AdS/CFT: Then and Now

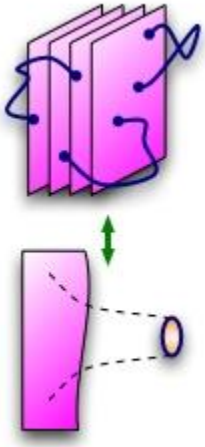
Igor Klebanov

Princeton University

Talk at

On June 30, 1997…

- A group of string theorists working on D-brane/black hole and D-brane/black brane correspondence. Polchinski; Strominger, Vafa; Callan, Maldacena; …
- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved RR charged background of type IIB theory of closed superstrings



$$ds^2 = \left(1+\frac{L^4}{r^4}\right)^{-1/2}\left(-(dx^0)^2+(dx^i)^2\right)+\left(1+\frac{L^4}{r^4}\right)^{1/2}\left(dr^2+r^2d\Omega_5^2\right)$$

- Matching the brane tensions gives $L^4 = g_{\rm YM}^2 N \alpha'^2$
- In addition to the 't Hooft large N limit, a new dramatic simplification for $g_{\rm YM}^2 N \gg 1$: the metric has small curvature everywhere.
- Bekenstein-Hawking entropy of near-extreme 3-brane

$$S_{BH} = \frac{2\pi A_h}{\kappa^2} = \frac{\pi^2}{2} N^2 V_3 T^3$$

- Agrees, up to a factor of 3/4, with that in weakly coupled SYM theory. Gubser, IK, Peet
- Low-energy absorption cross-sections agree exactly

$$\sigma_{SUGRA} = \frac{\pi^4}{8} \omega^3 L^8 = \frac{\kappa^2 \omega^3 N^2}{32\pi}$$

The AdS/CFT Duality

Maldacena; Gubser, IK, Polyakov; Witten

- The low-energy limit taken directly in the geometry. Maldacena
- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- Allows us to "solve" strongly coupled gauge theories, e.g. find operator dimensions $\Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$

Three Lessons Learned

- Lesson1: String theory can make definite, testable predictions!
- The dimensions of unprotected operators, which are dual to massive string states, grow at strong coupling as $2\left(ng_{\rm YM}\sqrt{N}\right)^{1/2}$
- Verified for the Konishi operator dual to the lightest massive string state (n=1) using the exact integrability of the planar $\mathcal{N}$=4 SYM theory. Gromov, Kazakov, Vieira; …
- Similar successes for the dimensions of high-spin operators, which are dual to spinning strings in AdS space.

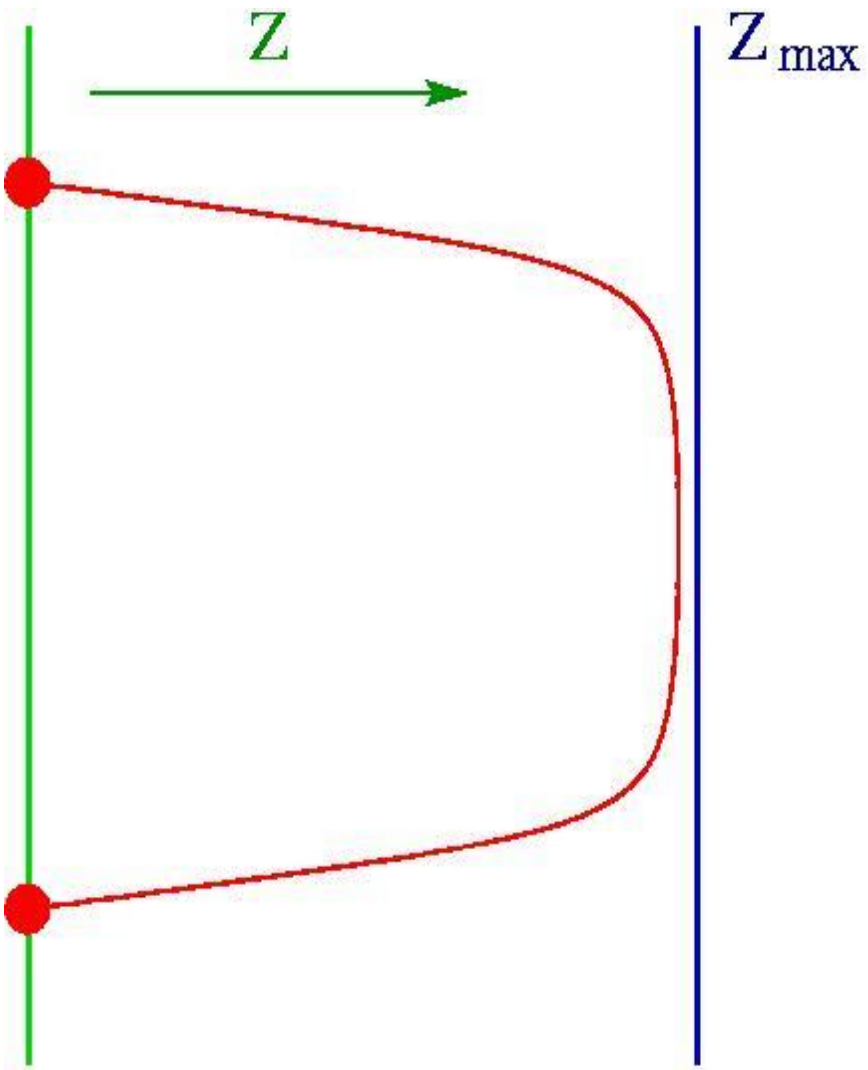
Lesson2: Color Confinement

- The quark anti-quark potential is linear at large distance but nearly Coulombic at small distance.
- The 5-d metric should have a warped form Polyakov

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\left(-(dx^0)^2 + (dx^i)^2\right)$$

- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

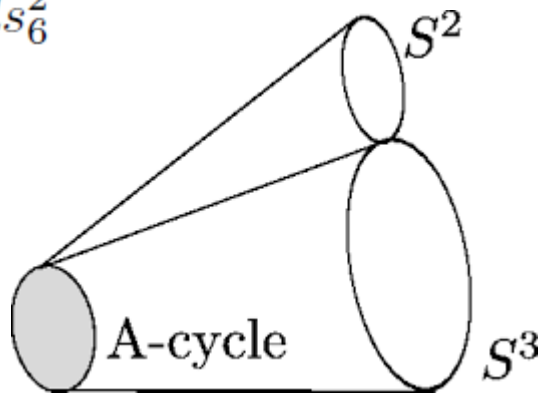
$$\frac{a^2(z_{\max})}{2\pi\alpha'}$$



- In some models, like the warped deformed conifold, the confinement happens dynamically through dimensional transmutation. IK, Strassler

$$ds_{10}^2 = h^{-1/2}(y)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(y)ds_6^2$$

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$



- However, the string dual of asymptotically free gauge theory remains elusive.

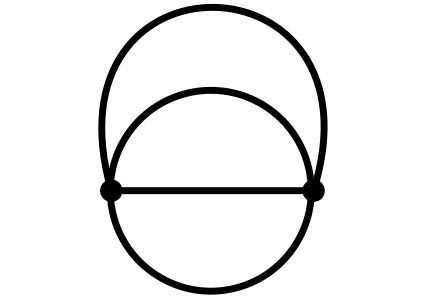
- Lesson 3: The whole thing has become WAY more than anyone expected 20 years ago.
- I am amazed by the range of applications of the gauge/gravity duality.
- In addition to the strongly coupled plasmas and many body physics, we have learned a lot about quantum entanglement and quantum information. Maldacena; Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi; IK, Kutasov, Murugan; Myers, …
- This is teaching us a lot about the mysteries of black holes and quantum gravity.
- We have also learned a great deal about the Chern-Simons matter CFTs using both the ABJM type models and the higher-spin AdS/CFT. Aharony et al; Giombi et al; …

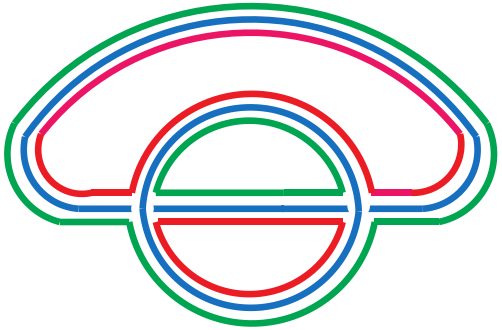
A Brief Wish List

- Getting better control over the regime where the coupling is not very large, but is of order 1. Most gauge theories, including the non-supersymmetric ones, are in this regime. This is crucial for understanding the large N QCD more quantitatively.
- A better understanding of the 1/N corrections to observables. This is crucial for the applications to quantum gravity.
- CFTs dual to de Sitter space.

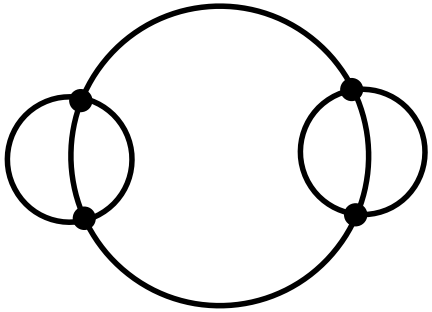
Another Wish: More Melons

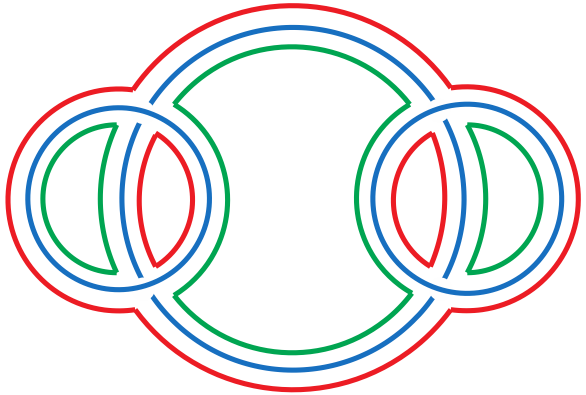
- The “melonic” large N limits, which appear in the tensor models, have already been connected with SYK-like models. Gurau; Witten; IK, Tarnopolsky;…
- Hopefully, the tensor models will find other uses.





$$g^2 N^6 \sim N^3 \lambda^2$$





$$g^4 N^9 \sim N^3 \lambda^4$$

- HAPPY 20TH BIRTHDAY, ADS/CFT!
- AND MANY HAPPY RETURNS!