SUSY N=1 ADE Dynamics

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arXiv: 1401.4168, 1402.5411

See also J. Lin's talk

Introduction

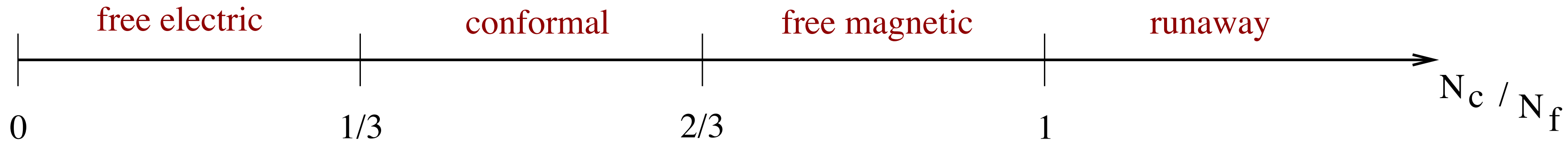
In the last twenty years there has been important progress in supersymmetric field theory. At the same time, many qualitative and quantitative phenomena remain mysterious. Today, I'd like to discuss an example of this, which involves a class of theories that naturally generalizes SQCD and follows an ADE classification.

Outline

- A_1 (N. Seiberg, 1994)
- A_k (DK, A. Schwimmer, N. Seiberg, 1995)
- D_k (J. Brodie, 1996)
- ADE (K. Intriligator, B. Wecht, 2003)
- E_7 (DK, J. Lin, 2014)

A_1

N=1 SQCD is a gauge theory with gauge group $SU(N_c)$, and N_f flavors of chiral superfields that transform in the fundamental representation of the gauge group, $Q, \tilde{Q}$. The low energy dynamics of this theory varies with N_f, N_c as follows:



Although the gauge coupling runs with the scale, one can think of the discrete parameter

$$x = N_c/N_f$$

as a `t Hooft coupling that measures the strength of gauge interactions  in the infrared (compare to the `t Hooft coupling λ of N=4 SYM, and to the discrete coupling N_c/k of CS theory).

- For x<1/3, the theory is not asymptotically free, so the IR dynamics is free, like in (massless) QED.

- For 1/3<x<2/3, the gauge interactions are non-vanishing in the IR, and the theory approaches a non-trivial fixed point. As x increases, this fixed point becomes more strongly coupled, which means that the scaling dimensions of operators deviate further from their free values.

- For x>2/3, the description of the IR theory in terms of the original $SU(N_c)$ degrees of freedom breaks down and one needs to find an alternative one.

Seiberg proposed such a description, in terms of a dual theory, similar to the original one, with gauge group $SU(N_f - N_c)$, similar charged matter, and singlet meson fields M, dual to the electric gauge invariant chiral operators $M = \tilde{Q}Q$, which are coupled to the magnetic quarks $q, \tilde{q}$ via the superpotential

$$W = Mq\tilde{q}$$

The rank of the magnetic gauge group implies that the magnetic `t Hooft coupling is

$$x_m = \frac{N_f - N_c}{N_f} = 1 - x$$

Thus, as the electric theory becomes more strongly coupled, the magnetic one becomes more weakly coupled. In particular, it provides a weakly coupled description of the problematic region x>2/3.

Conversely, the electric theory provides a weakly coupled description of the magnetic theory when the latter is strongly coupled.

A_k

N=1 SQCD has a family of generalizations obtained by adding to the theory an adjoint chiral superfield X with superpotential

$$W = \mathrm{Tr} X^{k+1}$$

with k=1, 2, 3, …

For k=1, the adjoint superfield is massive, and can be integrated out, leading back to SQCD.

For k=2, the superpotential W is marginal. Gauge interactions make it relevant for all x>1/2; thus adding W to the Lagrangian leads to a non-trivial fixed point.

For k>2, the superpotential is superficially irrelevant, however it turns out that for sufficiently large x, gauge interactions reduce its dimension enough that it become relevant in the IR for all k.

A stable supersymmetric vacuum only exists in the range

$$x \leq k$$

The strong coupling region is better described in terms of a dual theory with the following properties:

- Gauge group: $SU(kN_f - N_c)$

- Charged matter fields: $q, \tilde{q}, \widehat{X}$

- Gauge singlet mesons: $M_j \leftrightarrow \widetilde{Q} X^{j-1} Q$

- Magnetic superpotential:

$$\mathcal{W} \sim \mathrm{Tr} \widehat{X}^{k+1} + \sum_{j=1}^{k} M_j \widetilde{q} \widehat{X}^{k-j} q$$

- Magnetic `t Hooft coupling: $x_m = k - x$

The study of the theories with the adjoint X revealed a relation to mathematical singularities of type A_k. This point of view was particularly helpful when analyzing deformations of the adjoint superpotential.

J. Brodie further developed this relation by asking what happens if one replaces the A-series singularity with a D-series one.

D_{k+2}

There are now two adjoints, X and Y, and superpotential

$$\mathcal{W} \sim \mathrm{Tr}\left(X^{k+1} + XY^2\right)$$

Brodie found a very similar structure to the A-series, but with important new elements.

The similar part:

- For general k, the naively irrelevant superpotential for X actually becomes relevant for sufficiently strong coupling.

- An upper bound on the coupling x, above which no stable SUSY vacuum exists, $x \leq 3k$.

- A dual description of the infrared dynamics in terms of a gauge theory with gauge group $SU(3kN_f - N_c)$ charged fields $q, \widetilde{q}, \widehat{X}, \widehat{Y}$ and singlet mesons

$$M_{lj} = \widetilde{Q} X^{l-1} Y^{j-1} Q \; ; \quad l = 1, \cdots, k; \quad j = 1, 2, 3$$

The new elements:

- The matrix nature of the adjoint fields X, Y: In the A series, at low energies one can use the gauge symmetry and D-term constraints to diagonalize the adjoint field X, and study the dynamics of the eigenvalues. In the D series, we have two massless adjoints, which cannot be diagonalized at the same time. This leads to still unresolved complications in the analysis of the vacuum structure of the theory in the presence of general deformations of the superpotential.

- Quantum constraints on chiral operators: the F-term constraints of the D-series superpotential are

$$X^k = Y^2 \,; \qquad \{X, Y\} = 0 \,.$$

Naively, one can use these to construct chiral operators of the form $\widetilde{Q}\Theta_{lj}Q$ with

$$\Theta_{lj} = X^{l-1}Y^{j-1} \;; \quad l = 1, \cdots, k \;; \quad j = 1, 2, \cdots .$$

This looks incompatible with Brodie's duality, according to which only operators with j=1, 2, 3 should survive.

For odd k it's actually OK, since one can use the F-term equations to conclude that

$$Y^3 = Y \cdot Y^2 = Y \cdot X^k = -X^k \cdot Y = -Y^3 = 0$$

For even k, the situation is more puzzling. On the one hand, at least classically the constraint $Y^3 = 0$ is not valid, but on the other it is required by the duality.

Brodie proposed that in that case, the constraint appears quantum mechanically, although its origin is not well understood.

ADE

The understanding of the above theories improved significantly after the advent of a-maximization (by K. Intriligator and B. Wecht) in 2003. These authors classified all possible fixed points that can be obtained in N=1 supersymmetric gauge theory with SU(N) gauge group and matter in the fundamental and adjoint representations.

They showed that such fixed points have an ADE classification:

$\widehat{O}$	$W_{\widehat{O}} = 0$
$\widehat{A}$	$W_{\widehat{A}} = \mathrm{Tr} Y^2$
$\widehat{D}$	$W_{\widehat{D}} = \mathrm{Tr} XY^2$
$\widehat{E}$	$W_{\widehat{E}} = \mathrm{Tr} Y^3$
A_k	$W_{A_k} = \mathrm{Tr}(X^{k+1} + Y^2)$
D_{k+2}	$W_{D_{k+2}} = \mathrm{Tr}(X^{k+1} + XY^2)$
E_6	$W_{E_6} = \mathrm{Tr}(Y^3 + X^4)$
E_7	$W_{E_7} = \mathrm{Tr}(Y^3 + YX^3)$
E_8	$W_{E_8} = \mathrm{Tr}(Y^3 + X^5)\,.$

- The ADE classification is due to gauge dynamics.

- The $\widehat{O}, \widehat{A}, \widehat{D}, \widehat{E}$ theories are interesting, but we will not discuss them further today.

- The A_k, D_k theories are those reviewed above.

- Our goal in the rest of this talk will be to try to understand the exceptional theories.

E_7

The transformation properties of the various gauge theory fields under the symmetries are:

Field	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
Q	f	f	1	1	$1-\frac{1}{9}\frac{N_c}{N_f}$
$\widetilde{Q}$	$\overline{f}$	1	$\overline{f}$	-1	$1-\frac{1}{9}\frac{N_c}{N_f}$
V	adj.	1	1	0	0
X	adj.	1	1	0	$\frac{4}{9}$
Y	adj.	1	1	0	$\frac{2}{3}$

- The superpotential for the adjoints is

$$\mathcal{W} = \mathrm{Tr} Y^3 + \mathrm{Tr} Y X^3$$

- The F-term constraints that follow from this superpotential are

$$Y^2 = X^3$$

$$X^2 Y + XYX + YX^2 = 0$$

- Classical chiral meson operators take the form $\widetilde{Q} \Theta Q$, with

$$\Theta = X^n, YX^n, XYX^n, YXYX^n$$

- One can show that at large coupling the UV variables in terms of which the theory is defined must break down, like in the other examples.

- We assume that the strong coupling region is governed by a dual description similar to the other cases.

The quantum numbers of the dual fields are taken to be:

Field	$SU(\widetilde{N}_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
q	f	$\overline{f}$	1	$N_c/\widetilde{N_c}$	$1-\frac{1}{9}\frac{\widetilde{N}_c}{N_f}$
$\widetilde{q}$	$\overline{f}$	1	f	$-N_c/\widetilde{N_c}$	$1-\frac{1}{9}\frac{\widetilde{N}_c}{N_f}$
$\widetilde{V}$	adj.	1	1	0	0
$\widetilde{X}$	adj.	1	1	0	$\frac{4}{9}$
$\widetilde{Y}$	adj.	1	1	0	$\frac{2}{3}$
$M_j,\ j=1,\ldots\alpha$	1	f	$\overline{f}$	0	$2r_Q+r_j$

- The rank of the dual gauge group must take the general form

$$\widetilde{N}_c = \alpha N_f - N_c$$

where α is the number of gauge singlet mesons in the magnetic theory. This follows from $SU(N_f)^3$ `t Hooft anomaly matching. This number, as well as the R-charges of these mesons, r_j, are kept free.

- To determine them, we demand that the superconformal indices of the electric and magnetic theories coincide.

- In general, these indices are very complicated functions of the chemical potentials, but Dolan and Osborn observed that they simplify significantly in the large N Veneziano limit.

(See J. Lin's talk)

This gives a constraint of the form

$$\sum_{j=1}^{\alpha} t^{r_j} = \frac{1 + t^{\frac{1}{9}} + t^{\frac{2}{9}} + \ldots + t^{\frac{\alpha-1}{9}}}{1 + t^{\frac{1}{9}} - t^{\frac{1}{3}} - t^{\frac{4}{9}} - t^{\frac{5}{9}} + t^{\frac{7}{9}} + t^{\frac{8}{9}}} \ .$$

which determines α, r_j. One finds $\alpha = 30$, and a certain set of r_j, which can be thought of as arising from applying the constraint

$$aYX^6 + bXYX^5 = 0$$

to the full list of operators.

Thus, we conclude that the dual of a $SU(N_c)$ theory has gauge group $SU(30N_f - N_c)$. This proposal satisfies a number of detailed consistency conditions:

- There are precisely 30 mesons, and the list of r_j is such that one can write a magnetic superpotential for the magnetic meson fields.

- `t Hooft anomaly matching is non-trivially satisfied.

- Potential unitarity violations are resolved.

Open problems

- E_6, E_8 : we saw that the E_7 theory has a very similar structure to the A and D series ones. Using the superconformal index one can show that this cannot be the case for the remaining exceptional theories. Thus, in these cases there must be qualitative new elements. What are they?

- In some of the theories we found that there must be quantum constraints on the chiral ring. Can one derive them?

- The D and E series seem to involve some type of matrix singularity theory, which is important for studying deformations of the adjoint superpotential. How does it work?

- Can one relate the dynamical ADE structure that arises in these theories to a geometric or algebraic ADE structure, e.g. by embedding these theories in string theory?