Strings 2014, Princeton
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Recent Progress on the Abelian Sector of F-Theory

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arXiv:1303.6970 [hep-th]: M. Cvetič, D.K., H. Piragua
arXiv:1306.3987 [hep-th]: M. Cvetič, A. Grassi, D.K., H. Piragua
arXiv:1307.6425 [hep-th]: M. Cvetič, D.K., H. Piragua
arXiv:1310.0463 [hep-th]: M. Cvetič, D.K., H. Piragua, P. Song
arXiv:1407.nnnn : D.K., D. Mayorga Peña, P. Oehlmann, H. Piragua, J. Reuter
arXiv:14nn.nnnn : M. Cvetič, D.K, H. Piragua, W. Taylor

F-theory & U(1)-symmetries

INTRODUCTION

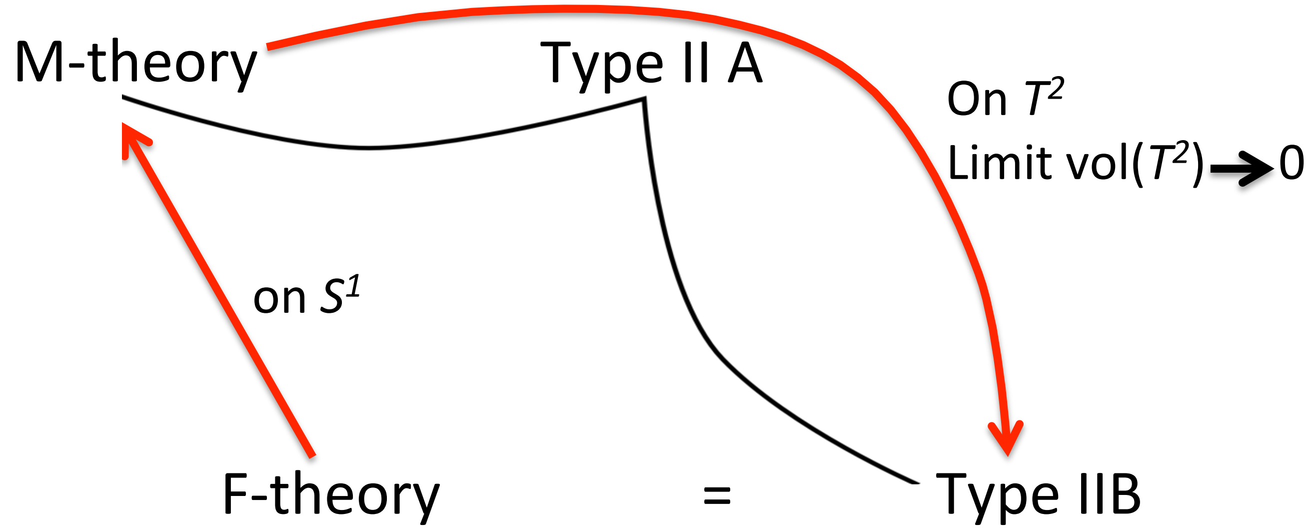
Why F-theory?

F-theory = Type IIB

- elliptically fibered **Calabi-Yau manifold**

- back-reacted **(p,q)7-branes**
- regions with **large g_s** on **non-CY space**

Why F-theory?



F-theory
- elliptically fibered **Calabi-Yau manifold**

Type IIB
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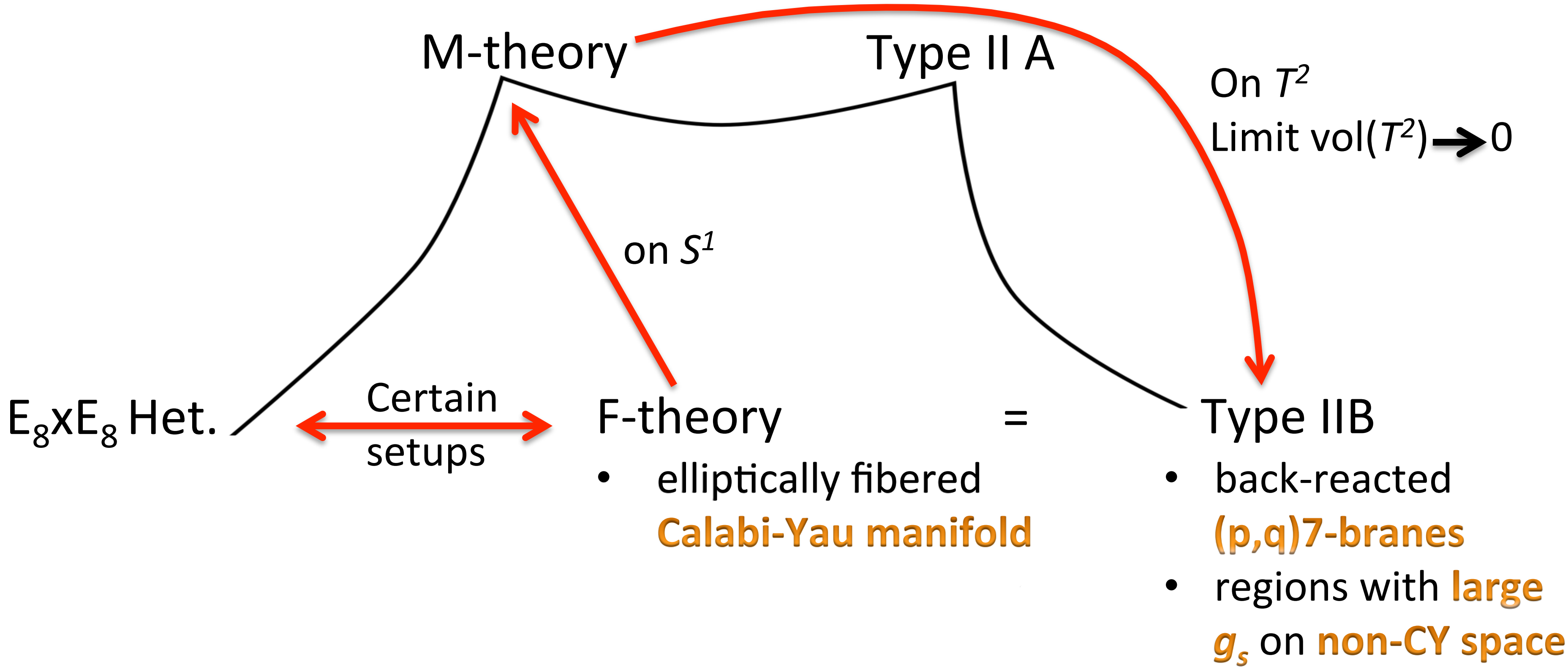
M-theory
Type II A
On T^2
Limit vol(T^2) → 0
on S^1
E_8xE_8 Het.
Certain setups
F-theory
=
Type IIB
• elliptically fibered Calabi-Yau manifold
• back-reacted (p,q)7-branes
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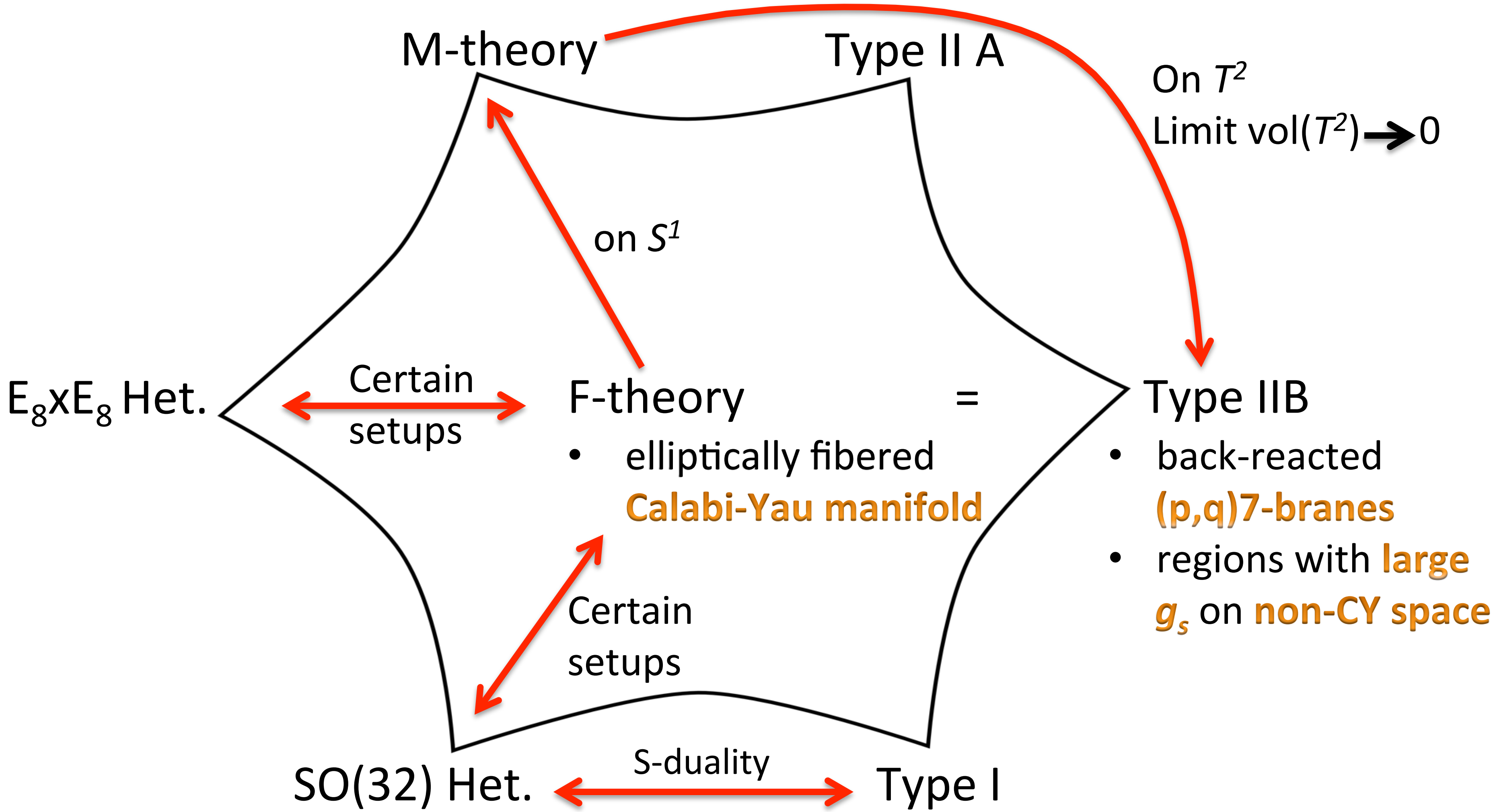


Het/F-theory duality: see Anderson's talk

Effective theories of F-theory

Use F-theory to engineer effective theories:

F-theory
Calabi-Yau geometry
(+ G_4-flux,…)

Geometry ⟷ **Physics**

N=1 SUGRA effective theories in 6D & 4D

Classification of 6D (1,0) SCFTs via F-theory: see Vafa's talk

Since **F-theory vacua** are non-perturbative, they **have novel features**

- **effective theories** different from those of **pert. vacua**
- used for models of **particle physics & cosmology**

local models: **[Donagi,Wijnholt; Beasley,Heckman,Vafa; Bouchard,Heckman,Kane,Seo,Shao,Tavanfar,Vafa; Font,Ibanez; Randall,Simmons-Duffin; Hayashi,Kawano,Tsuchiya,Watari,Yamazaki; Dudas,Palti; Cecotti,Cheng,Heckman,Vafa; Marchesano,Martucci… many works]**

global models: **[Blumenhagen,Grimm,Jurke,Weigand;Marsano,Saulina,SchäferNameki; Cordova; Grimm,Krause,Weigand… many works]**

Goals of this talk

Develop & **extend geometry/physics dictionary** of F-theory:

Arithmetic of elliptically fibred CY: **Mordell-Weil group**	⟺	**Abelian sector** of F-theory **effective theories**

[Morrison,Vafa]

The Abelian sector of F-theory has been rather **unexplored**:

only **few concrete examples** **Few early examples:** [Aldazabal,Font,Ibanez,Uranga; Klemm Mayr,Vafa]

Torsion part: [Aspinwall,Morrison; Mayrhofer,Morrison,Till,Weigand]

A lot of recent progress: [Grimm,Weigand;Esole,Fullwood,Yau;Morrison,Park; Cvetič,Grimm,DK; Braun,Grimm,Keitel; Lawrie,Schäfer-Nameki; Borchmann,Mayrhofer,Palti,Weigand; Cvetič,DK,Piragua; Grimm,Kapfer,Keitel;Braun,Grimm, Keitel; Cvetič,Grassi,DK,Piragua; Borchman,Mayrhofer,Palti,Weigand; Cvetič,DK,Piragua; Cvetič,DK,Piragua,Song; Braun,Collinucci,Valandro; Morrison,Taylor; Kuntzler,Schäfer-Nameki]

Unlike well-studied **non-Abelian case**

[Kodaira; Tate;Morrison,Vafa; Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa; Candelas,Font,…]

Recently: [Esole,Yau;Marsano,Schäfer-Nameki; Morrison,Taylor; Cvetič,Grimm,DK,Piragua; Braun,Grimm,Kapfer,Keitel; Borchman,Krause,Mayrhofer,Palti,Weigand; Hayashi,Lawrie,Morrison, Schäfer-Nameki; Esole,Shao,Yau]

Outline & Results

Systematic construction of Abelian sectors in F-theory

1. **Engineering** of general **rank *n* Abelian sector** in **global F-theory**
   - Exemplify explicitly for **$U(1)^2$** gauge group.
2. Develop **toolbox** to study such geometries
   - **matter** spectra **in 6D** (also with non-Abel. groups).
   - **Yukawas, 4D chiralities:** G_4-flux.
3. **Application** of toolbox:
   - CICY in $\mathbb{P}^3$ (**$U(1)^3$** group), elliptic curves in 16 2D **toric varieties.**
   - moduli space of F-theory: **U(1)-enhancements**.

A very brief summary

F-THEORY COMPACTIFICATIONS

F-theory = geometry/physics dictionary

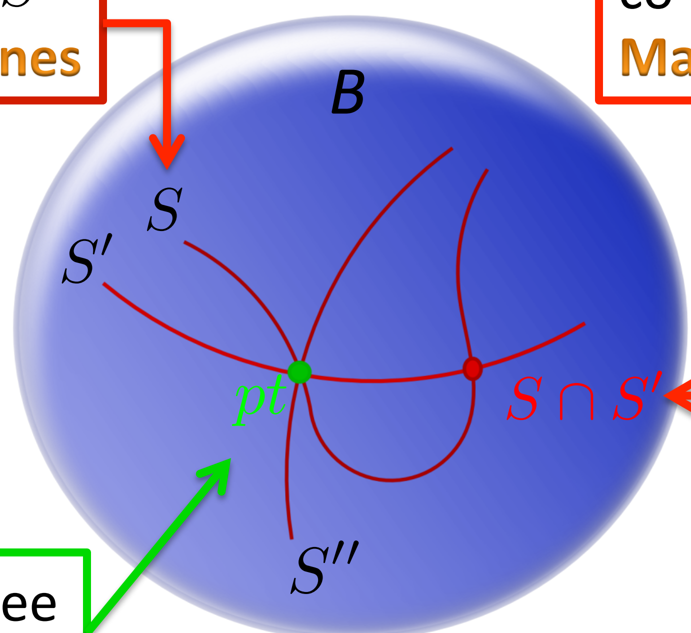
F-theory specified by **elliptically fibered Calabi-Yau** manifold $\pi: X \rightarrow B$

Singularities of elliptic fibration ⟺ **setup of intersecting 7-branes**

co-dim. one sing. over S
Gauge theory on **7-branes**

co-dim. two sing. $S \cap S'$
Matter: intersec. 7-branes

[Katz,Vafa]



4D Yukawa: co-dim three
$pt = S \cap S' \cap S''$

4D **chiral** matter:
G_4-flux

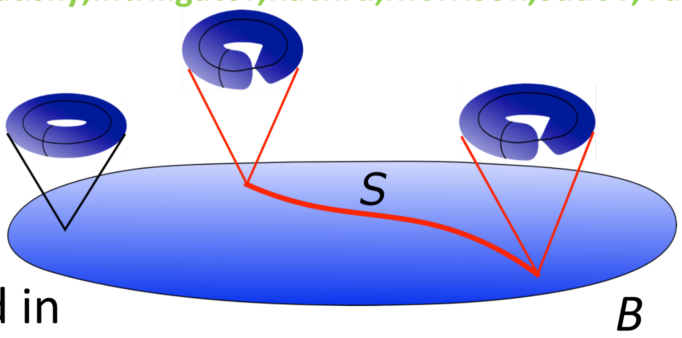
Structure of elliptic fibrations with Mordell-Weil group

U(1)-GAUGE SYMMETRIES IN F-THEORY

Non-Abelian gauge symmetry in F-theory

[Kodaira;Tate;Vafa;Morrison,Vafa;Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa]

Gauge group from singularities of X



1. **Weierstrass form** for elliptic fibration of X

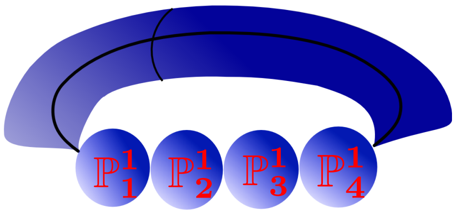
$$y^2 = x^3 + fxz^4 + gz^6$$

2. **Severity of singularity** over divisor S encoded in

> **Singularity type** = orders of vanishing of f, g, $\Delta=4f^3+27g^2$

3. Singularity type ⇒ structure of **tree of $\mathbb{P}^1$'s** over S in **resolution**

resolved I_4-singularity:



- **Cartan generators**: KK-reduction along (1,1)-form $\omega_i \leftrightarrow \mathbb{P}^1_i$

$$C_3 \supset A^i \omega_i$$

- **Enhancement**: light M2-branes on resolving $\mathbb{P}^1$'s [Witten]

U(1)'s in F-theory & the Mordell Weil group

- U(1)'s should arise like Cartans by **KK-reduction** $C_3 \supset A^m \omega_m$.
- **Forbid enhancement** by M2's: **only *I_1*-fibers** at codimension 1.

(1,1)-form ω_m ⟺ rational section

[Morrison,Vafa II]

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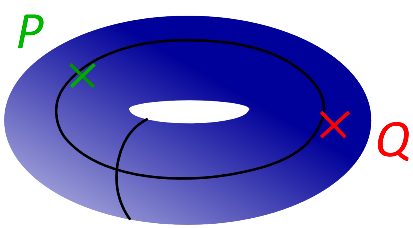
[Morrison,Vafa II]

1. **Rational point *Q*** on elliptic curve *E* with zero point *P*
   - is solution $[x_Q : y_Q : z_Q]$ **in field *K*** of Weierstrass form,

$$y^2 = x^3 + fxz^4 + gz^6$$

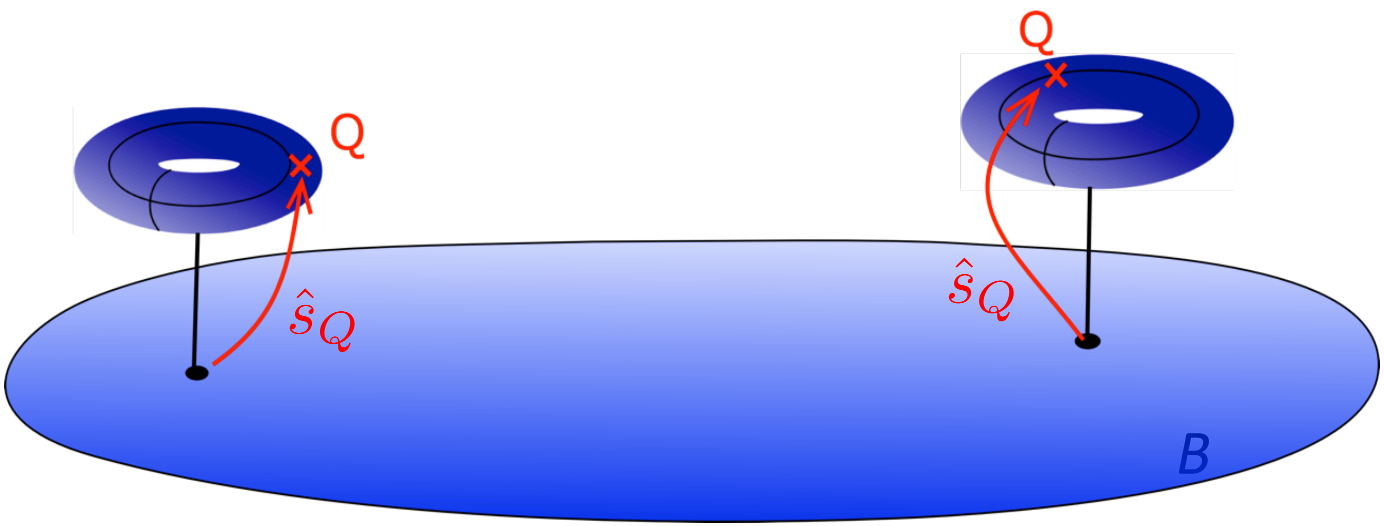


   - Rational points form **Abelian group** under addition ***Q+R***.

⇒ **Mordell-Weil group of rational points**

U(1)'s in F-theory & the Mordell Weil group

2. *Q* on *E* induces **rational section** $\hat{s}_Q : B \to X$ of elliptic fibration



- $\hat{s}_Q$ gives rise to a second copy of *B* in *X*: new divisor B_Q in *X*

⇒ (1,1)-form ω_m **constructed from** divisor $\boldsymbol{B_Q}$.

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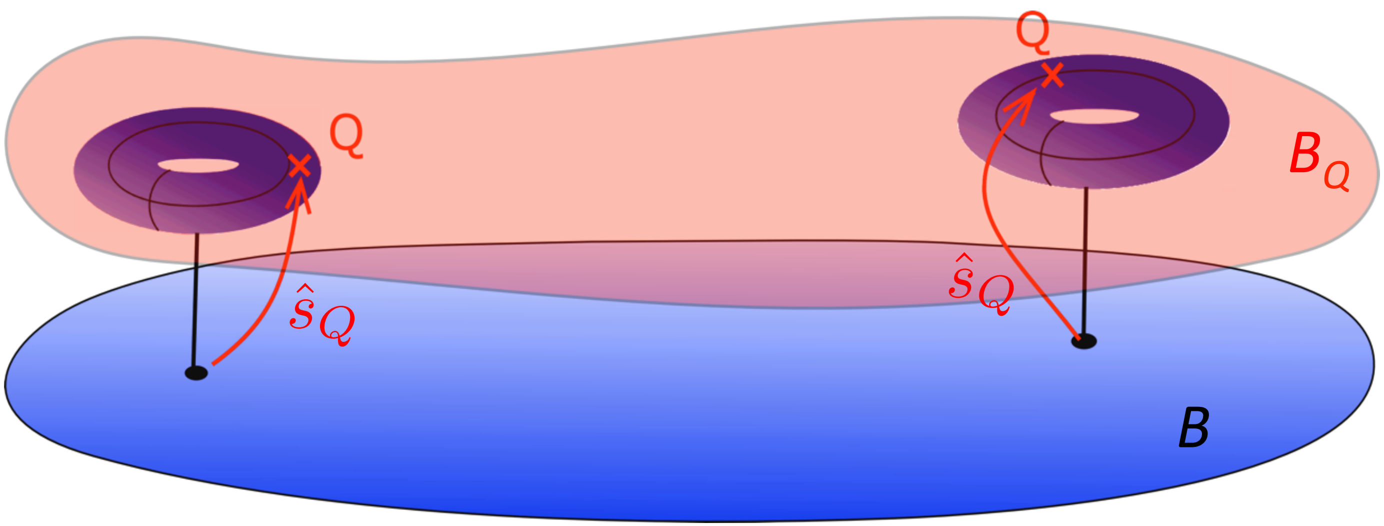

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Structure of elliptic fibrations with rational points

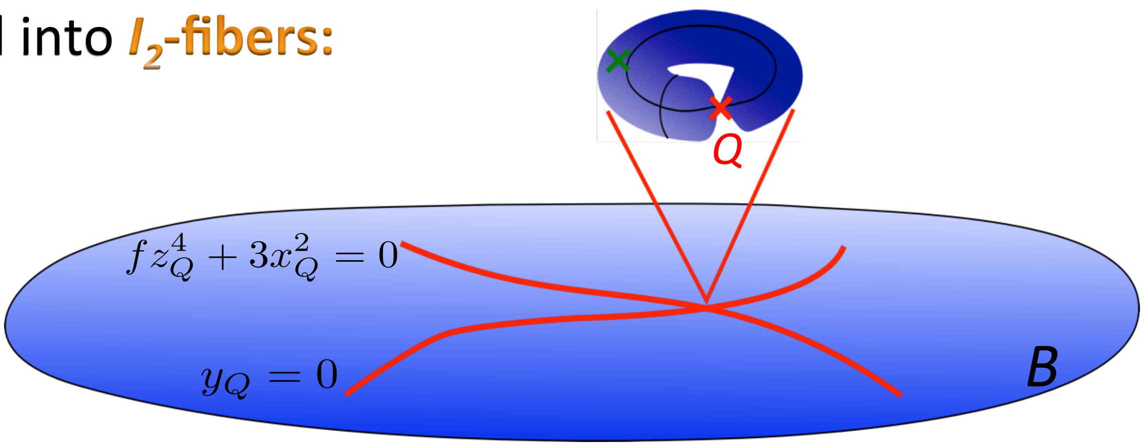
Consequences for Weierstrass form: rat. point Q= $[x_Q : y_Q : z_Q]$

1. **Implies constraints:** relation between *f, g*

$$g z_Q^6 = y_Q^2 - x_Q^3 - f x_Q z_Q^4$$

2. **Implies singularity** at **codimension two** in *B:*
   - Factorization: $(y - y_Q)(y + y_Q) = (x - x_Q)(x^2 + x_Q x + f z_Q^4 + x_Q^2)$
   - Singularity: $y_Q = f z_Q^4 + 3x_Q^2 = 0$ ⇨ WSF **singular at *Q***

- Resolved into ***I_2*-fibers:**



⇨ Section $\hat{s}_Q$ implies **U(1)-charged matter**, only I_1-fiber in codim. 1

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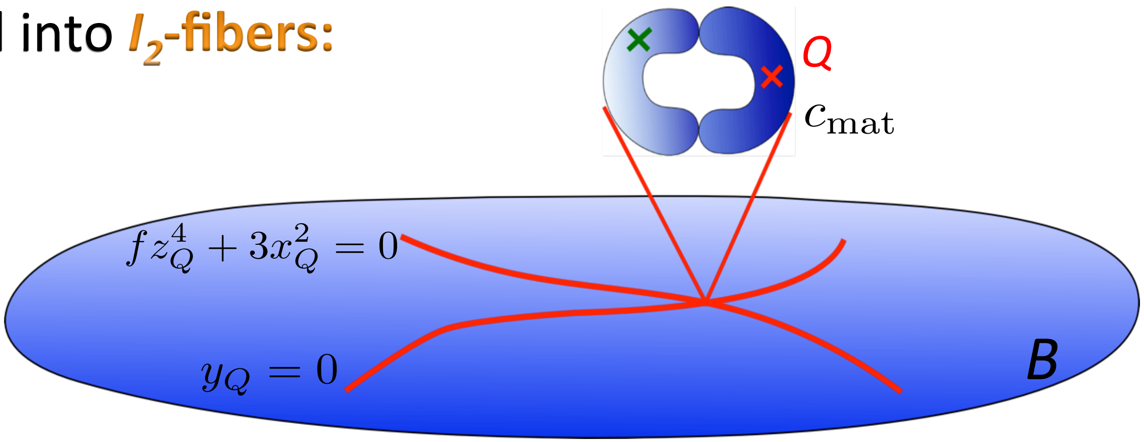

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The task

Provide **general construction** of ell. curve ***E* with *n* rational points**:

- **Solve the constraints** in Weierstrass form.
- Provide **resolved** Calabi-Yau **elliptic fibrations**.

⇨ Addressing these points requires **departure from Weierstrass form.**

Elliptic curves with *n* rational points Q_i

Elliptic curve ***E*** with points *P*, Q_i **has canonical embedding** in $W\mathbb{P}^m$

⇨ degree *n+1* line bundle ***M=O(P+Q₁+…+Qₙ)*** on *E*: $W\mathbb{P}^m$ determined by

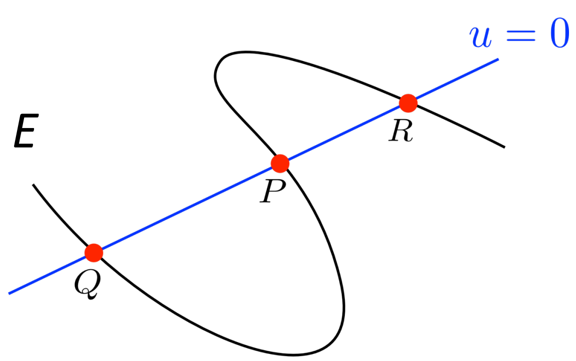
$$M = \mathcal{O}_{W\mathbb{P}^m}(1)|_E$$

<u>Example *n*=2:</u> points *P, Q, R*

1. *M=O(P+Q+R)* degree three: **three sections** (*u,v,w*) = $\mathbb{P}^2$-coordinates

$\Rightarrow$ ***E* is cubic curve** in $\mathbb{P}^2[u:v:w]$.

2. Existence of **points *P, Q, R*:** ***E* non-generic**, cubic has to factorize as



$$uf_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

Degree two polynomial $f_2(u,v,w)$

$\Rightarrow$ *E* **generic Calabi-Yau in blow-up** of $\mathbb{P}^2$ at points *Q, R* = ***dP₂***.

Explicit examples

n=0: **Tate form** in $\mathbb{P}^2(1,2,3)$.

n=1: *E* with *P, Q* is generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$. **[Morrison,Park]**

n=2: *E* with *P, Q, R* is **generic CY in *dP₂***. **[Borchmann,Mayerhofer,Palti,Weigand; Cvetič,D.K.,Piragua]**

n=3: *E* with *P, Q, R, S* is **CICY in** $\mathrm{Bl}_3\mathbb{P}^3$. **[Cvetič,D.K.,Piragua,Song]**

n≥4: *E* is **Pfaffian variety** in $\mathbb{P}^4$ (*n=4*), *E* is determinantal (*n>4*).

Work in progress: **[Cvetič,D.K.,Piragua,Song]**

Key properties of this construction:

- CY-**elliptic fibrations** are automatically **smooth**.
- For *n=0,1,2,3*: **zero-section** $\hat{s}_P$ **is non-holomorphic**

⇒ still **valid F-theory background.**

Illustration: dP_2-elliptic fibrations with two rational sections

TOOLBOX FOR STUDYING F-THEORY WITH U(1)'S

Construction of CY-elliptic fibrations

[Cvetič, D.K.,Piragua; Cvetič,Grassi,D.K.,Piragua]

<u>Illustration:</u> CY-fibrations *X* with rank 2 **curve in *dP*$_2$**

Related: [Borchmann,Mayrhofer,Palti,Weigand]

$$E \subset dP_2 \longrightarrow X$$
$$\downarrow$$
$$B$$

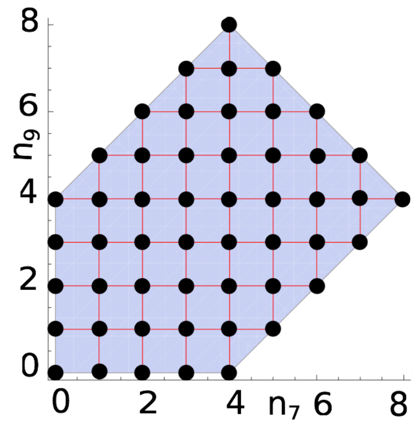
three sections from *P*, *Q*, *R*

$$u f_2(u,v,w) + vw(s_7 v + s_9 w) = 0$$

- **$\mathbb{P}^2$-coordiantes** [*u* : *v* : *w*] and **coefficients *s$_i$*** lifted to **sections on *B***.
- CY manifold ***X* topologically determined by** divisors $\mathcal{S}_7 = (s_7)$, $\mathcal{S}_9 = (s_9)$ that can be varied: **new degrees of freedom**.

<u>For $B = \mathbb{P}^3$:</u>

$$\mathcal{S}_7 = n_7 H_{\mathbb{P}^3}$$
$$\mathcal{S}_9 = n_9 H_{\mathbb{P}^3}$$



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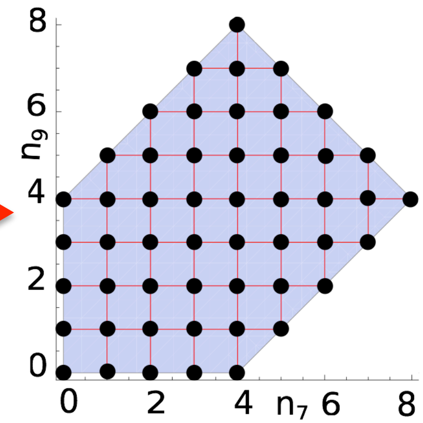

Space of all 4D F-theory vacua for fixed fiber *E* in *dP*$_2$ & base *B*= $\mathbb{P}^3$

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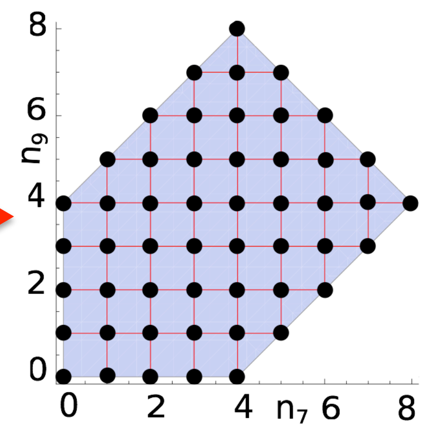


Can **construct** and **study** entire **family of CY's explicitly.**

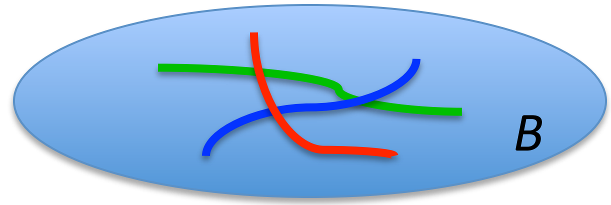
Space of all 4D F-theory vacua for fixed fiber *E* in *dP₂* & base *B*= $\mathbb{P}^3$

Charged matter: codimension two singularities

[Cvetič, D.K.,Piragua; Cvetič,Grassi,D.K.,Piragua]

All **charged matter** at codimension two in *B*

Matter locus: $y_Q = f z_Q^4 + 3x_Q^2 = 0$



- Problem: is **reducible variety** with many components.
- We are interested in **irreducible components** ⟺ **individual matter loci.**

➡ Need **ideal techniques: irred. components** described only by **prime ideals.**

- **Computationally hard** to find ass. prime ideals for **polys of high degree**.

➡ Need bootstrap: some **matter** loci have **geometry/physics interpretation** which makes them **easier to find**.

1) Matter at loci in *B* where the **sections are ill-defined.**
2) Matter at loci characterized by **additional constraints**

➡ matter with **multiple U(1)-charges**.

6D matter spectrum & Anomalies

[Cvetič, D.K., Piragua]

Derived **closed formula for 6D matter multiplicities** for entire class of F-theory vacua **over any base *B*.**

- <u>Example:</u> *E* in dP_2, $B = \mathbb{P}^2$ (n_7, n_9 specify fibration of *X* over $\mathbb{P}^2$)

(q_1, q_2)	Multiplicity
$(1,0)$	$54 - 15n_9 + n_9^2 + (12 + n_9)\, n_7 - 2n_7^2$
$(0,1)$	$54 + 2\left(6n_9 - n_9^2 + 6n_7 - n_7^2\right)$
$(1,1)$	$54 + 12n_9 - 2n_9^2 + (n_9 - 15)\, n_7 + n_7^2$
$(-1,1)$	$n_7\,(3 - n_9 + n_7)$
$(0,2)$	$n_9 n_7$
$(-1,-2)$	$n_9\,(3 + n_9 - n_7)$

- <u>Consistency check:</u> spectrum proven to **cancel 6D anomalies**! ✓

Non-Abelian gauge groups, codimension 3

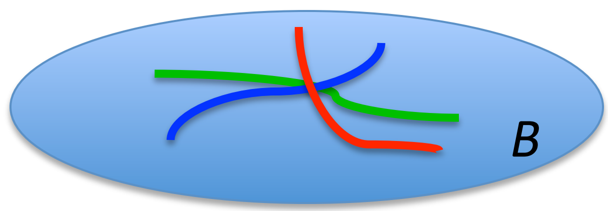
1) Can systematically **add non-Abel groups** to model with U(1)'s:

- Toric **tops**. [Candelas,Font; Bouchard,Skarke; Braun,Grimm,Keitel; Borchmann,Mayrhofer,Palti,Weigand]
- **Tate algorithm** for other ell. fiber types. **Fiber in *P(1,1,2)*:** [Kuntzler,Schäfer-Nameki]

Can still obtain **full 6D spectrum**, e.g. ***dP_2*-fiber with SU(5)**, arbitrary base *B*

⇨ closed formulas for **6D matter multiplicities.** [Cvetič,D.K.,Piragua]

2) **Yukawa couplings** at codimension 3:



- **intersections of three matter loci** computed using their **prime ideals**

⇨ ***dP_2*-fiber**, also with SU(5): **all** gauge invariant **Yukawas realized.**

in progress: [Cvetič, D.K., Langacker, Piragua]

G_4-flux & 4D matter chiralities

[Cvetič,Grassi,D.K.,Piragua]

3) **New ingredient** for F-theory on **CY 4-folds** X_4: **G_4-flux**

Geometry: (vertical) G_4-flux in $H_V^{(2,2)}(X_4,\mathbb{Z}/2)$ [Witten]

→ requires computation of **primary vertical cohomology** of CY 4-fold.

Example: ***E* in *dP*$_2$**

- ***$H_V^{(2,2)}(X_4)$*** explicitly **computed** for family of **all *X_4*** with ***B*=$\mathbb{P}^3$.**

G_4-flux is **M-theory concept:** properly studied in M-theory.

3D M-/F-theory duality: F-theory on S^1 = M-theory

- In **3D effective theory, CS-terms encode** information of vertical ***G_4*-flux:**

$$\Theta^M_{AB} = \int_{X_4} G_4 \wedge \omega_A \wedge \omega_B$$

[Gukov,Vafa,Witten; Haack,Louis]

- M-/F-duality for CS-terms relates ***G_4*-flux to 4D chirality** of F-theory
  - use to **compute 4D chiralities.**
  - use to derive ***G_4*-flux conditions for F-theory.**

See also: [Hayashi,Grimm; Cvetič,Grimm,D.K.; Grimm,Kapfer,Keitel; Braun,Grimm,Keitel]

extend earlier conditions: [Dasgupta,Rajesh,Sethi]

Application of toolbox

APPLICATION 1: RANK THREE CURVES

Elliptic fibrations with three rational points

[Cvetič,D.K.,Piragua,Song]

Similarly **explicit results** for elliptic fibrations with 3 U(1)'s:

- Elliptic curve with rank 3 Mordell-Weil group: **CICY in** $\mathrm{Bl}_3\mathbb{P}^3$
- **All** Calabi-Yau **elliptic fibrations** of *E* over given base *B* **classified**

F-theory vacua ⟺ points in **polytopes**.

- **Matter representations determined**
  - **14 representations**.
  - miraculous structure of singularities: **Tri-fundamental matter**
- **Matter multiplicities** in 6D found for general base B

⇒ 6D Anomalies cancelled. ✓

6D matter spectrum with U(1)^3

Charges	Multiplicities
(1,1,-1)	$[s_8]\cdot[s_{18}]$
(0,1,2)	$[s_9]\cdot[s_{19}]$
(1,0,2)	$[s_{10}]\cdot[s_{20}]$
(-1,0,1)	$[\tilde{s}_3]\cdot\tilde{\mathcal{S}}_7-[s_8]\cdot[s_{18}]$
(0,-1,1)	$[\hat{s}_3]\cdot\hat{\mathcal{S}}_7-[s_8]\cdot[s_{18}]$
(-1,-1,-2)	$[\tilde{s}_8]\cdot\mathcal{S}_9-[s_{10}]\cdot[s_{20}]$
(0,0,2)	$\tilde{\mathcal{S}}_7\cdot\mathcal{S}_9-[s_{19}][s_9]$
(1,1,1)	$4[K_B^{-1}]^2-3([p_2]^b)^2-2[K_B^{-1}]\hat{\mathcal{S}}_7-3([p_2]^b)\hat{\mathcal{S}}_7-2[K_B^{-1}]\tilde{\mathcal{S}}_7-3([p_2]^b)\tilde{\mathcal{S}}_7$ $-2\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7+2[K_B^{-1}]\mathcal{S}_9+9([p_2]^b)\mathcal{S}_9+5\hat{\mathcal{S}}_7\mathcal{S}_9+5\tilde{\mathcal{S}}_7\mathcal{S}_9-8\mathcal{S}_9^2$
(1,1,0)	$2[K_B^{-1}]^2+3([p_2]^b)^2+2[K_B^{-1}]\hat{\mathcal{S}}_7+3([p_2]^b)\hat{\mathcal{S}}_7+2[K_B^{-1}]\tilde{\mathcal{S}}_7+3([p_2]^b)\tilde{\mathcal{S}}_7$ $+\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7-3[K_B^{-1}]\mathcal{S}_9-9([p_2]^b)\mathcal{S}_9-4\hat{\mathcal{S}}_7\mathcal{S}_9-4\tilde{\mathcal{S}}_7\mathcal{S}_9+7\mathcal{S}_9^2$
(1,0,1)	$2[K_B^{-1}]^2+3([p_2]^b)^2+2[K_B^{-1}]\hat{\mathcal{S}}_7+3([p_2]^b)\hat{\mathcal{S}}_7-3[K_B^{-1}]\tilde{\mathcal{S}}_7+3([p_2]^b)\tilde{\mathcal{S}}_7$ $+2\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7+\tilde{\mathcal{S}}_7^2+2[K_B^{-1}]\mathcal{S}_9-9([p_2]^b)\mathcal{S}_9-5\hat{\mathcal{S}}_7\mathcal{S}_9-4\tilde{\mathcal{S}}_7\mathcal{S}_9+6\mathcal{S}_9^2$
(0,1,1)	$2[K_B^{-1}]^2+3([p_2]^b)^2-3[K_B^{-1}]\hat{\mathcal{S}}_7+3([p_2]^b)\hat{\mathcal{S}}_7+\hat{\mathcal{S}}_7^2+2[K_B^{-1}]\tilde{\mathcal{S}}_7$ $+3([p_2]^b)\tilde{\mathcal{S}}_7+2\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7+2[K_B^{-1}]\mathcal{S}_9-9([p_2]^b)\mathcal{S}_9-4\hat{\mathcal{S}}_7\mathcal{S}_9-5\tilde{\mathcal{S}}_7\mathcal{S}_9+6\mathcal{S}_9^2$
(1,0,0)	$4[K_B^{-1}]^2-3([p_2]^b)^2-2[K_B^{-1}]\hat{\mathcal{S}}_7-3([p_2]^b)\hat{\mathcal{S}}_7+2[K_B^{-1}]\tilde{\mathcal{S}}_7-3([p_2]^b)\tilde{\mathcal{S}}_7$ $-\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7-2\tilde{\mathcal{S}}_7^2-2[K_B^{-1}]\mathcal{S}_9+9([p_2]^b)\mathcal{S}_9+4\hat{\mathcal{S}}_7\mathcal{S}_9+5\tilde{\mathcal{S}}_7\mathcal{S}_9-6\mathcal{S}_9^2$
(0,1,0)	$4[K_B^{-1}]^2-3([p_2]^b)^2+2[K_B^{-1}]\hat{\mathcal{S}}_7-3([p_2]^b)\hat{\mathcal{S}}_7-2\hat{\mathcal{S}}_7^2-2[K_B^{-1}]\tilde{\mathcal{S}}_7$ $-3([p_2]^b)\tilde{\mathcal{S}}_7-\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7-2[K_B^{-1}]\mathcal{S}_9+9([p_2]^b)\mathcal{S}_9+5\hat{\mathcal{S}}_7\mathcal{S}_9+4\tilde{\mathcal{S}}_7\mathcal{S}_9-6\mathcal{S}_9^2$
(0,0,1)	$4[K_B^{-1}]^2-4([p_2]^b)^2+2[K_B^{-1}]\hat{\mathcal{S}}_7-4([p_2]^b)\hat{\mathcal{S}}_7-2\hat{\mathcal{S}}_7^2+2[K_B^{-1}]\tilde{\mathcal{S}}_7-4([p_2]^b)\tilde{\mathcal{S}}_7$ $-2\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7-2\tilde{\mathcal{S}}_7^2+2[K_B^{-1}]\mathcal{S}_9+12([p_2]^b)\mathcal{S}_9+6\hat{\mathcal{S}}_7\mathcal{S}_9+6\tilde{\mathcal{S}}_7\mathcal{S}_9-10\mathcal{S}_9^2$

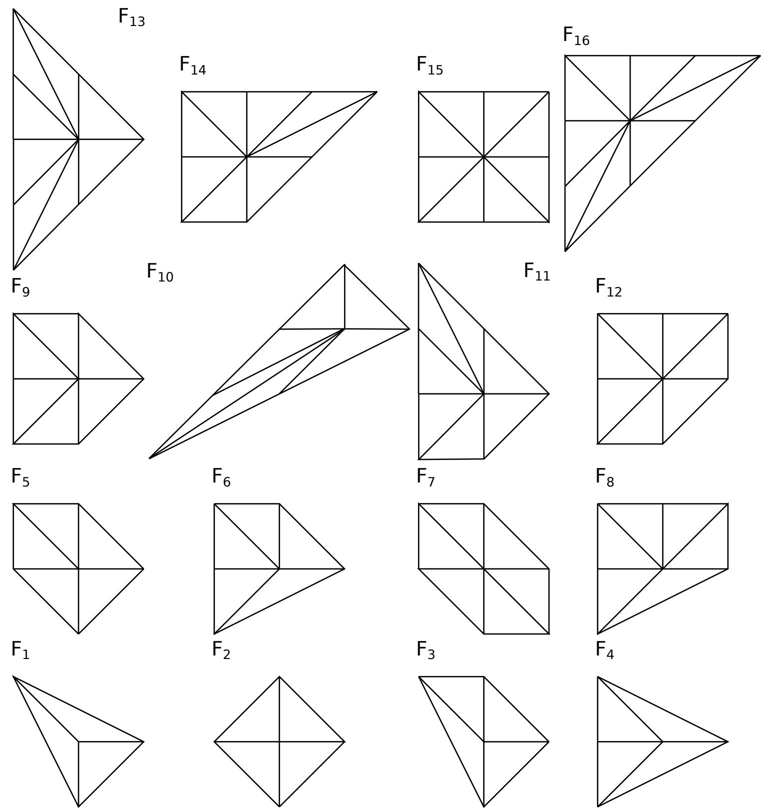
Application of toolbox

APPLICATION 2: ELLIPTIC CURVES IN TORIC VARIETIES

Elliptic fibrations with toric elliptic fibers

[D.K., Mayorga Peña, Oehlmann, Piragua, Reuter]

Elliptic fiber as hypersurface in 2d toric varieties associated to 16 reflexive polytopes:



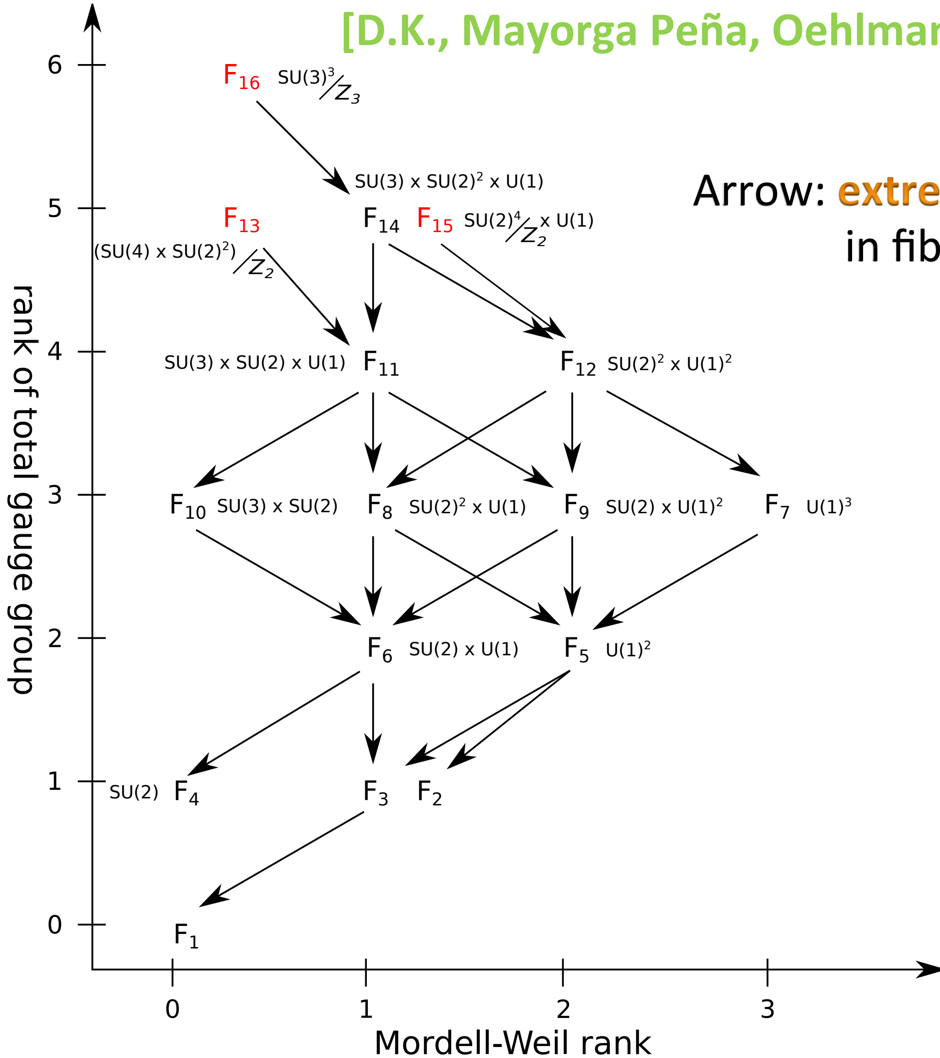
For algorithmic approach to toric models & toric Mordell-Weil, see: [Braun,Grimm,Keitel]

Applying presented techniques:

- **classify all CY-fibrations** with given *E* & arbitrary base *B.*
- determine **gauge group**.
- compute **matter spectra**: matter reps, 6D multiplicities; 4D **Yukawa couplings.**

Elliptic fibrations with toric elliptic fibers

[D.K., Mayorga Peña, Oehlmann, Piragua, Reuter]



Arrow: **extremal transitions** in fiber

- Up to **rank three Mordell-Weil group**, Mordell-Weil **torsion**, only **multi-sections** (need Jacobian fibrations).
- Extremal transitions in fiber = **Higgsing in eff. theory**: worked out.

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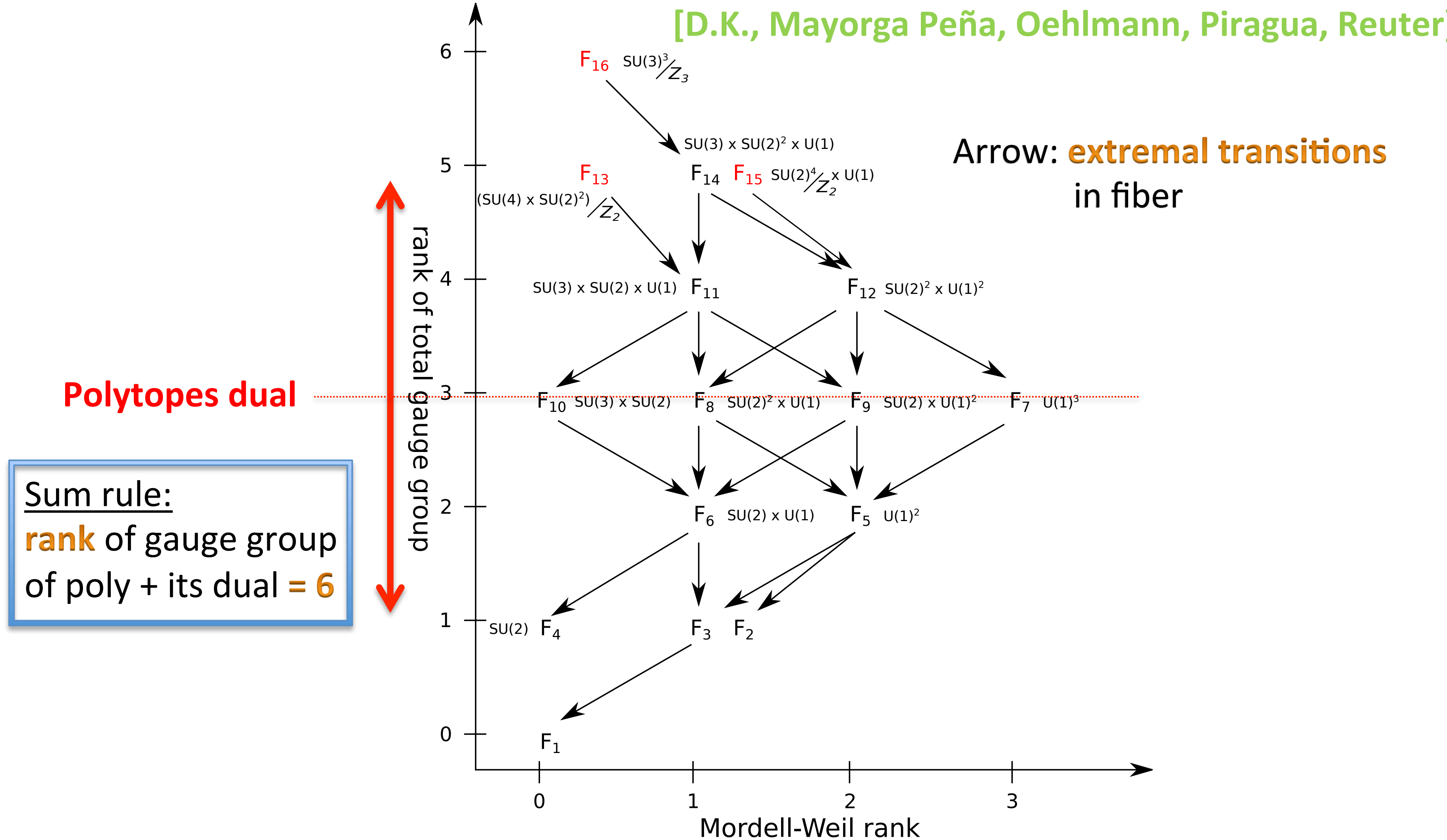

- Up to **rank three Mordell-Weil group**, Mordell-Weil **torsion**, only **multi-sections** (need Jacobian fibrations).
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Application of toolbox

APPLICATION 3: ENHANCING $U(1)^2$

Higgs-Transitions in F-theory: U(1)'s $\rightarrow G_{nA}$

Elliptic fibrations with higher rank Mordell-Weil group crucial for understanding the **moduli space of F-theory** compactifications.

➡ Can we tune complex structure to **enhance U(1)'s** to non-Abel. ***G_{nA}*?**

Rank 1 case understood: **[Morrison, Taylor]**

Every 6D F-theory with **single U(1)** comes **from Higgsed SU(2).**

SU(2) on Riemann surface Σ_g	adjoint VEV → ← tuning moduli	U(1)
rk(MW)=0		rk(MW)=1

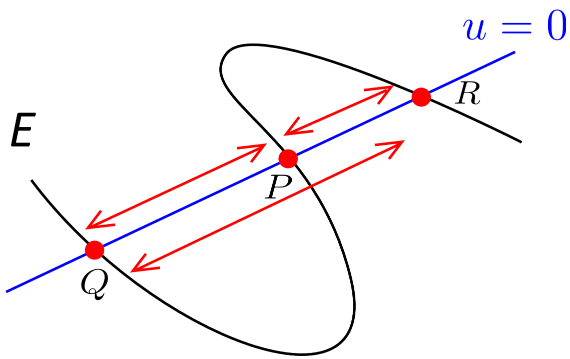
Geometrically: transition of **vertical divisor into rational section.**

Higgs-Transitions in F-theory: $U(1)^2 \to G_{nA}$

[Cvetič, D.K., Piragua, Taylor]

Enhancement of U(1)xU(1): **different** types of possible **enhancements**

- Reduce MW-rank to zero by **merging rational points** *Q*, *R* with origin *P*



$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

Higgs-Transitions in F-theory: $U(1)^2 \rightarrow G_{nA}$

[Cvetič, D.K., Piragua, Taylor]

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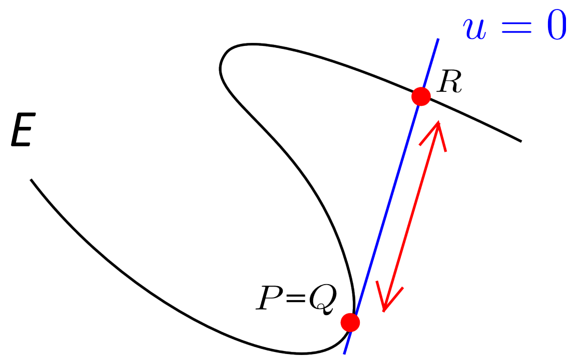

$$u f_2(u,v,w) + \lambda_1 (a_1 v + b_1 w)^2 (a_3 v + b_3 w) = 0$$

- rk(MW)=2 → 1: $\overline{PQ} \longrightarrow 0$

Higgs-Transitions in F-theory: $U(1)^2 \rightarrow G_{nA}$

[Cvetič, D.K., Piragua, Taylor]

<u>Enhancement of U(1)xU(1):</u> **different** types of possible **enhancements**

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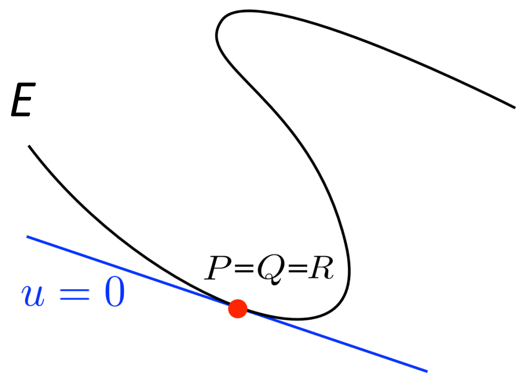

$$u f_2(u,v,w) + \lambda_1 \lambda_2 (a_1 v + b_1 w)^3 = 0$$

- rk(MW)=2 → 1: $\overline{PQ} \longrightarrow 0$
- rk(MW)=1 → 0: $\overline{PR} \longrightarrow 0$

This tuned fibration has **codimension 1 singularities** build in:

1. U(1)xU(1)→ SU(3): set $\lambda_i = 1$, at **locus** $f_2(0,-b_1,a_1) = 0$ **in *B***

   ⇨ ***I_3*-singularity** in *E* at $P=[0,-b_1,a_1]$.

2. U(1)xU(1)→ SU(2)xSU(2): set $f_2(0,-b_1,a_1) = 1$

   ⇨ ***I_2*-fiber at $\lambda_i = 0$** in *B*: $u f_2(u,v,w) = 0$.

3. general case **not rank preserving**: $U(1)^2 \rightarrow SU(3) x SU(2)^2$.

Summary

- **Construction** of elliptic fibrations with Mordell-Weil group.
- Developed **toolbox** to analyze these models
  - 6D matter spectrum & 4D Yukawas: **ideal techniques.**
  - 4D chiralities & G_4-flux: CY **4-fold cohomology** & **flux conditions.**
- Applied tools to **elliptically fibered CY's** with ell. fibers as
  - hypersurface in dP_2: $U(1)^2$, also with SU(5)
  - **complete intersection CY** in $\mathrm{Bl}_3(\mathbb{P}^3)$: $U(1)^3$
  - **all hypersurfaces** in 16 2d toric varieties.
- **$U(1)^2$** can be **enhanced** into G_{nA}, **not** always in **rank preserving** way.

Outlook

- Classification of *n>3* U(1)'s [Cvetič, DK, Piragua, Peng Song]: **work in progress**
- Heterotic dual of F-theory w/ U(1)'s [Cvetič, Grassi, DK, Piragua, Song]: **work in progress**

Thank you