

On the dual flow of slow-roll Inflation

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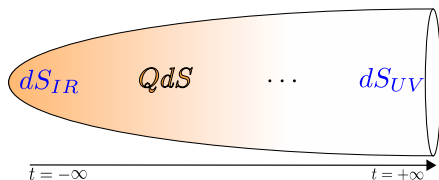
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Slow-roll Inflation and the dS/CFT correspondence

The inflationary background interpolates between two *de Sitter* phases with different *Hubble* radii.

The famous calculation of the **power spectrum** of fluctuations is an approximation in the *Quasi de Sitter* region, which gives the *Slow-roll* corrections to the leading *scale-invariant* spectrum.



$$\mathcal{P}(k) \sim \frac{1}{k^{d+2\epsilon+\eta}}$$

Scale-invariant Slow-roll corrections

The dS/CFT correspondence suggests a duality between a gravitational theory in *de Sitter* spacetime and a CFT living on a constant time slice at temporal infinity. The dictionary is

$$Z[g] = \Psi[g] \sim e^{iS_{cl}}$$

The central charge of each fixed point is related to the corresponding *Hubble* constant

$$a \sim \frac{1}{H^{d-1}}$$

The dual process to slow-roll Inflation is then interpreted as an RG flow between two fixed points.

Goals and Strategy

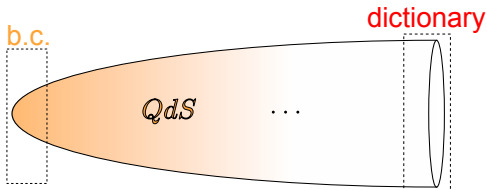
The goal:

Calculate correlation functions in the dual field theory beyond the Quasi de Sitter approximation

$$\langle OO \rangle$$

The challenge:

The **dictionary** relates the behavior of bulk fields near the boundary (**late times**) to the dual field theory while the **Bunch-Davies boundary conditions** are imposed at **early times**.



The strategy: Find a solution in the bulk that interpolates between early and late times using a matching procedure.

Byproduct: The solution at all times can also be used to study bulk properties (Inflation).

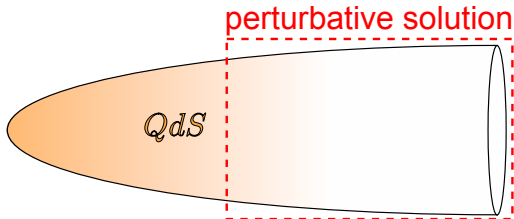
The matching procedure

The matching is between two different solutions which are separately valid at different regions of the space of parameters but together cover all of it:

- **Perturbative solution.** The expansion parameter is the momentum over the inverse comoving Hubble radius

$$\frac{k}{A'}$$

- Solution in the **Quasi de Sitter** region.



For sufficiently low momentum the perturbative solution will be valid all the way to the Quasi de Sitter region and we can match the two solutions.

The results

The comoving curvature perturbation ζ has two solutions:

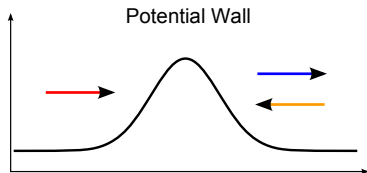
$$\zeta(t) \simeq D_0 + D_1 e^{-dHt}$$

The Bunch-Davies boundary conditions at early times fix the Quasi de Sitter wave function

$$D_0^{(QdS)} \sim k^{-\frac{d+(2\epsilon+\eta)}{2}}, \quad \frac{D_1^{(QdS)}}{D_0^{(QdS)}} \equiv G(k) \sim k^{d+(2\epsilon+\eta)}$$

The matching procedure transfers the information about the boundary conditions in the IR to the late time solution and the result is a mixing between the amplitudes:

$$\begin{pmatrix} D_0 \\ D_1 \end{pmatrix}_{QdS} \sim \begin{pmatrix} 1 & c_2(k) \\ k^2 c_1(k) & 1 \end{pmatrix} \begin{pmatrix} D_0 \\ D_1 \end{pmatrix}_{late}$$



This result is expected since this problem is similar to solving the Schrödinger equation with a potential wall, where the modes in one side of the wall are superposition of the modes on the other side.

In the explicit breaking phase

$$\langle \mathcal{O}\mathcal{O} \rangle \sim G(k) \sim k^{d+2\epsilon+\eta} + \dots$$

where "..." corresponds to subleading and local terms.

This spectrum corresponds to a perturbation of the IR fixed point by a *quasi-marginal* operator of dimension

$$\Delta_{IR} = d + \frac{2\epsilon + \eta}{2}$$

In particular, the mixing coefficients appear only at subleading orders.

Spontaneous breaking phase

In the spontaneous breaking phase we find

$$\langle \mathcal{O} \mathcal{O} \rangle \sim \frac{1 - c_2(k)G(k)}{G(k) - k^2 c_1(k)} \sim \frac{1}{c_1(0)} \frac{1}{k^2 + \dots + k^2 \Delta_{IR}^{-d} + \dots} \sim \frac{1}{c_1(0)} \frac{1}{k^2} + \dots$$

The leading term at low momentum is a **massless pole** with a residue $\frac{1}{c_1(0)}$.

The field theory interpretation - a **dilaton** interacting with a **perturbed CFT**. Schematically, the Lagrangian is of the form

$$\mathcal{L} = \frac{1}{2}(\partial\tau)^2 + \mathcal{L}_{IR} + g\tau\mathcal{O}_{IR}$$

We have to sum over all the following diagrams

$$\langle \tau\tau \rangle = \tau \text{---} \tau + \tau \text{---} \overset{\mathcal{O}_{IR}}{\text{---}} \text{---} \tau + \dots$$

with arbitrary number of intermediate \mathcal{O}_{IR} states. The result is

$$\langle \tau\tau \rangle \sim \frac{1}{k^2 - g^2 \langle \mathcal{O}_{IR} \mathcal{O}_{IR} \rangle}$$

Example: Exact solution in $(2 + 1)$ -dimensions

In $(2 + 1)$ -dimensions we were able to calculate the residue exactly

$$c_1(0) \sim H_{UV}^{-1} - H_{IR}^{-1}$$

which leads to the following two-point function

$$\langle \mathcal{O} \mathcal{O} \rangle \sim \frac{1}{H_{UV}^{-1} - H_{IR}^{-1}} \frac{1}{k^2}$$

In agreement with the expectation from the dilaton effective action

$$S_{\text{dilaton}} = \frac{\Delta c}{24\pi} \int d^2x \sqrt{-g} (\tau R + (\partial\tau)^2)$$

which gives in flat space

$$\langle \tau \tau \rangle \sim \frac{1}{\Delta c} \frac{1}{k^2}$$

(recall that in 2D $c \sim H^{-1}$).

- Calculate more mixing coefficients (beyond 2D).
- The effect of the mixing on cosmological observables in the bulk, like the [bispectrum](#) (the three-point function).
- Relate the mixing coefficients, in a systematic way, to the [parameters of the flow](#) (running couplings or terms in the dilaton effective action).
- Applications to [mathematics](#) - the mixing coefficients are related to the solutions of the [Heun](#) equation, which has many applications in physics.