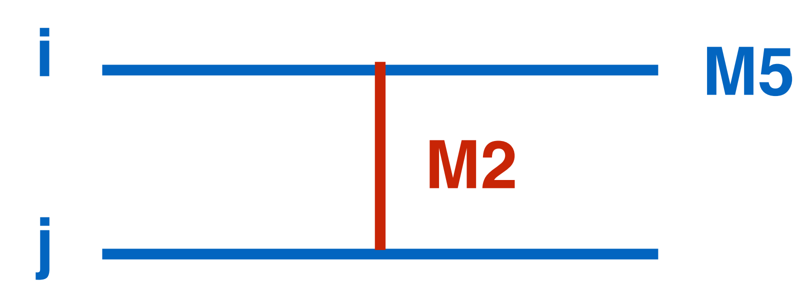
KL,Yee’06,Bolognesi,KL’11,Kim^2,Koh,Lee^2’11,H.C.Kim,KL’12,KIm^3,KL’13

Witten,Seiberg,Douglas,Lambert,Papageorgakis,Schmidt-Sommerfeld,Kimmey,Maldacena,Minwalla,Raju,J.Bhattacharya,S. Bhattacharya

A-type SCFTs

- on A_{N-1} singularity in type IIB **(Witten'95)**
- on N M5 branes **(Strominger'95Witten'95)**
- on a single M5 brane
  - (2,0) tensor multiplet: B, Φ_I , Ψ : $\gamma^6 \Psi=\Psi$ (chiral) with field strength: H=dB=*H (self-dual)
- The source for the tensor is a M2 brane ending on the M5 brane: selfdual strings *d*H=J
- tensionless strings at the symmetric phase

	0	1	2	3	4	5	6	7	8	9	11
M5	●	●	●	●	●	●					
M2	●					●	●				



A-type SCFTs

- Non-Lagrangian theory
- degrees of freedom in large N: N^3
- In tensor branch
  - N 1/2 BPS massless tensor multiplets
  - N(N-1)/2 1/2 BPS self-dual strings
  - N(N-1)(N-2)/6 1/4 BPS self-dual junctions

Klebanov,Tseytilin'96

compactification to 5d

- compactify on a circle of radius R: $x^5 \sim x^5 + 2\pi R$
- 5d N=2 super Yang-Mills theory with coupling constant $8\pi^2/g^2 = 1/R$
- instantons: the Kaluza-Klein modes:
- instanton dynamics: threshold bound states

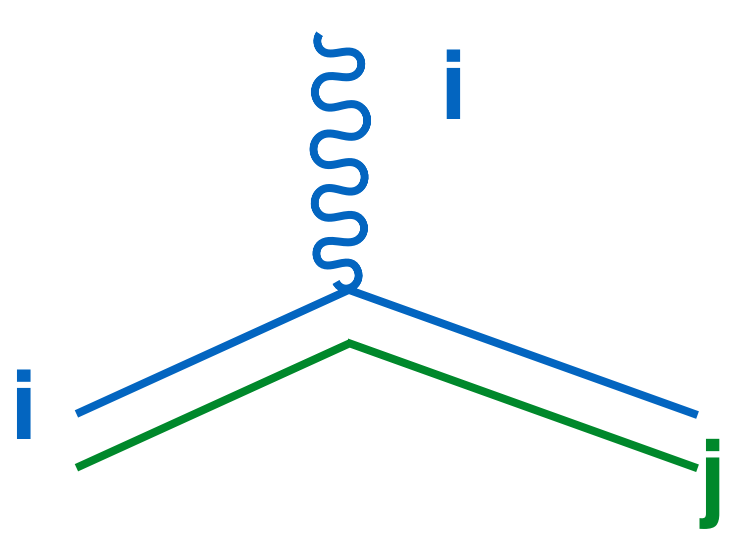
H.C.Kim,S.Kim,E.Koh,KL,S.Lee'11

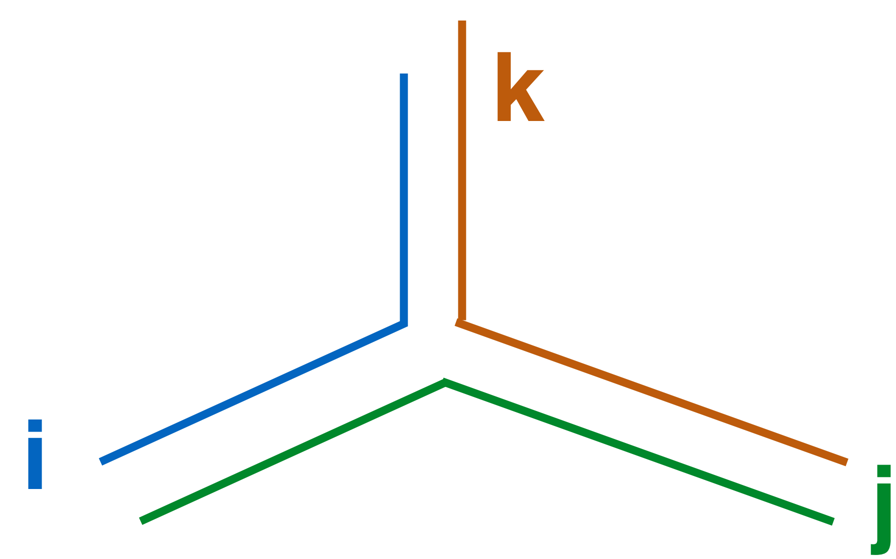
Nekrasov'02,Nekrasov,Okounkov'03

Douglas'11,Lambert,Parpageorgakis,Schmidt-Sommerfeld'11

5d YM coupling

- Cartan: massless tensor multiplets on M5 branes
- W-bosons: self-dual strings between two M5 branes
- Interaction
  - $f_{\alpha\text{-}\alpha i}$: root root cartan = tensor-selfdual string
  - $f_{\alpha\beta\gamma}$: three roots: junction of three self-dual strings





6d Kapustin-Witten equations

- KW-equation: 1/16 BPS equation for monopole string junctions in 4d: lock $SO(4)_{\text{spatial}}$ to $SO(4)_R$ subgroup

$$F_{ab} = \epsilon_{abcd} D_c \phi_d - i[\phi_a, \phi_b], \;\; D_a \phi_a = 0$$

- dyonic one in 5d N=2 SYM

$$F_{a0} = D_a \phi_5, \;\; D_a^2 \phi_a = [\phi_a, [\phi_a, \phi_5]]$$

- 6d abelian equation: lock $SO(5)_{\text{spatial}}$ to $SO(5)_R$

$$H_{abc} = \epsilon_{abcde} \partial_d \phi_e = \frac{1}{2} \epsilon_{abcde} H_{de0}, \quad D_a \phi_a = 0$$

$$\Gamma^{0a} \rho^a \epsilon = 0 \text{ for } a = 1, 2, 3, 4, 5$$

5 dim web of self-dual strings in the tensor phase

chiral primary operators?

- The way to calculate chiral primary operator of SCFT on R^6 is to calculate the Witten index on S^5 x R. We choose supercharge Q and S so that

$$\mathcal{Q}^2 \sim E - 2(R_1 + R_2) - j_1 - j_2 - j_3$$

- We define the Witten index with $a_1+a_2+a_3=0$ as

$$Z_{S^5\times S^1}(\beta, m, a_i) \equiv \mathrm{Tr}\left[(-1)^F e^{-\beta(E-\frac{R_1+R_2}{2})} e^{-\beta a_i j_i} e^{\beta m \frac{R_1-R_2}{2}}\right]$$

- Express this in a path integral, and evaluate using the localization.

$$Z_{S^5\times S^1}(\mu) = \int [d\phi] e^{-S_0(\phi)} Z^{(1)}_{\mathbb{R}^4\times T^2}(\phi,\mu) Z^{(2)}_{\mathbb{R}^4\times T^2}(\phi,\mu) Z^{(3)}_{\mathbb{R}^4\times T^2}(\phi,\mu)$$

Kim^2'12,Lockhart,Vafa'12,Kim^3'12

$S^1 x S^5/Z_K$

- We compactify the Euclidean time circle $\tau \sim \tau + \beta$. The metric for $S^5 x S^1$ is

$$ds^2_{S^1\times S^5} = d\tau^2 + ds^2_{CP^2} + (dy+V)^2, \;\; J = \frac{1}{2}dV = \text{Kahler form}$$

- S^5 is a circle fibered over CP^2.

- In the large β limit, the index function is clearer.

- Z_K-modding along the fiber direction y with a R-charge twist, preserving some supersymmetry.

- On $S^1 x CP^2$, there is a Yang-Mills + Chern-Simons term $J\wedge$ tr (AdA+…), quantized overall coupling constant $K/4\pi^2$

Kim,KL'12,Kim[3]KL'13

on $S^1 x CP^2$

$$Q = Q^{++}_{---},\, S = Q^{--}_{+++}$$

- Lagrangian on R x CP^2 with 2 supersymmetries for any p:

$$\begin{aligned} S \;=\; & \frac{K}{4\pi^2}\int_{\mathrm{R\times CP^2}} d^5x\sqrt{|g|}\;\mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2\sqrt{|g|}}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta-\frac{2i}{3}A_\rho A_\sigma A_\eta\Big) \\ & -\frac{1}{2}D_\mu\phi_I D^\mu\phi_I+\frac{1}{4}[\phi_I,\phi_J]^2-2\phi_I^2-\frac{1}{2}(M_{IJ}\phi_J)^2-i(3-p)[\phi_1,\phi_2]\phi_3-i(3+p)[\phi_4,\phi_5]\phi_3 \\ & -\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda-\frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda]-\frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn}+\frac{1}{8}\bar\lambda M_{IJ}\rho_{IJ}\lambda\Big], \end{aligned} \tag{2.27}$$

- Supersymmetry Transformation

$$\begin{aligned} \delta A_\mu &= +i\bar\lambda\gamma_\mu\epsilon = -i\bar\epsilon\gamma_\mu\lambda, \quad \delta\phi_I = -\bar\lambda\rho_I\epsilon = \bar\epsilon\rho_I\lambda, \\ \delta\lambda &= +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD_\mu\phi_I\rho_I\gamma^\mu\epsilon - \frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon - 2\phi_I\rho_I\tilde\epsilon - M_{IJ}\phi_I\rho_J\epsilon. \end{aligned}$$

- p/2=-1/2 : $k = j_1+j_2+j_3+ R_1+ 2R_2$
  - additional supersymmetries: Total 8 supersymmetries

$$Q^{+-}_{-++},\; Q^{+-}_{+-+},\; Q^{+-}_{++-} \quad \text{conjugates}$$

Expected: K=3: 10, K=2: 16, K=1: 32

on $S^1 x CP^2$

- Unrefined index with m= 1/2 - a_3, and we get the exact partition function

$$e^{\beta\omega_3\left(\frac{N(N^2-1)}{6}+\frac{N}{24}\right)}\prod_{s=0}^{\infty}\prod_{d=1}^{N}\frac{1}{1-e^{-\beta\omega_3(d+s)}}\ .$$

$$e^{\beta\omega_3(\frac{N(N^2-1)}{6}+\frac{N}{24})}\mathrm{PE}\left(\frac{q+q^2+\cdots q^N}{1-q}\right)$$

Kim[3]'12,Beem,Rastelli,vanRees'14

on $S^1 x CP^2$

- K=1 case
- Ground state is $F= 2J(s_1,s_2,\ldots,s_N)=2J(N-1,N-3,\ldots,-(N-3),-(N-1))$.
  Instanton number is $-1/2\ \Sigma_i\ s_i^2 = N(N^2-1)/6$.
- Vacuum Energy: $E = -N(N^2-1)/6 - N/24$
- Excited states can be obtained by add instantons in three fixed and reducing the uniform fluxes by $2J(\ldots -1,\ldots,1)$.

index function

$$Z_{S^5\times S^1} = 1 + qy + q^2\left[2y^2 + y(y_1+y_2+y_3) - (y_1^{-1}+y_2^{-1}+y_3^{-1}) + y^{-1}\right] + \mathcal{O}(q^3)$$

$$\begin{aligned} U(2) \;&:\; q^3\left[2y^3 + 2y^2(y_1+y_2+y_3) + y\left(y_1^2+y_2^2+y_3^2 - \frac{1}{y_1} - \frac{1}{y_2} - \frac{1}{y_3}\right)\right. \\ &\left. - \left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\frac{y_2}{y_3}+\frac{y_3}{y_2}+\frac{y_3}{y_1}+\frac{y_1}{y_3}\right) + y^{-1}(y_1+y_2+y_3)\right] \\ U(3) \;&:\; q^3\left[3y^3 + 2y^2(y_1+y_2+y_3) + y\left(y_1^2+y_2^2+y_3^2 - \frac{1}{y_1} - \frac{1}{y_2} - \frac{1}{y_3}\right)\right. \\ &\left. - \left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\frac{y_2}{y_3}+\frac{y_3}{y_2}+\frac{y_3}{y_1}+\frac{y_1}{y_3}\right) + y^{-1}(y_1+y_2+y_3)\right] \end{aligned}$$

6d (1,0) SCFTs

Kim,Kim,KL,Park,Vafa,'14,Kim,Kim,Lee'15

6d (1,0) SCFTs

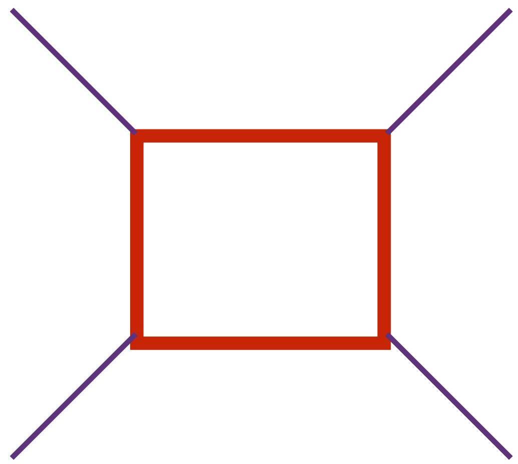
Seiberg'96,Danielsson et.al.'97

Heckman,Morrison,Vafa(Heckman('13),Morrison, Rudelius, Vafa ('15)

- vector multiplet ($A\mu,\lambda$)
- hyper multiplet (ϕ,ψ)
- tensor multiplet (B,Ψ,Φ)
- fermion helicity

	helicity
vector	(0,1)
hyper	(1,0)
tensor	(1,0)
Q	(1,0)

Gauge Anomaly



$$\mathrm{Tr}_R F^4 = \alpha_R \mathrm{tr} F^4 + c_R (\mathrm{tr} F^2)^2$$

$$\alpha_{\mathrm{tot}} = (\alpha_{\mathrm{adj}} - \sum_{\mathrm{hyper}\ R} \alpha_R) = 0$$

$$c_{\mathrm{tot}} = (c_{\mathrm{adj}} - \sum_{\mathrm{hyper}\ R} c_R) \geq 0$$

- The gauge anomaly polynomial is made of two pieces.
  - α_{tot}=0 : vector + hyper
  - c_{tot}=0 : vector+hyper+ tensor
    - tensor-vector coupling via the Green-Schwartz mechanism

$$H^2 + \sqrt{c_R}(B \wedge \mathrm{tr} F \wedge F + \Phi F^2)$$

Example

- Start from a M5 near E8 wall
- Add A_{N-1} —type singularities
- Equivalent to O8+8D8+ NS5 + N D6 branes

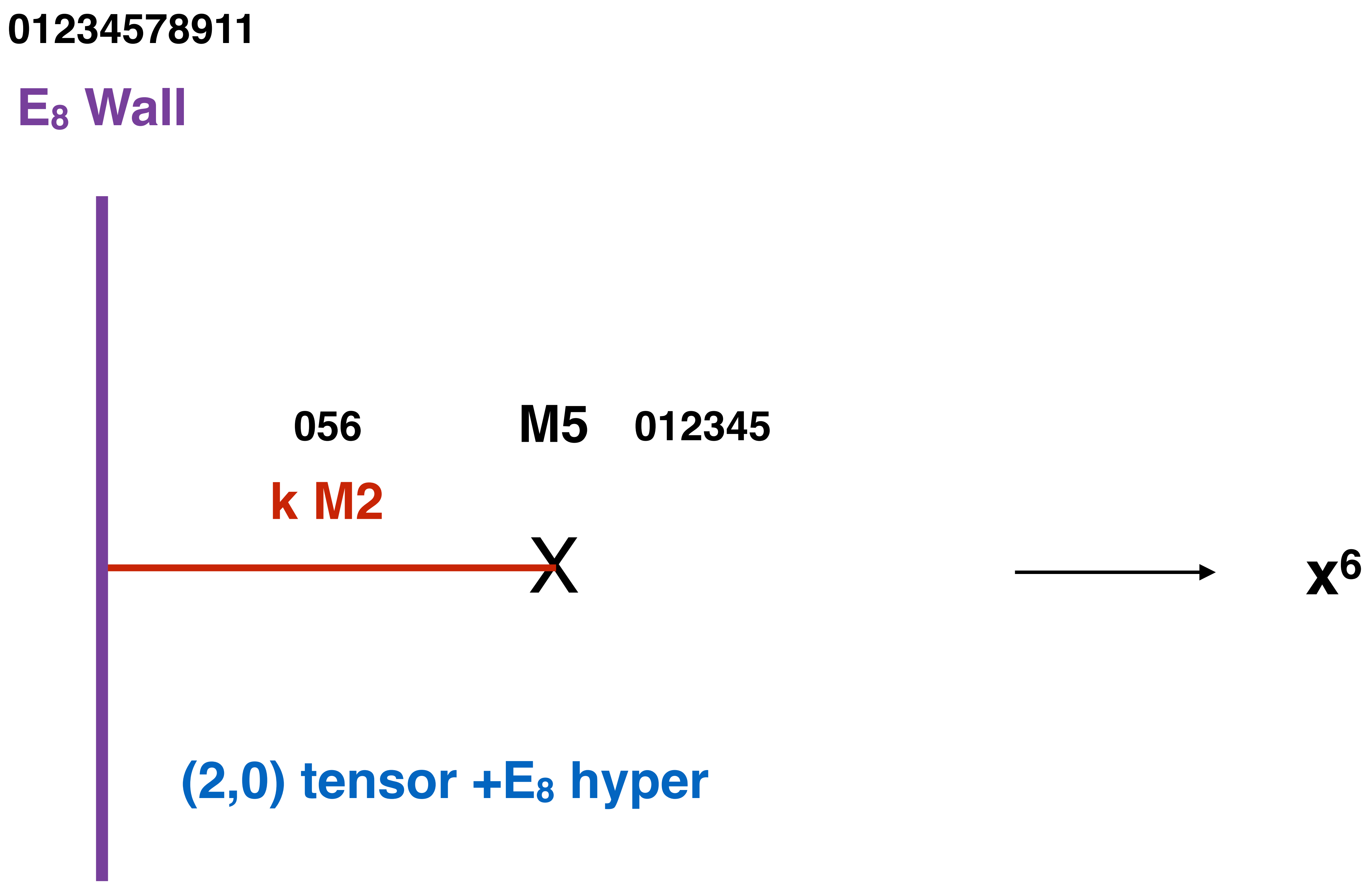
	0	1	2	3	4	5	6	7	8	9	11
E_8 wall	●	●	●	●	●	●		●	●	●	●
M5 brane	●	●	●	●	●	●					
M2 brane	●					●	●				
$\mathbb{C}^2/\Gamma_{A_{N-1}}$								x	x	x	x

Example

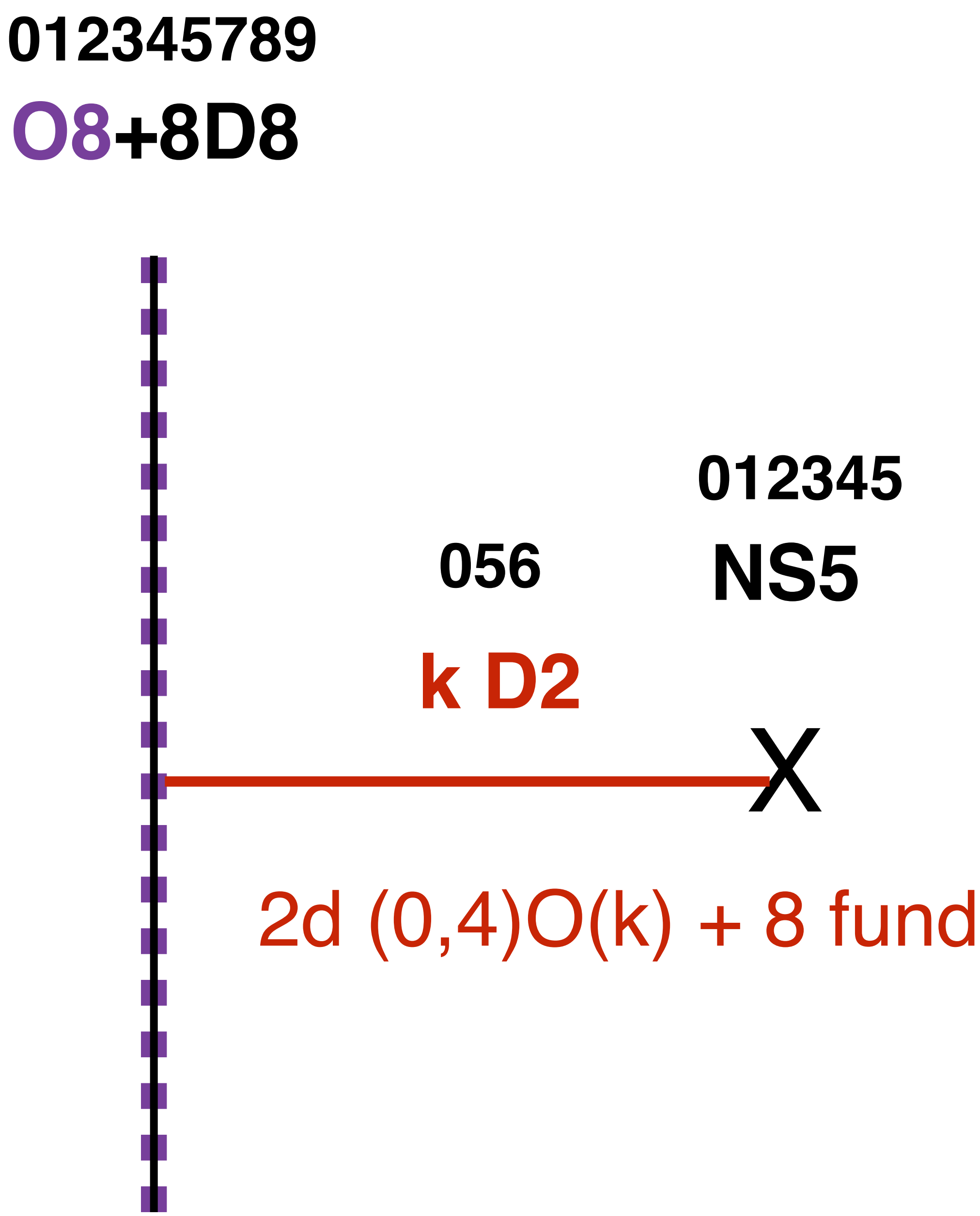
- Start from a M5 near E8 wall
- Add A_{N-1}—type singularities
- Equivalent to O8+8D8+ NS5 + N D6 branes

	0	1	2	3	4	5	6	7	8	9
O8+8D8	•	•	•	•	•	•		•	•	•
NS5 brane	•	•	•	•	•	•				
D2 brane	•					•	•			
D6 brane	•	•	•	•	•	•	•			

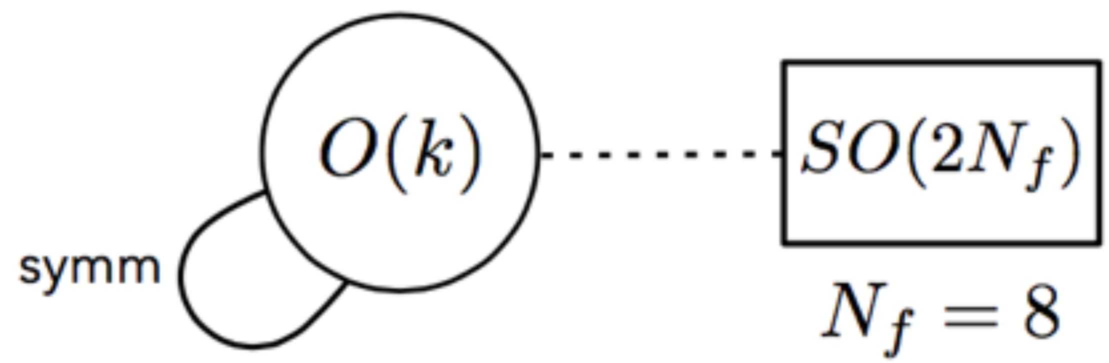
rank 1 with E_8 global symmetry



rank 1 with E_8 global symmetry



Kim,Kim,KL,Park,Vafa'14

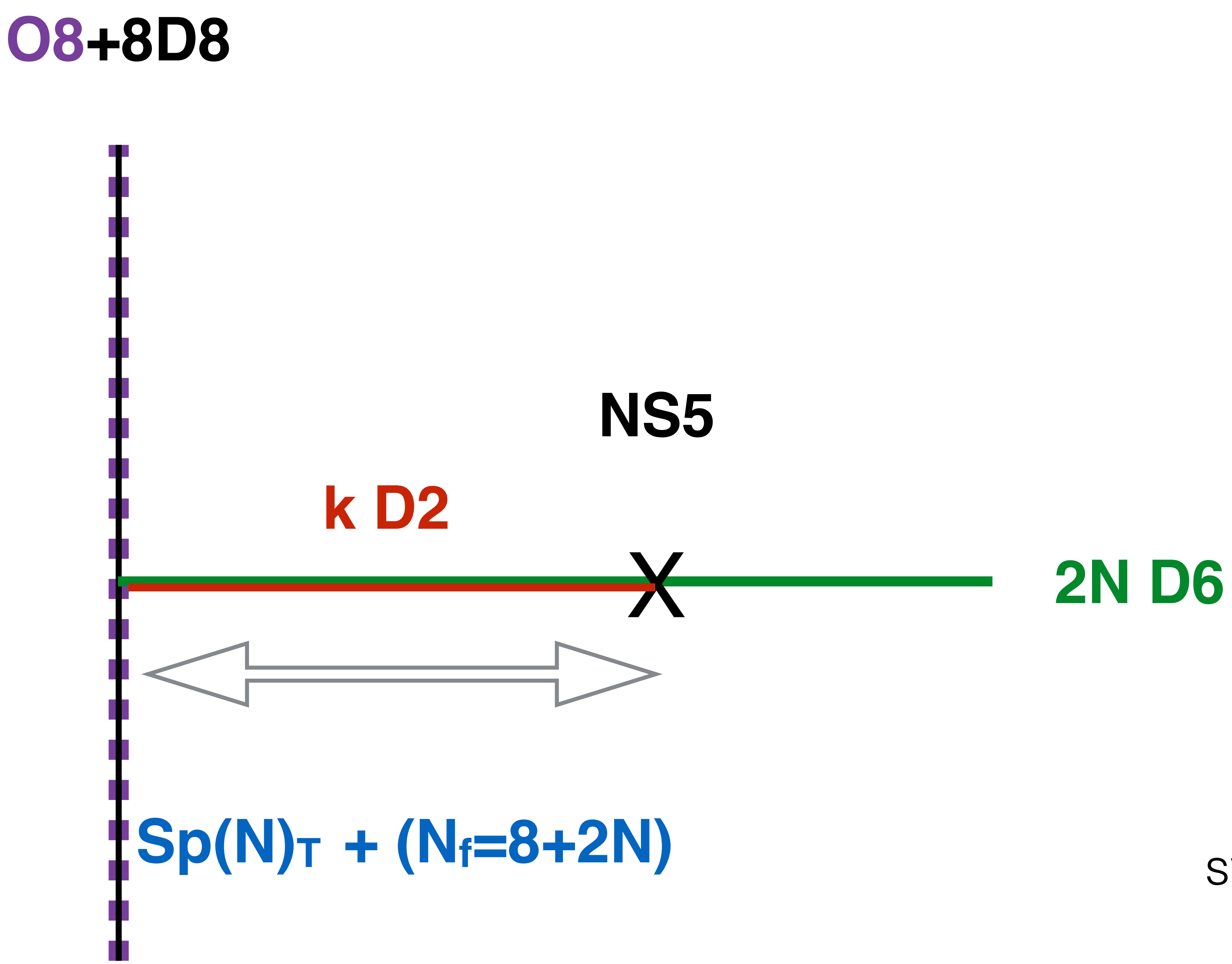


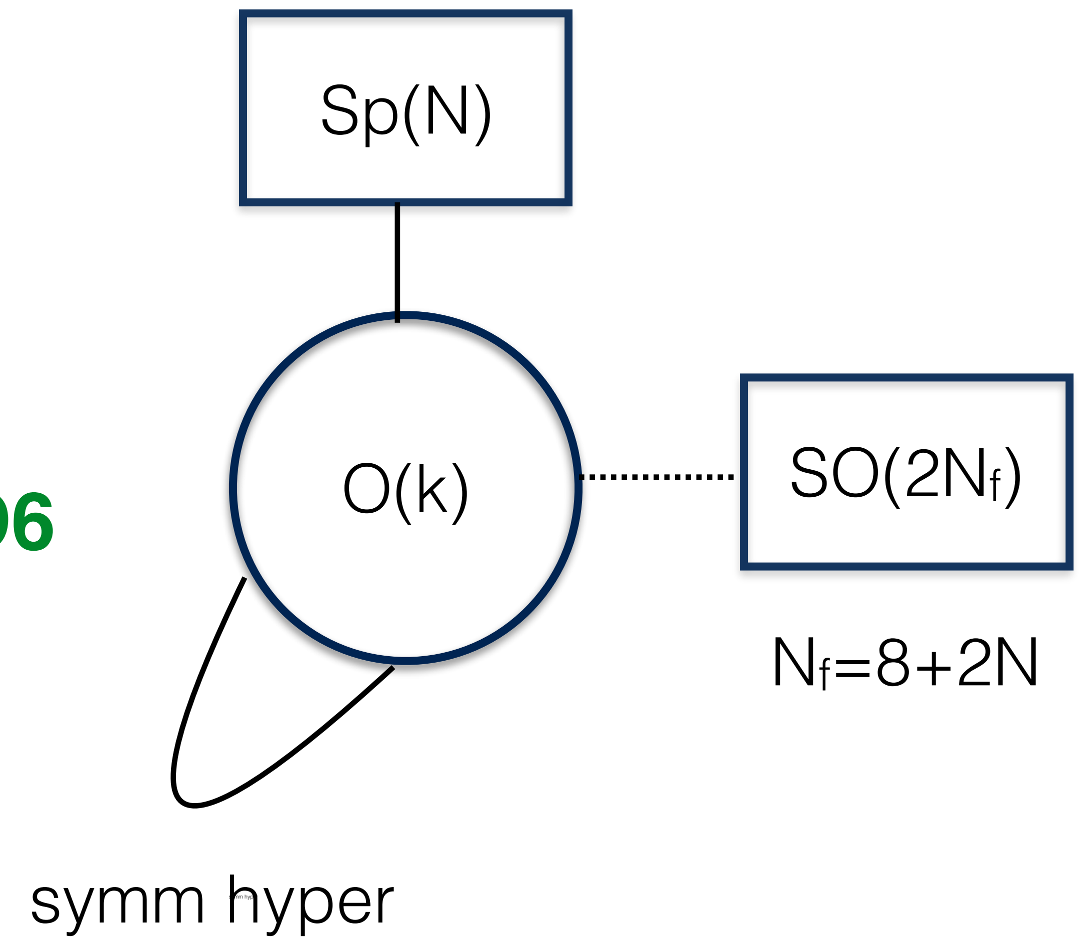
(b) E-string

Elliptic genus calculation

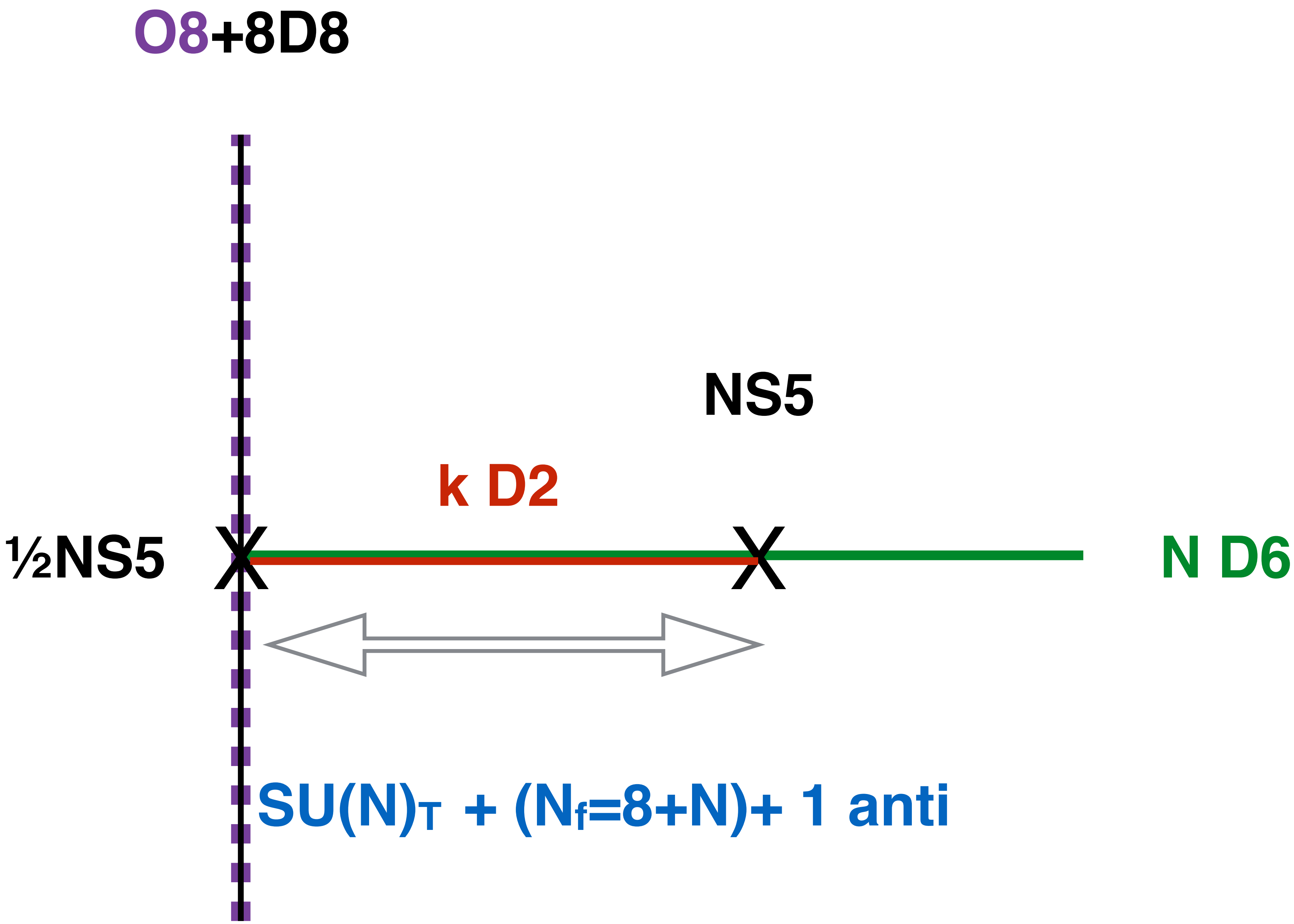
Multiple M5 branes: one less chemical potential

rank 1 with SO(16+4N) symmetry





6d $SU(N)_T$ + (8+N) fund + 1 antisym

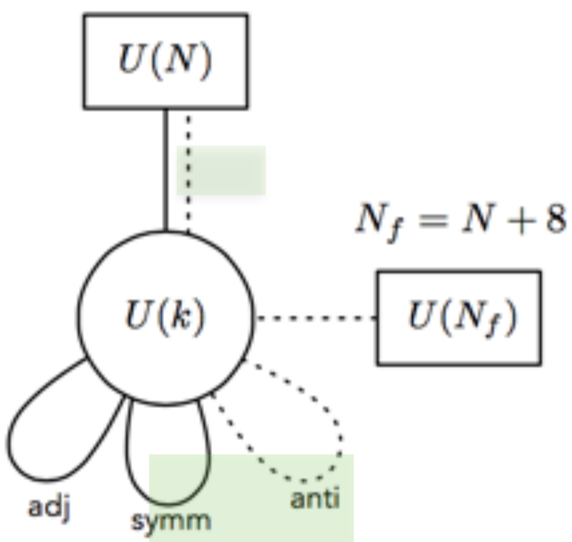


Brunner,Karch'98,Hanany,Zaffaroni'98

2d (0,4) string dynamics

Kim,Kim,KL'15

- String dynamics can be written by a quiver-diagram. Elliptic genus can be calculated. The enhancement of the global symmetry can be tested.



(a)

Field	Type	$U(k)$	$U(N)$	$U(N_f)$	$U(1)_A$
$(A_\mu, \lambda^{\dot{\alpha}A})$	vector	**adj**	–	–	0
$(a_{\alpha\dot{\beta}}, \chi_\alpha^A)$	hyper	**adj**	–	–	0
$(q_{\dot{\alpha}}, \psi^A)$	hyper	**k**	$\overline{\mathbf{N}}$	–	0
(Ξ_l)	Fermi	**k**	–	$\overline{\mathbf{N_f}}$	0
$(\varphi_A, \Phi^{\dot{\alpha}})$	twisted hyper	**sym**	–	–	+1
(Ψ_α)	Fermi	**anti**	–	–	+1
(ψ)	Fermi	**k**	**N**	–	+1

(b)

Index function

- Choose charge one of $Q_{+\dot{\alpha}A}$ to define the index function
- BPS: $H=P^5$ along a compactified circle $x^5 \sim x^5+2\pi R$
- ε_1: 1-2, ε_2: 3-4, ε_3: 6-7, ε_4: 8-9
- Calculating the Witten index function of each LST on a circle.

$$Z = \mathrm{Tr}(-1)^F q^{(H+P^5)/2} e^{2\pi i \epsilon_+ (J_+ + J_R)} e^{2\pi i \epsilon_- J_-} e^{2\pi i m J_m} e^{2\pi i \alpha_r \mathcal{F}_r}$$

$$\epsilon_\pm = (\epsilon_1 \pm \epsilon_2)/2, \epsilon_R = (\epsilon_3 + \epsilon_4)/2, \ m = \epsilon_3 - \epsilon_4)/2$$

String Elliptic Genus in (0,4) GLSM

- $Z_n = Tr(-1)^F q^{(H+P5)/2}$ ….. for n-strings
- express it in the path integral
- localize it on holonomy zero modes
- $Z_{1\text{-loop}}=Z_{vec}\, Z_{hyper}\, Z_{fermi}\, Z_{twist}$
- Do the holonomy integral in Jeffrery-Kirwan residue prescription

Benini,Eager,Hori,Tachikawa I,II(13)

single string

$$\oint d\phi\, \frac{\eta^3\,\theta_1(2\epsilon_+)}{i\theta_1(\epsilon_1)\theta_1(\epsilon_2)} \cdot \prod_{i=1}^{N} \frac{\eta\,\theta_1(\phi+a_i+M)}{\theta_1(\epsilon_+\pm(\phi-a_i))} \cdot \frac{\eta^2}{\theta_1(-\epsilon_+\pm(2\phi+M))} \cdot \prod_{l=1}^{N+8} \frac{\theta_1(\phi-m_l)}{\eta}$$

JK prescription: with n>0, we choose the poles of positive charge Q

$$\epsilon_+ + \phi - a_j = 0 \quad (j=1,\cdots,N), \qquad -\epsilon_+ + 2\phi + M = 0,$$

- $\phi = a_j - \epsilon_+ \quad (j=1,\cdots,N)$

$$-\sum_{j=1}^{N} \frac{\eta^{-6}\prod_{l=1}^{N+8}\theta_1(a_j-\epsilon_+-m_l)}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)\theta_1(2a_j-3\epsilon_++M)} \cdot \prod_{i\neq j} \frac{\theta_1(a_i+a_j-\epsilon_++M)}{\theta_1(a_j-a_i)\theta_1(2\epsilon_+-(a_j-a_i))}$$

- $\phi = \frac{\epsilon_+-M}{2} + \ell_I$ for $\ell = \{0, \frac{1}{2}, \frac{1+\tau}{2}, \frac{\tau}{2}\} \quad (I=1,2,3,4)$

$$-\frac{1}{2}\frac{\eta^{-6}}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)}\left[\frac{\prod_{l=1}^{N+8}\theta_1(\frac{\epsilon_+-M}{2}-m_l)}{\prod_{i=1}^{N}\theta_1(\frac{3\epsilon_+-M}{2}-a_i)} + (-1)^N\sum_{I=2}^{4}\frac{\prod_{l=1}^{N+8}\theta_I(\frac{\epsilon_+-M}{2}-m_l)}{\prod_{i=1}^{N}\theta_I(\frac{3\epsilon_+-M}{2}-a_i)}\right]$$

two strings

$$\oint \frac{d\phi_{1,2}}{2} \frac{-\eta^6\theta_1(2\epsilon_+)^2}{\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2} \prod_{i\neq j} \frac{\theta_1(\phi_{ij})\theta_1(\phi_{ij}+2\epsilon_+)}{\theta_1(\phi_{ij}+\epsilon_1)\theta_1(\phi_{ij}+\epsilon_2)} \prod_{l=1}^{N+8} \frac{\theta_1(\phi_{1,2}-m_l)}{\eta^2} \prod_{i=1}^{N} \frac{\eta^2\theta_1(\phi_{1,2}+a_i+M)}{\theta_1(\epsilon_+ \pm (\phi_{1,2}-a_i))}$$
$$\times \frac{\eta^4\theta_1(\epsilon_- \pm (\phi_1+\phi_2+M))}{\theta_1(-\epsilon_+ \pm (\phi_1+\phi_2+M))\theta_1(-\epsilon_+ \pm (2\phi_{1,2}+M))}.$$

We adopt the concise notations such as $\phi_{ij} \equiv \phi_i - \phi_j$, $a_{mn} \equiv a_m - a_n$, $\theta_I(\phi_{i,j}+b) \equiv \theta_I(\phi_i+b)\,\theta_I(\phi_j+b)$, $\theta_I(a_{m,n}+b) \equiv \theta_I(a_m+b)\,\theta_I(a_n+b)$, $\theta_{I,J}(b) \equiv \theta_I(b)\,\theta_J(b)$. The Weyl group $W \subset U(2)$ is $\mathbb{Z}_2$. After picking an auxiliary vector $\mathfrak{n}$ to be $(+1,+1)$, we collect all contributing residues given as follows.

poles

$(\phi_1, \phi_2) = (a_m - \epsilon_+, a_n - \epsilon_+)$ for $m \neq n$.

$(\phi_1, \phi_2) = (\frac{\epsilon_+ - M}{2} + \ell_I, a_m - \epsilon_+)$ and $(\phi_1, \phi_2) = (a_m - \epsilon_+, \frac{\epsilon_+ - M}{2} + \ell_I)$

more…..

Enhanced global symmetry on strings

- SU(1) case: D6 brane disappears in the strong coupling limit.
  - U(N_f=9) gets enhanced to E_8 global symmetry. With multiplet M5s, this formalism provides one more chemical potential along the transverse to M5 branes.
- SU(2) case: U(10) gets enhanced to SO(20)
- SU(3) case: (N_f=11) + (N_a=1) with symmetry U(11)xU(1) get enhanced to U(12) as the anti-symmetric hyper is equivalent to fundamental hyper
  - It is not clear from the instanton string dynamics. Elliptic

testing SO(20) with Sp(1)=SU(2)

Approach 2d O(1) for 6d Sp(1) in Example I

$$-\frac{\eta^2}{\theta_1(\epsilon_{1,2})}\sum_{I=1}^{4}\frac{\eta^2}{\theta_I(\epsilon_+\pm a)}\prod_{l=1}^{10}\frac{\theta_I(m_l)}{\eta}$$

Approach 2d U(1) for 6d SU(2) in Example II

$$-\frac{\eta^{-6}}{\theta_1(\epsilon_{1,2})}\left[\frac{\prod_{l=1}^{10}\theta_1(a-\epsilon_+-m_l)}{\theta_1(2a-3\epsilon_++M)}\frac{\theta_1(-\epsilon_++M)}{\theta_1(2a)\theta_1(2\epsilon_+-2a)}+(\pm a\to\mp a)\right]-\frac{\eta^{-6}}{\theta_1(\epsilon_{1,2})}\sum_{I=1}^{4}\frac{\prod_{l=1}^{10}\theta_I(\frac{\epsilon_+-M}{2}-m_l)}{2\theta_I(\frac{3\epsilon_+-M}{2}\pm a)}$$

Expand in q power

$$t=e^{2\pi i\epsilon_+},\ u=e^{2\pi i\epsilon_-},\ y_i=e^{2\pi i\tilde{m}_i},\ \overline{y}=e^{2\pi i\overline{m}},\ Y=e^{2\pi iM},\ w_i=e^{2\pi i\tilde{a}_i},\ \overline{w}=e^{2\pi i\overline{a}}.$$

$$\begin{aligned}&\frac{t}{(1-tu)(1-tu^{-1})}\Big[q^{-1/2}+\frac{q^{1/2}\cdot t^2}{(1-t^2w_1^2)(1-t^2w_1^{-2})}\Big(-\chi^{SO(20)}_{\mathbf{512}}\chi^{SU(2)}_{1/2}(w_1)+\chi^{SO(20)}_{\mathbf{512}}\chi^{SU(2)}_{1/2}(t)\\&+\chi^{SO(20)}_{\mathbf{20}}\chi^{SU(2)}_{1/2}(t)\chi^{SU(2)}_{3/2}(w_1)-\chi^{SO(20)}_{\mathbf{20}}\chi^{SU(2)}_{3/2}(t)\chi^{SU(2)}_{1/2}(w_1)-\chi^{SO(20)}_{\mathbf{190}}\chi^{SU(2)}_{1}(w_1)+\chi^{SU(2)}_{1/2}(t)\chi^{SU(2)}_{1/2}(u)\\&+\chi^{SU(2)}_{3/2}(t)\chi^{SU(2)}_{1/2}(u)-\chi^{SU(2)}_{1/2}(t)\chi^{SU(2)}_{1/2}(u)\chi^{SU(2)}_{1}(w_1)+\chi^{SU(2)}_{2}(t)\chi^{SU(2)}_{1}(w_1)-\chi^{SU(2)}_{1}(t)\chi^{SU(2)}_{2}(w_1)\\&+\chi^{SO(20)}_{\mathbf{190}}\chi^{SU(2)}_{1}(t)\Big)+\mathcal{O}(q^{3/2})\Big].\end{aligned}$$

$q^{-1/2}$: zero point energy

testing SU(12) with single string for SU(3)

$$\mathbf{12} \longrightarrow \mathbf{1}_{-11} + \mathbf{11}_{+1}$$
$$\overline{\mathbf{12}} \longrightarrow \mathbf{1}_{+11} + \overline{\mathbf{11}}_{-1}$$
$$\mathbf{143} \longrightarrow \mathbf{1}_0 + \mathbf{11}_{12} + \overline{\mathbf{11}}_{-12} + \mathbf{120}_0,$$

Expand in q power

$$\frac{t^2}{(1-tu)^2(1-tu^{-1})^2}\Big[q^{-1}\cdot\frac{t\,\chi^{\mathrm{SU}(2)}_{1/2}(t)}{(1+tu^{-1})(1+tu)} + q^0\cdot\Big(t^{-2}\chi^{SU(3)}_{\mathbf{8}} + t^{-1}(\chi^{\mathrm{SU}(2)}_{1/2}(u) - \chi^{SU(3)}_{\mathbf{3}}\chi^{\mathrm{SU}(12)}_{\overline{\mathbf{12}}} + \chi^{SU(3)}_{\bar{\mathbf{3}}}\chi^{\mathrm{SU}(12)}_{\mathbf{12}}) + \chi^{\mathrm{SU}(12)}_{\mathbf{143}} + 1 + \chi^{SU(3)}_{\mathbf{8}} + \mathcal{O}(t^1)\Big) + \mathcal{O}(q^1)\Big]. \quad (3.26)$$

5d reduction of 6d (1,0) SCFTs

Hayashi,S.S.Kim,KL,Taki,F.Yagi'15'15,Hayashi,S.S.Kim,KL,F.Yagi'15,'16,Kim,Kim,KL'13

5d reduction of 6d SCFT with E_8 symmetry: $Sp(0)_T$+SO(16)

- 5d N=1 SU(2) gauge theory + 8 fundamental hyper (+ 1 (rank 2) anti-symmetric hyper)
- decoupling hyper and reducing flavor symmetry: 5d
- In infinite coupling limit, $SU(2) + N_f$ fundamental hyper with $SO(2N_f) \times U(1)_I$ gets enhanced E_{N_f+1} global symmetry
- 5d rank 1 SCFTs: $E_0, E_1, \tilde{E}_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8$

Seiberg,

5d reduction of 6d $Sp(1)_T$+SO(20)

- 5d SU(3) + 10 fund hyper: $U(10)_f \times U(1)_I$
- 5d Sp(2) + 10 fund hyper with SO(20)x$U(1)_I$
- 5d SU(3) + 9 fund flavor + 1 anti-sym
- 5d SU(3) + 8 fund flavor + 2 anti-sym
- [4]-SU(2)-SU(2)-[4]
- Decoupling flavors and 5d SCFTs

Hayashi,Kim,KM,Taki,Yagi'15, Yonekura'15, Gaiotto,Kim'15, Zafrir'15, Hayashi,Kim,KL,Yagi'15

Bergman,Zafrir'14

5d N=1 SCFTs with $SU(3)_\kappa + N_f$ fundamental

$N_f \leqq 9$, $N_f + 2|\kappa| \leqq 10$

	N_f	$G_{\lvert\kappa\rvert}$ (κ is the Chern-Simons level)
6-dim	10	$SO(20)_0$
	9	$SO(20)_{\frac{1}{2}}$
	8	$SU(10)_0$, $[SO(16) \times SU(2)]_1$
	7	$[SU(8) \times SU(2)]_{\frac{1}{2}}$, $SO(14)_{\frac{3}{2}}$
	6	$[SU(6) \times SU(2) \times SU(2)]_0$, $SU(7)_1$, $SO(12)_2$
	5	$[SU(5) \times SU(2)]_{\frac{1}{2}}$, $SU(6)_{\frac{3}{2}}$, $SO(10)_{\frac{5}{2}}$
	4	$SU(4)_0$, $[SU(4) \times SU(2)]_1$, $SU(5)_2$, $SO(8)_3$
	3	$SU(3)_{\frac{1}{2}}$, $[SU(3) \times SU(2)]_{\frac{3}{2}}$, $SU(4)_{\frac{5}{2}}$, $SO(6)_{\frac{7}{2}}$
	2	$SU(2)_0$, $SU(2)_1$, $[SU(2) \times SU(2)]_2$, $SU(3)_3$, $SO(4)_4$
	1	$SU(2)_{\frac{5}{2}}$, $SU(2)_{\frac{7}{2}}$
	0	$SU(2)_3$

Hayashi,Kim,KL,Taki,F.Yagi'15,Yonekura'15,Ki,Gaitto'15

5d-6d check: Hayashi,Kim,KL,F.Yagi'16,Yun'16

generalization to 6d $Sp(N)_T$+(8+2N) fund

- 5d SU(n+2) + (8+2n) fund hyper:
- 5d Sp(n+1) + (8+2n) fund hyper:
- Decoupling flavors and 5d SCFTs
- SU(N+1) with $\kappa = N+3-N_f/2$ and N_f is equal to Sp(N)+ N_f

Hayashi,SS.Kim,KM,Taki,Yagi'15,Yonekura'15, Gaiotto,Kim'15, Zafrir'15, Hayashi,SSKim,KL,Yagi'15

5d N=1 SU(n) +$N_f \leqq$ (2n+4)

$N_f \leqq 2n+3$, $N_f + 2|\kappa| \leqq 2n+4$

	N_f	$G_{\lvert\kappa\rvert}$		
6 dim	$2n+4$	$SO(4n+8)_0$		
	$2n+3$	$SO(4n+8)_{\frac{1}{2}}$		
	$2n+2$	$SU(2n+4)_0,$	$[SO(4n+4)\times SU(2)]_1$	
	$2n+1$	$[SU(2n+2)\times SU(2)]_{\frac{1}{2}},$	$SO(4n+2)_{\frac{3}{2}}$	
	$2n$	$[SU(2n)\times SU(2)\times SU(2)]_0,$	$SU(2n+1)_1,$	$SO(4n)_2$

Hayashi,S.Kim,KM,Taki,Yagi'15,Yonekura'15,Kim,
Gaiotto'15

6d (2,0) and (1,1) LSTs

J.M.Kim,S.Kim,KL'15,J.Kim,KL to appear

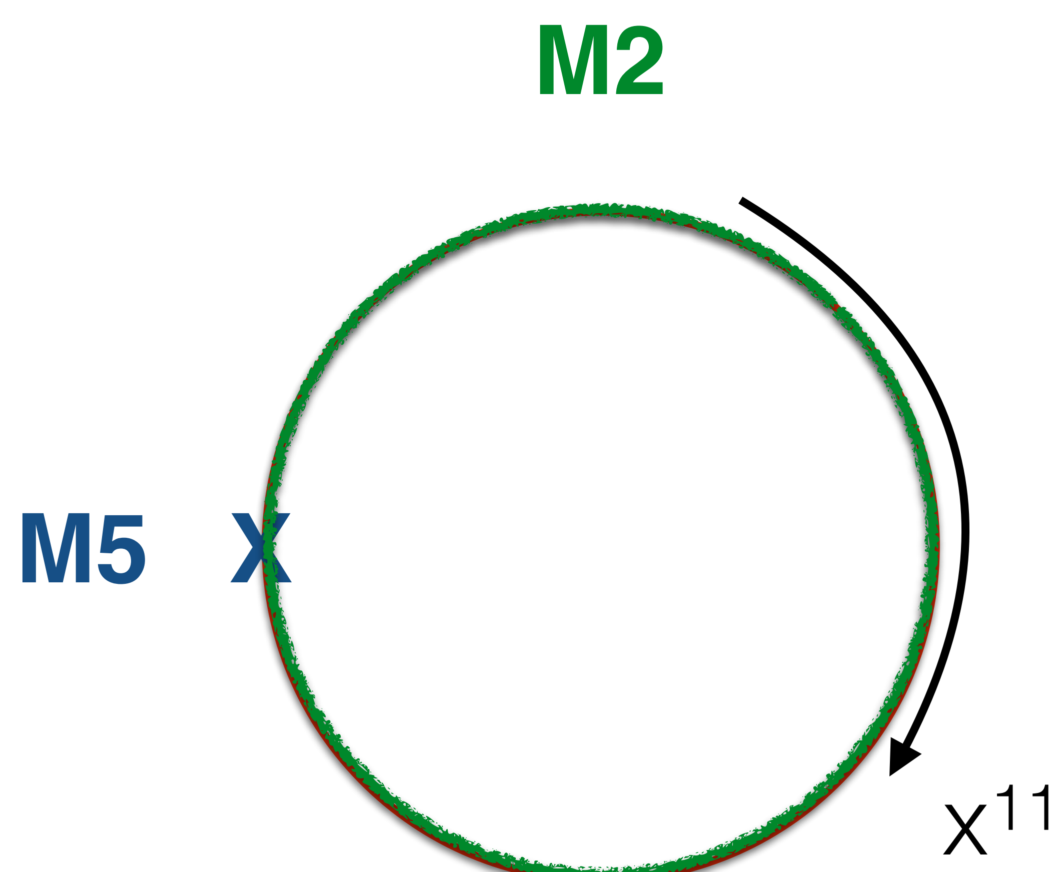
(2,0) & (1,1) LSTs

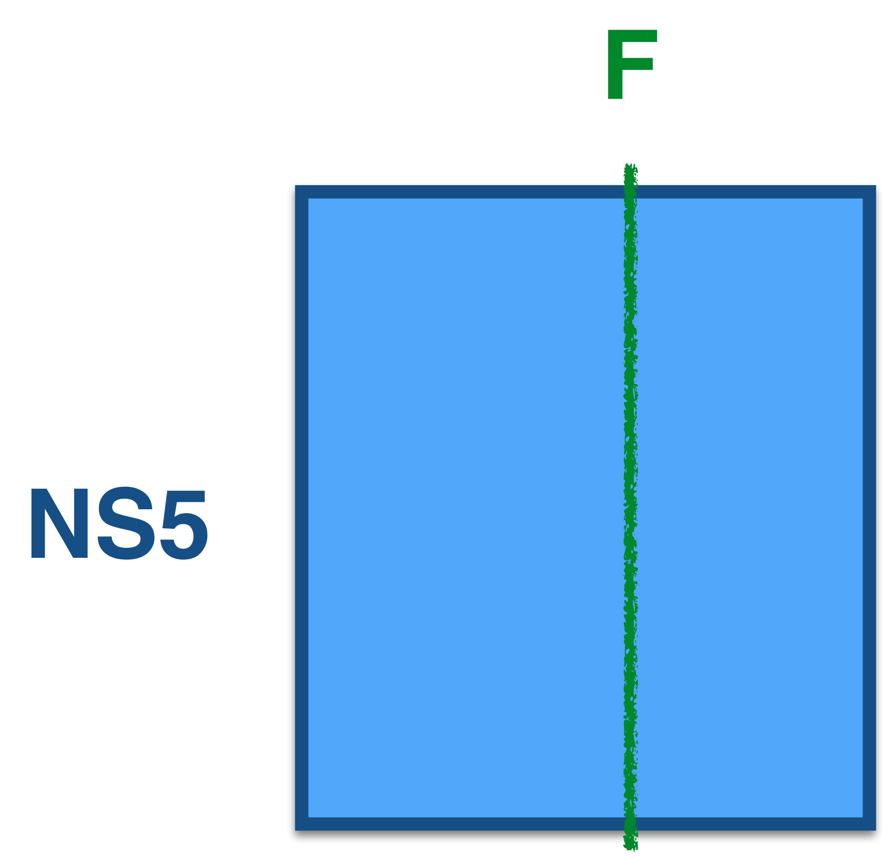
- type IIA & IIB: N NS5 branes + fundamental strings

		ε_1		ε_2			ε_3		ε_4	
	0	1	2	3	4	5	6	7	8	9
NS5	●	●	●	●	●	●				
F1	●					●				

(2,0) LSTs

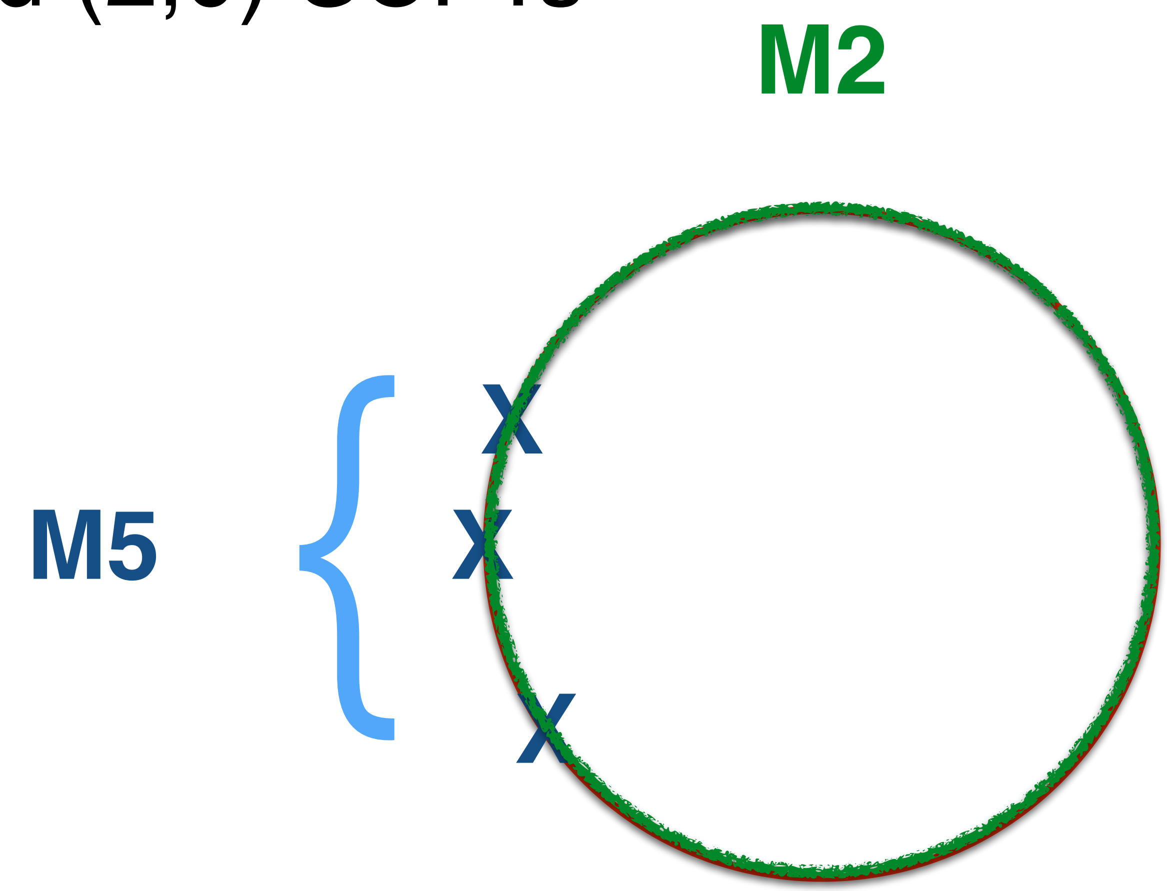
- Physics on N type IIA NS5 branes.
- Vacuum moduli is $(R^4 x S^1)^N/S_N$ with radius $\sim 1/l_s2$
- Abelian case: fundamental string on NS5 brane





(2,0) LSTs

- multiple NS5 branes
- M2 branes connecting M5 branes on M-circle= fractional fundamental strings
- Low energy dynamics = 6d (2,0) SCFTs



(1,1) LSTs

- type IIB NS5 branes which is S-dual to D5 branes
- 6d (1,1) super Yang-Mills theory with $8\pi^2/g^2=1/l_s^2$
- Instanton strings = fundamental strings on NS5 branes
- the vacuum moduli for N NS5 branes or 6d (1,1) SYM is $(R^4)^N/S_N$.

Index function

- Choose charge one of $Q_{+\dot{\alpha}A}$ to define the index function

- BPS: $H=P^5$ along a compactified circle $x^5 \sim x^5+2\pi R$

- ε_1: 1-2, ε_2: 3-4, ε_3: 6-7, ε_4: 8-9 Witten'97,Ahanorny,Berkooz'99

- Calculating the Witten index function of each LST on a circle.

$$Z = \mathrm{Tr}(-1)^F q^{(H+P^5)/2} e^{2\pi i\epsilon_+(J_+ + J_R)} e^{2\pi i\epsilon_- J_-} e^{2\pi i m J_m} e^{2\pi i\alpha_r \mathcal{F}_r}$$

$$\epsilon_\pm = (\epsilon_1 \pm \epsilon_2)/2, \epsilon_R = (\epsilon_3 + \epsilon_4)/2,\ m = \epsilon_3 - \epsilon_4)/2$$

Index function

- BPS states are made of momentum carried by perturbative modes and by strings along x5

- Index functions is a product of two contributions:

$$Z_{LST}(q,w) = Z_{pert}(q) Z_{string}(q,w),$$

$$Z_{string}(q,w) = 1 + \sum_{n_w=1}^{\infty} w^n Z_{n\ strings}(q)$$

$$Z_{LST}(q,w) = \sum_{k,l} q^k w^l Z_{k,l},\ Z_{0,0} = 1$$

T-duality along x^5

- Exchange the momentum and winding modes of (2,0) and (1,1) LSTs with $R_A = \alpha'/R_B$

$$Z^{(1,1)}_{(\text{momentum}=k,\text{winding number}=l)} = Z^{(2,0)}_{(\text{momentum}=l,\text{winding number}=k)}$$

$$Z^{(1,1)}_{LST}(q,w) = Z^{(2,0)}_{LST}(q' = w, w' = q)$$

Z_{pert}

$$I_{pert}^{tensor} = \mathrm{PE}\left[\frac{-t(u+u^{-1})}{(1-tu)(1-tu^{-1})}\frac{q}{1-q}\right]$$

$$I_{pert}^{vector} = \mathrm{PE}\left[\frac{-1-t^2}{(1-tu)(1-tu^{-1})}\frac{q}{1-q}\right]$$

$$I_{pert}^{hyper} = \mathrm{PE}\left[\frac{t(y+y^{-1})}{(1-tu)(1-tu^{-1})}\frac{q}{1-q}\right]$$

$$t = e^{2\pi i \epsilon_+},\ u = e^{2\pi i \epsilon_-},\ y = e^{2\pi i m},$$

$$PE(x) = \frac{1}{1-x} = \exp\left[\sum_{n=1}^{\infty}\frac{x^n}{n}\right]$$

On (2,0) strings

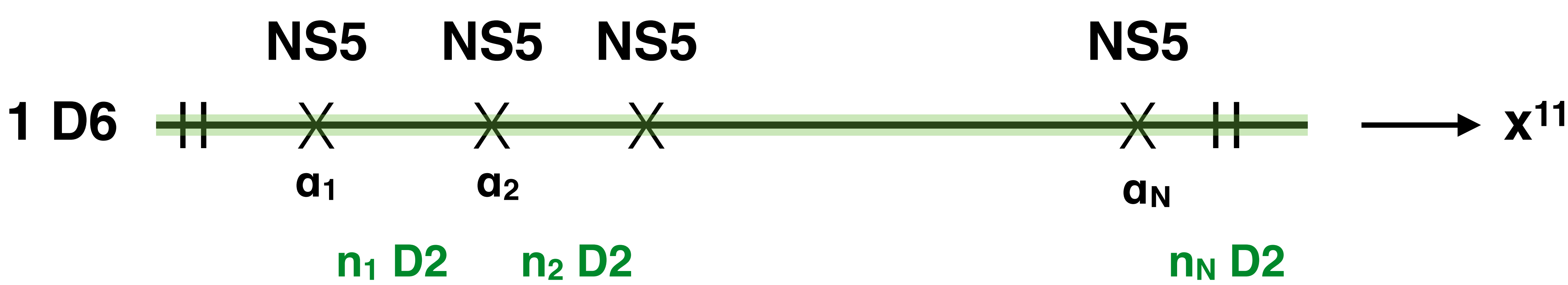
Aharony-Berkooz'99,J.Kim,S.Kim,KL'15

- NS5 branes on M-circle= M5 branes at position $(a_1, a_2, \ldots, a_N)$ on M-circle

- Include a single A_0 branes and S-dual which exchange x^{11} and x^9 : Introduce a single D6 brane

Haghighat,Iqbal,Kozcaz,Lockhart,Vafa (2013)

- Easy to write down the theory on D2 brane segments

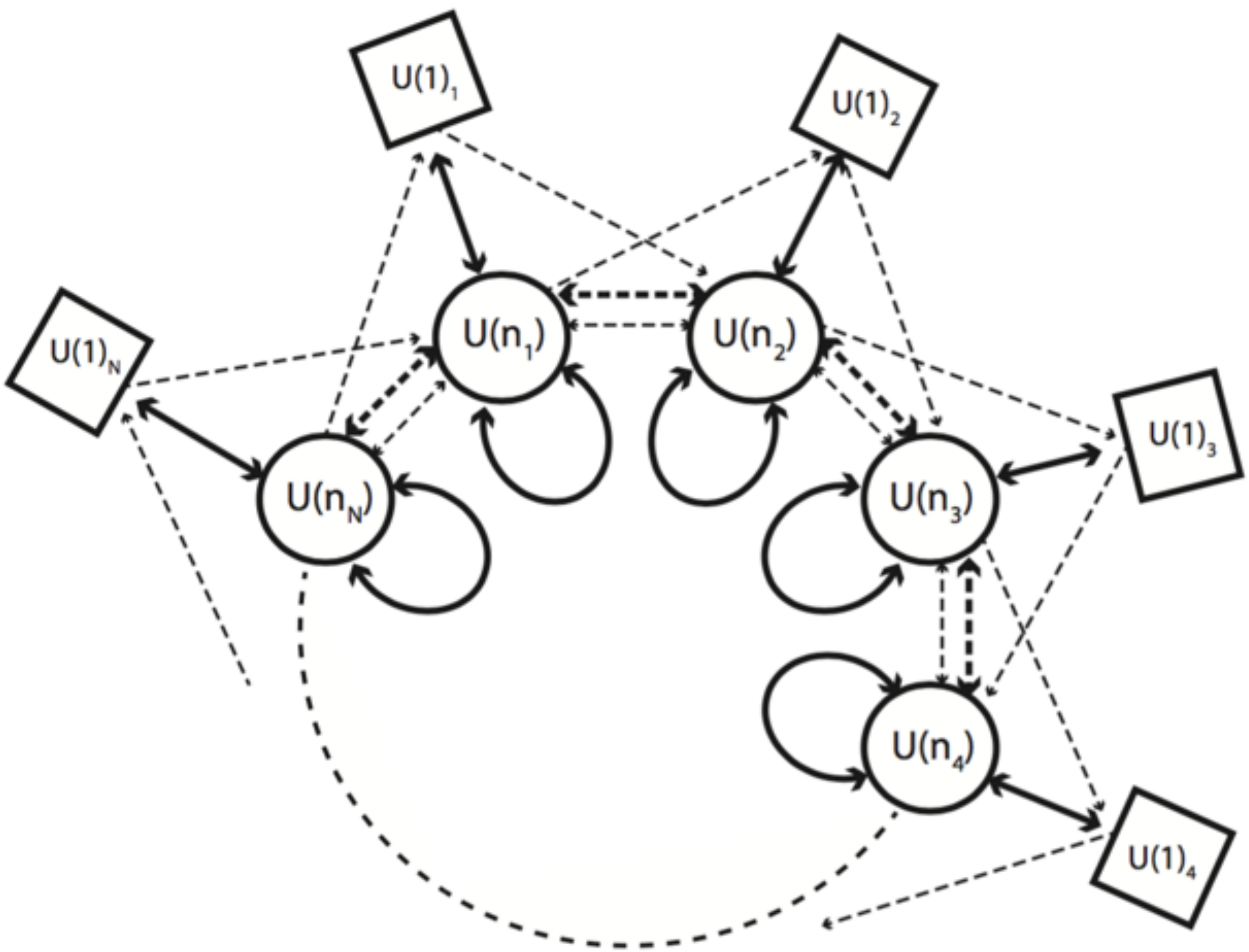


On (2,0) strings

Multiplet	Fields	$U(n_i)$	$U(1)_m$
Vector	$A_\mu^{(i)}, \bar{\lambda}_+^{(i)A\dot{\alpha}}$	adj_i	0
Hyper	$q_{\dot{\alpha}}^{(i)},\ \psi_-^{(i)A}$	$\mathbf{n}_i$	0
Hyper	$a_{\alpha\dot{\beta}}^{(i)},\ \lambda_{\alpha-}^{(i)A}$	adj_i	0
Twisted hyper	$\Phi_A^{(i)},\ \Psi_-^{(i)\dot{\alpha}}$	$(\mathbf{n}_{i-1}, \bar{\mathbf{n}}_i)$	1
Fermi	$\Psi_{\beta+}^{(i)}$	$(\mathbf{n}_{i-1}, \bar{\mathbf{n}}_i)$	1
Fermi	$\psi_+^{(i)}$	$\mathbf{n}_i$	1
Fermi	$\tilde{\psi}_+^{(i)}$	$\bar{\mathbf{n}}_i$	-1

2d (0,4) QFT on fractional D2 strips

On (2,0) strings



On (2,0) strings

- $Z^{(2,0)}{}_{\text{LST}} = Z_{\text{pert}}\, Z_{\text{string}}$

$$Z^{\text{IIA}}_{\text{string}}(\alpha_i, \epsilon_\pm, m; q', w') = \sum_{n_i=0}^{\infty} e^{2\pi i \sum_{i=1}^{N} n_i \alpha_{i,i+1}} Z^{(n_1,\ldots,n_N)}_{\text{string}}(\epsilon_\pm, m; q')$$
$$= \sum_{n_i=0}^{\infty} (v_1)^{n_1} (v_2)^{n_2} \cdots (v_N)^{n_N} Z^{(n_1,\ldots,n_N)}_{\text{string}}(\epsilon_\pm, m; q').$$

$$Z^{(n_1,\ldots,n_N)}_{\text{string}}(\epsilon_\pm, m; q') = \sum_{\{Y_1,\cdots,Y_N\};|Y_i|=n_i} \prod_{i=1}^{N} \prod_{(a,b)\in Y_i} \frac{\theta_1(q'; E^{(a,b)}_{i,i+1} - m + \epsilon_-)\theta_1(q'; E^{(a,b)}_{i,i-1} + m + \epsilon_-)}{\theta_1(q'; E^{(a,b)}_{i,i} + \epsilon_1)\theta_1(q'; E^{(a,b)}_{i,i} - \epsilon_2)}$$

$$E^{(a,b)}_{ij} = (Y_{i,a} - b)\epsilon_1 - (Y^T_{j,b} - a)\epsilon_2\,, \quad E^{(a,b)}_{i,N+1} = E^{(a,b)}_{i,1}$$

On (1,1) strings

- self-dual strings= SU(N) instanton strings
- 2d (4,4) ADHM dynamics on instanton strings
- fractionalization of momentum
- $\alpha_1, \alpha_2, \ldots, \alpha_N, \alpha_{N+1} = \alpha_1 + 2\pi R_A$: the gauge holonomy of YM along x^5 of IIB = the position of M5 branes along x^{11}

On (1,1) strings

- 2d (4,4) ADHM dynamics

$\mathcal{N}=(4,4)$	$\mathcal{N}=(0,4)$	Fields	$U(k)$	$U(N)$
vector	vector	$A_\mu, \bar{\lambda}_+^{A\dot{\alpha}}$	adj	1
	twisted hyper	$\varphi_{aA},\ \bar{\lambda}_{a-}^{\dot{\alpha}}$	adj	1
hyper	hyper	$a_{\alpha\dot{\beta}},\ \lambda_{\alpha-}^{A}$	adj	1
	Fermi	$\lambda_{a\beta+}$	adj	1
hyper	hyper	$q_{\dot{\alpha}},\ \psi_-^A$	$\bar{\mathbf{k}}$	$\mathbf{N}$
	Fermi	ψ_{a-}	$\bar{\mathbf{k}}$	$\mathbf{N}$

On (1,1) strings

Sum over N Young diagrams whose total size is k.

$$Z_k(\alpha_i, \epsilon_\pm, m; q) = \sum_{Y:\sum_i |Y_i|=k} \prod_{i,j=1}^{N} \prod_{s\in Y_i} \frac{\theta_1(q; E_{ij} + m - \epsilon_-)\,\theta_1(q; E_{ij} - m - \epsilon_-)}{\theta_1(q; E_{ij} - \epsilon_1)\,\theta_1(q; E_{ij} + \epsilon_2)}$$

$$E_{ij} = \alpha_i - \alpha_j - \epsilon_1 h_i(s) + \epsilon_2 v_j(s).$$

Testing T-duality

- The extra sector due to FI term (D6 brane) $\hat{Z}_{IIA} = Z_{IIA}/Z_{extra}$
- The wrapped string or D2 brane in IIA brane setup which moves away from NS5 brans.

U(1) LSTs

- Both cases, the string dynamics is free and so the multi-winding string partition function is given by the Hecke transformation of that of a single string partition function. Both cases, one gets the identical partition function.

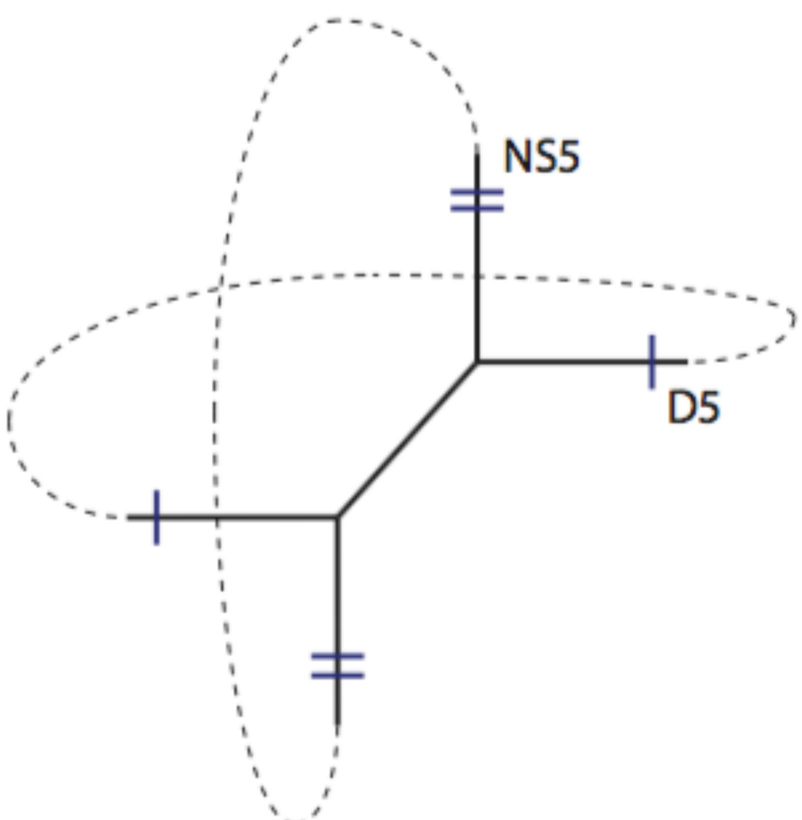
- Taking care of the extra states in IIA and the difference in the perturbative part, T-duality leads to

  - $Z^{IIA}(\epsilon_\pm, m; q', w')$ is symmetric function under the exchange of q' and w'

- pq5 brane picture implies triality between

Exchange Symmetry of q' and w'

$$Z_{\rm IIA}(\epsilon_\pm, m; q', w') = PE\Big[I_-(\epsilon_\pm, m) z_{\rm sp}(\epsilon_\pm, m, q', w')\Big]$$



under S-duality, it is symmetric

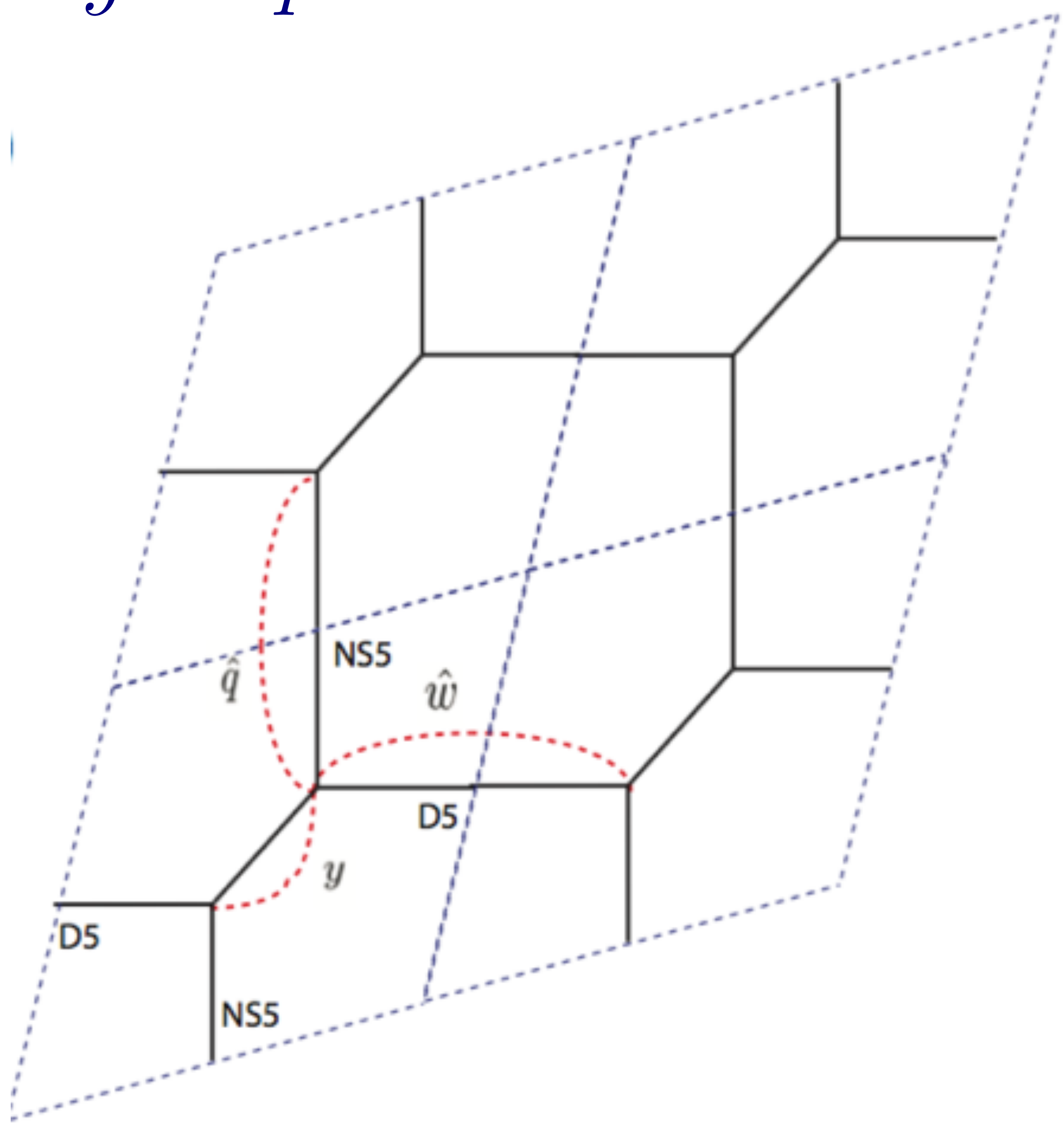
Exchange Symmetry of q' and w'

$$z_{\rm sp}(\epsilon_\pm, m; q', w') = (q'+w') + (q'^2+w'^2) + (q'w')\left[tu + \frac{t}{u} + \frac{1}{tu} + \frac{u}{t} - uy - \frac{y}{u} - \frac{u}{y} - \frac{1}{uy}\right]$$
$$+ q'^3 + w'^3 + (q'^2w' + q'w'^2)\left[t^2u^2 + \frac{t^2}{u^2} + \frac{u^2}{t^2} + \frac{1}{t^2u^2} + t^2 + \frac{1}{t^2} - tu^2y - \frac{ty}{u^2} - \frac{tu^2}{y} - \frac{t}{u^2y}\right.$$
$$- \frac{y}{tu^2} - \frac{u^2}{ty} - \frac{1}{tu^2y} - \frac{u^2y}{t} + tu + \frac{t}{u} + \frac{1}{tu} + \frac{u}{t} - 2ty - \frac{2t}{y} - \frac{2}{ty} - \frac{2y}{t} + 2u^2 + \frac{2}{u^2} - uy - \frac{y}{u}$$
$$\left. - \frac{u}{y} - \frac{1}{uy} + y^2 + \frac{1}{y^2} + 4\right] + (q'^4 + w'^4) + (q'^3w' + q'w'^3)\left[t^3u^3 + \frac{t^3}{u^3} + \frac{u^3}{t^3} + \frac{1}{t^3u^3} + t^3u + \frac{t^3}{u}\right.$$
$$+ \frac{u}{t^3} + \frac{1}{t^3u} - t^2u^3y - \frac{t^2y}{u^3} - \frac{t^2u^3}{y} - \frac{t^2}{u^3y} - \frac{u^3y}{t^2} - \frac{y}{t^2u^3} - \frac{u^3}{t^2y} - \frac{1}{t^2u^3y} + t^2u^2 + \frac{t^2}{u^2} + \frac{u^2}{t^2}$$
$$+ \frac{1}{t^2u^2} - 2t^2uy - \frac{2t^2y}{u} - \frac{2t^2u}{y} - \frac{2t^2}{uy} - \frac{2uy}{t^2} - \frac{2y}{t^2u} - \frac{2u}{t^2y} - \frac{2}{t^2uy} + 2t^2 + \frac{2}{t^2} + 2tu^3 + \frac{2t}{u^3}$$

Triality

Hollowood,Iqbal,Vafa'03

$$\hat{q} \leftrightarrow \hat{w} \leftrightarrow y \leftrightarrow \hat{q}$$



$$\hat{q} = qy^{-1} \ , \quad \hat{w} = wy^{-1}$$

Triality

Add the contribution from the massive hyper in 5d due to mass term

$$\tilde{Z}(\epsilon_\pm; \hat{q}, \hat{w}, y) = PE\left[I_{\mathrm{com}}(\epsilon_\pm) y\right] Z_{\mathrm{IIA}} \ .$$

$$I_{\mathrm{com}}(\epsilon_\pm) = \frac{1}{2\sinh\frac{2\pi i\epsilon_1}{2} 2\sinh\frac{2\pi i\epsilon_2}{2}} = \frac{t}{(1-tu)(1-tu^{-1})}.$$

$$\tilde{Z}(\epsilon_\pm; \hat{q}, \hat{w}, y) = PE\Big[I_{\mathrm{com}} \tilde{z}_{\mathrm{sp}}(\epsilon_\pm; \hat{q}, \hat{w}, y)\Big],$$

Triality

$$
\begin{aligned}
\hat{z}_{sp}(\epsilon_\pm; \hat{q}, \hat{w}, y) &= \hat{q} + \hat{w} + y - (u + u^{-1})(\hat{q}\hat{w} + \hat{q}y + \hat{w}y) + \frac{(1+u^2)(t+u+t^2u+tu^2)}{tu^2}\hat{q}\hat{w}y \\
&+ (\hat{q}^2\hat{w} + \hat{q}\hat{w}^2 + \hat{q}^2y + \hat{q}y^2 + \hat{w}^2y + \hat{w}y^2) - (u+u^{-1})(\hat{q}^2\hat{w}^2 + \hat{q}^2y^2 + \hat{w}^2y^2) \\
&- \frac{(u^2+1)\left(t^2\left(u^2+1\right)+2tu+u^2+1\right)}{tu^2}\hat{q}\hat{w}y(\hat{q}+\hat{w}+y) \\
&+ (\hat{q}^3\hat{w}^2 + \hat{q}^2\hat{w}^3 + \hat{q}^3y^2 + \hat{q}^2y^3 + \hat{w}^3y^2 + \hat{w}^2y^3) + \frac{(1+u^2)(t+u+t^2u+tu^2)}{tu^2}\hat{q}\hat{w}y(\hat{q}^2+\hat{w}^2+y^2) \\
&+ \frac{t^4\left(u^5+u^3+u\right)+t^3\left(u^6+4u^4+4u^2+1\right)}{t^2u^3}\hat{q}\hat{w}y(\hat{q}\hat{w}+\hat{q}y+\hat{w}y) \\
&+ \frac{t^2\left(3u^4+7u^2+3\right)u+t\left(u^6+4u^4+4u^2+1\right)+u^5+u^3+u}{t^2u^3}\hat{q}\hat{w}y(\hat{q}\hat{w}+\hat{q}y+\hat{w}y) \\
&- (u+u^{-1})(\hat{q}^3\hat{w}^3 + \hat{q}^3y^3 + \hat{w}^3y^3) \\
&- \frac{(u^2+1)\left(t^4\left(u^4+u^2+1\right)+3t^3\left(u^3+u\right)\right)}{t^2u^3}\hat{q}\hat{w}y(\hat{q}^2\hat{w}+\hat{q}\hat{w}^2+\hat{q}^2y+\hat{q}y^2+\hat{w}^2y+\hat{w}y^2) \\
&- \frac{(u^2+1)\left(2t^2\left(u^4+3u^2+1\right)+3t\left(u^3+u\right)+u^4+u^2+1\right)}{t^2u^3}\hat{q}\hat{w}y(\hat{q}^2\hat{w}+\hat{q}\hat{w}^2+(\text{cyclic}))
\end{aligned}
$$

U(2) LSTs

- fractionalization of strings in IIA and momentums in IIB
- IIB,IIA $\quad v_1 = e^{2\pi i \alpha_{12}}, \ v_2 = q v_1^{-1}$

$$Z_{\rm IIB}(\alpha_i, \epsilon_\pm, m; w, v_i) = PE\left[I_{\rm com}(t,u) \sum_{i,j,k=0}^{\infty} F_{ijk}^{\rm IIB}(t,u,y) w^i v_1^j v_2^k \right]$$

$$\hat{Z}_{\rm IIA}(\alpha_i, \epsilon_\pm, m; w, v_i) = PE\left[I_{\rm com}(t,u) \sum_{i,j,k=0}^{\infty} F_{ijk}^{\rm IIA}(t,u,y) w^i v_1^j v_2^k \right]$$

$$q' = w, \ w' = q$$

T-duality $\quad F_{ijk}^{\rm IIA} = F_{ijk}^{\rm IIB} \equiv F_{ijk}.$

U(2) IIA & IIB LSTs

$$F_{000} = 1\,, \quad F_{010} = -t - \frac{1}{t} + y + \frac{1}{y}\,, \quad F_{011} = -2t - \frac{2}{t} + 2y + \frac{2}{y}$$

$$F_{020} = 0\,, \quad F_{021} = -t - \frac{1}{t} + y + \frac{1}{y}\,, \quad F_{022} = -2t - \frac{2}{t} + 2y + \frac{2}{y}$$

$$F_{100} = -2u - \frac{2}{u} + 2y + \frac{2}{y}\,,$$

$$F_{110} = -t^2u - \frac{t^2}{u} - \frac{u}{t^2} - \frac{1}{t^2u} + t^2y + \frac{t^2}{y} + \frac{y}{t^2} + \frac{1}{t^2y} + tuy + \frac{ty}{u} + \frac{tu}{y} + \frac{t}{uy} + \frac{y}{tu} + \frac{u}{ty}$$
$$+ \frac{1}{tuy} + \frac{uy}{t} - ty^2 - \frac{t}{y^2} - \frac{1}{ty^2} - \frac{y^2}{t} - 2t - \frac{2}{t} - 2u - \frac{2}{u} + 2y + \frac{2}{y} \quad ($$

Conclusion

- Any thing unknown happens at the symmetric phase of SCFTs and LSTs?
    - dynamics of self-dual strings
- Other approaches: bootstrap, effective action…
- Other partition functions?