General Relativity and the Cuprates

Gary Horowitz

UC Santa Barbara

GH, J. Santos, D. Tong,

1204.0519, 1209.1098

GH and J. Santos, 1302.6586

Gauge/gravity duality can reproduce many properties of condensed matter systems, even in the limit where the bulk is described by classical general relativity:

1) Fermi surfaces
2) Non-Fermi liquids
3) Superconducting phase transitions
4) é

It is not clear why it is working so well.

Can one do more than reproduce qualitative features of condensed matter systems?

Can gauge/gravity duality provide a quantitative explanation of some mysterious property of real materials?

We will argue that the answer is yes!

Plan: Calculate the optical conductivity of a simple holographic conductor and superconductor with lattice included.

Earlier work on the effects of a lattice by many groups, e.g., Kachru et al; Maeda et al; Hartnoll and Hofman; Zaanen et al.; Siopsis et al., Flauger et al

Main result: We will find surprising similarities to the optical conductivity of some cuprates.

$$\mathrm{Re}(\sigma) = \frac{K\tau}{1+(\omega\tau)^2}, \quad \mathrm{Im}(\sigma) = \frac{K\omega\tau^2}{1+(\omega\tau)^2}$$

Note:

(1) For $\omega\tau \gg 1, \quad |\sigma| \approx K/\omega$

(2) In the limit $\tau \to \infty$:

$$\mathrm{Re}(\sigma) \propto \delta(\omega), \quad \mathrm{Im}(\sigma) = K/\omega$$

This can be derived more generally from Kramers-Kronig relation.

Our gravity model

We start with just Einstein-Maxwell theory:

$$S = \int d^4x\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right]$$

This is the simplest context to describe a conductor. We require the metric to be asymptotically anti-de Sitter (AdS)

$$ds^2 = \frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2}$$

Want finite temperature: Add black hole

Want finite density: Add charge to the black hole. The asymptotic form of A_t is

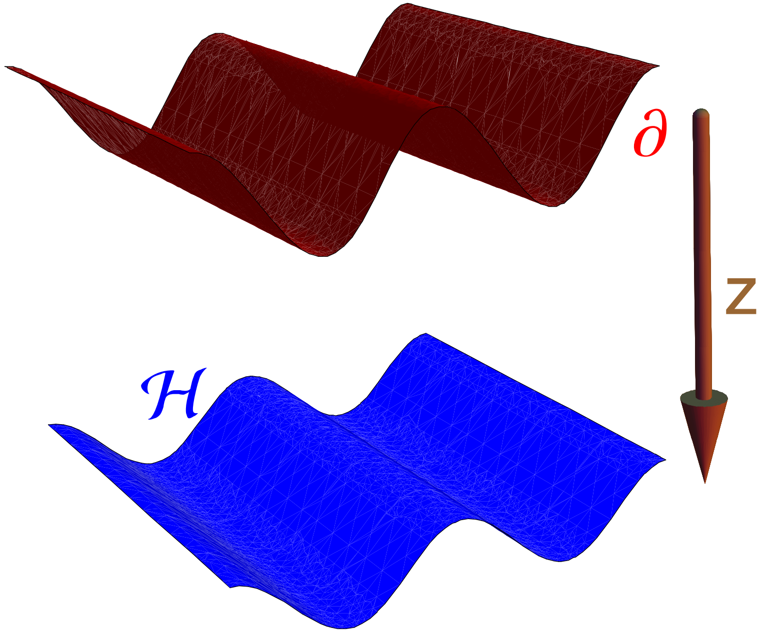
$$A_t = \mu - \rho z + O(z^2)$$

ε is the chemical potential and ɟ is the charge density in the dual theory.

Introduce the lattice by making the chemical potential be a periodic function:

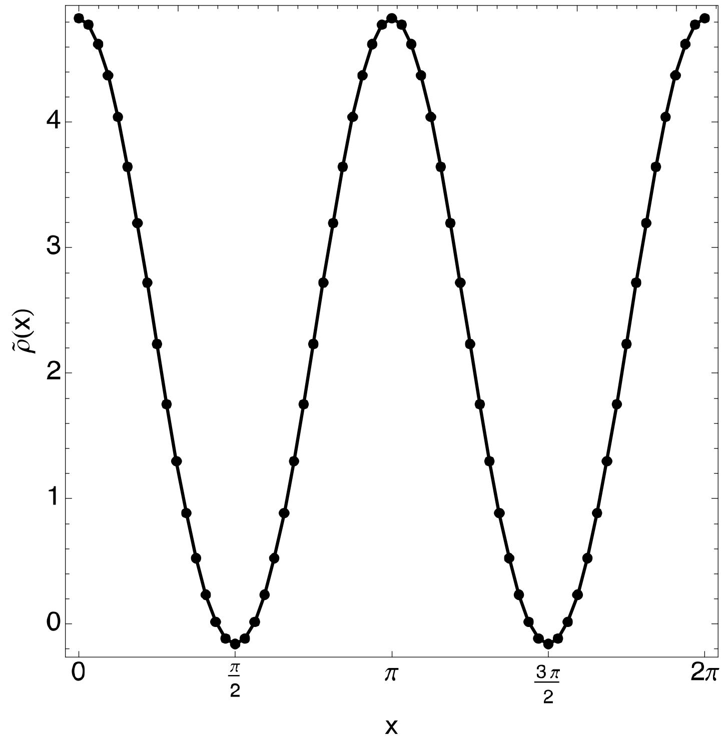
$$\mu(x) = \bar{\mu}\left[1 + A_0 \cos(k_0 x)\right]$$

We numerically find solutions with smooth horizons that are static and translationally invariant in one direction.



Solutions are rippled charged black holes.

Charge density for
$A_0 = ½$, $k_0 = 2$,
$T/ε = .055$



Conductivity

To compute the optical conductivity using linear response, we perturb the solution

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \delta g_{\mu\nu}, \qquad A_\mu = \hat{A}_\mu + \delta A_\mu$$

Boundary conditions:

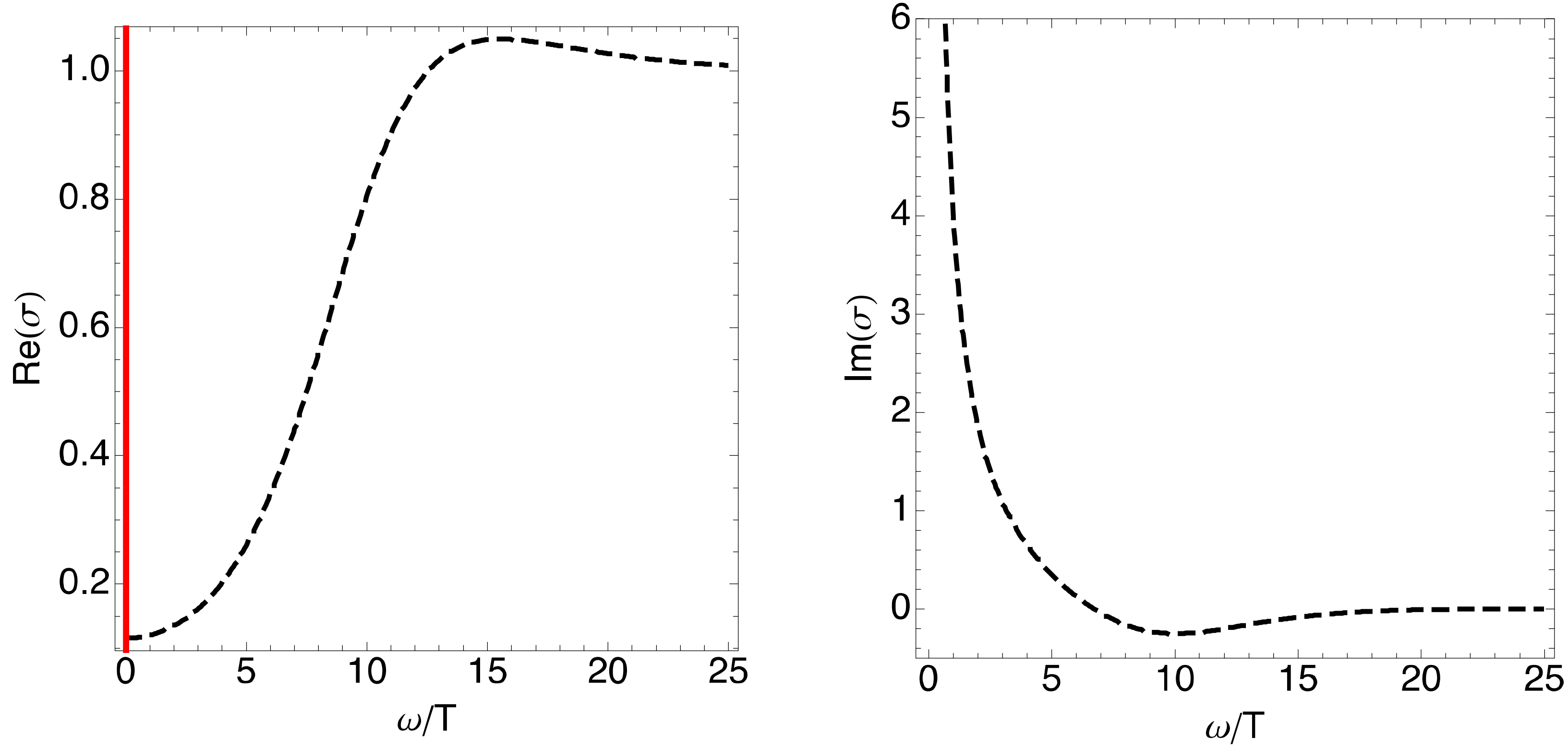
ingoing waves at the horizon

ŭg$_{ε3}$ normalizable at infinity

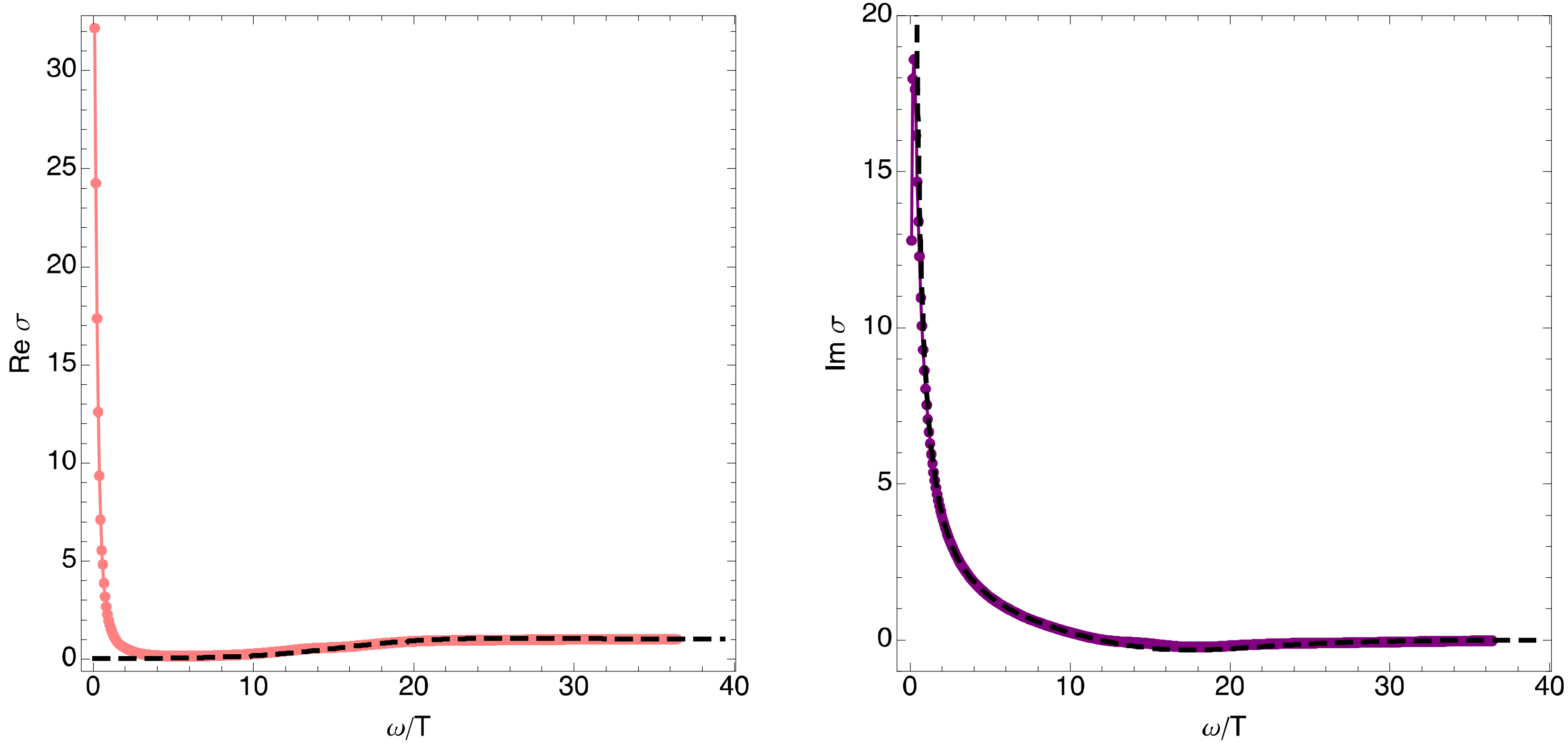
ŭA$_t$ ~ O(z), ŭA$_x$ = e$^{-iɤ t}$ [E/iɤ + J z + é]

induced current

Review: optical conductivity with no lattice
(T/ε = .115)

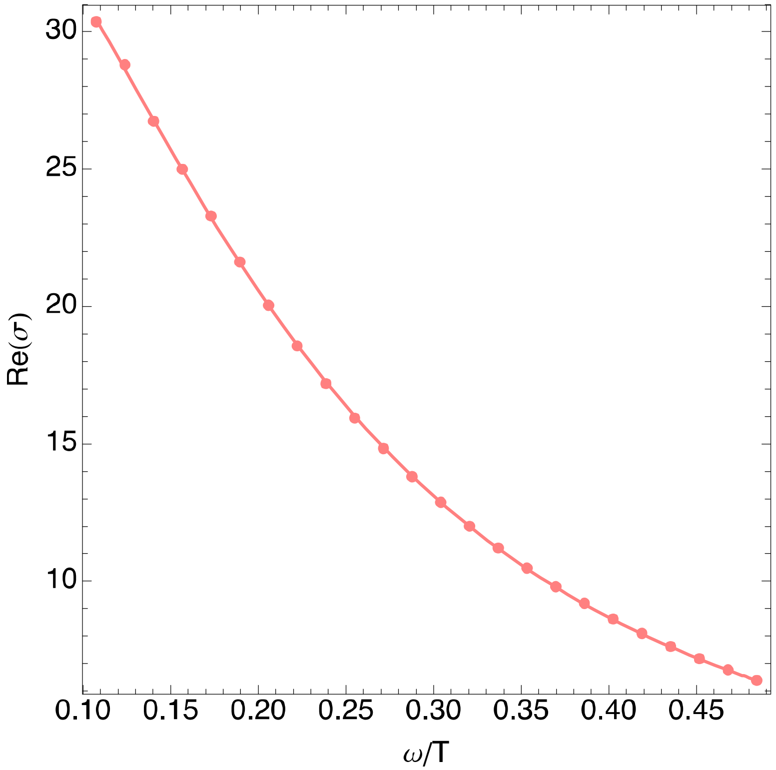


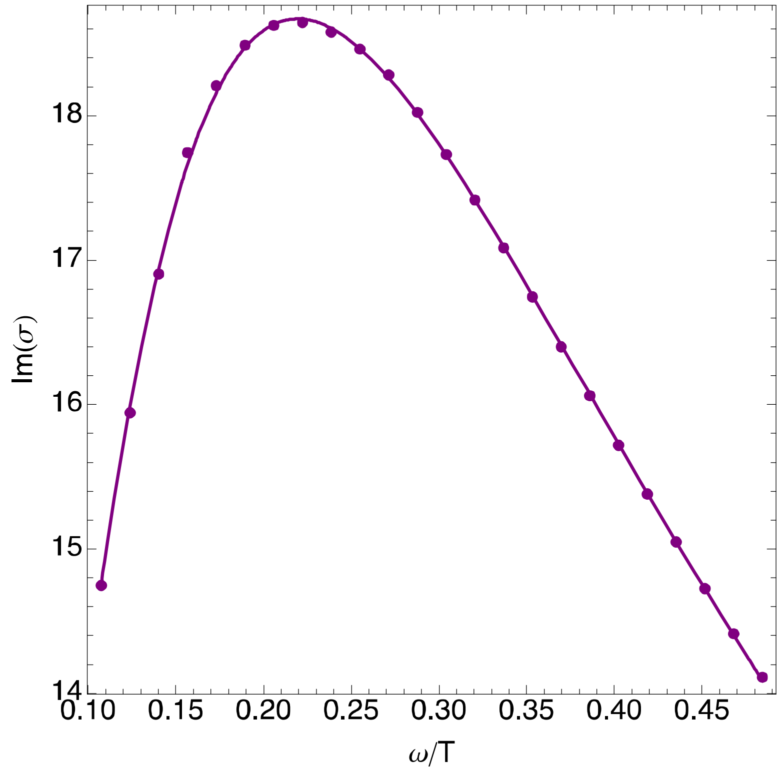
With the lattice, the delta function is smeared out



The low frequency conductivity takes the simple Drude form:

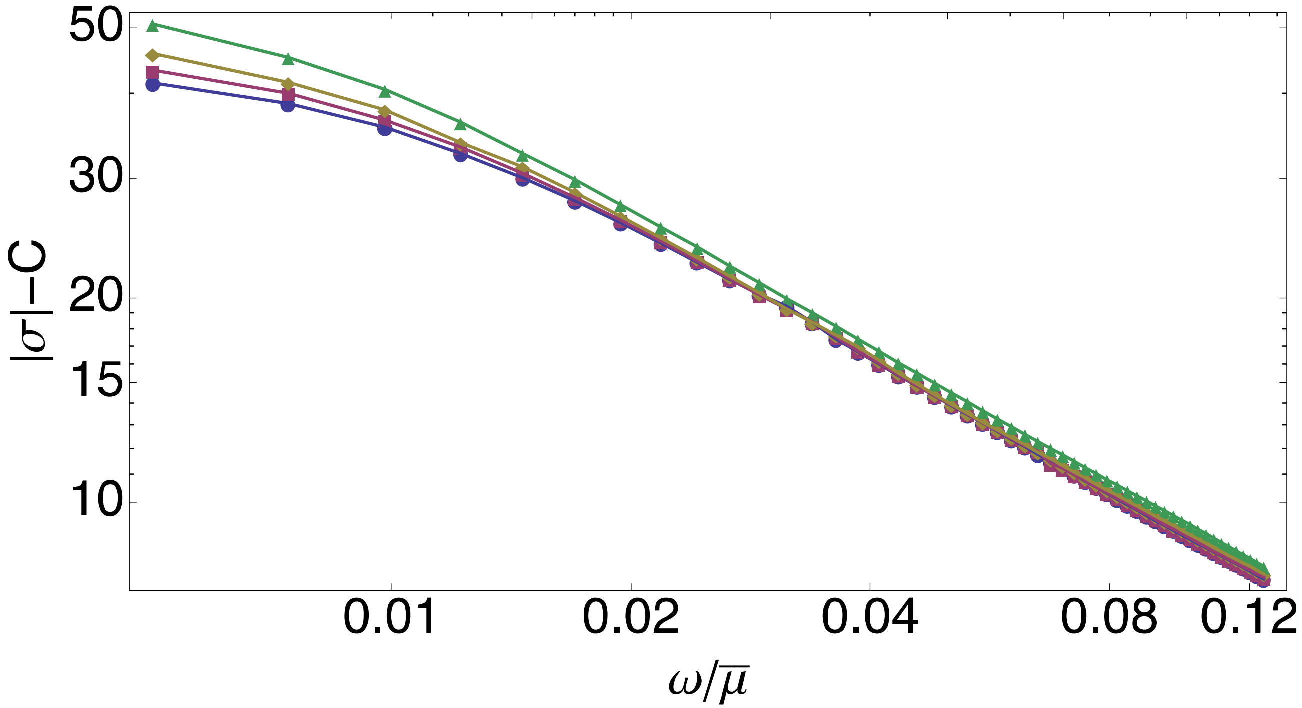
$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$





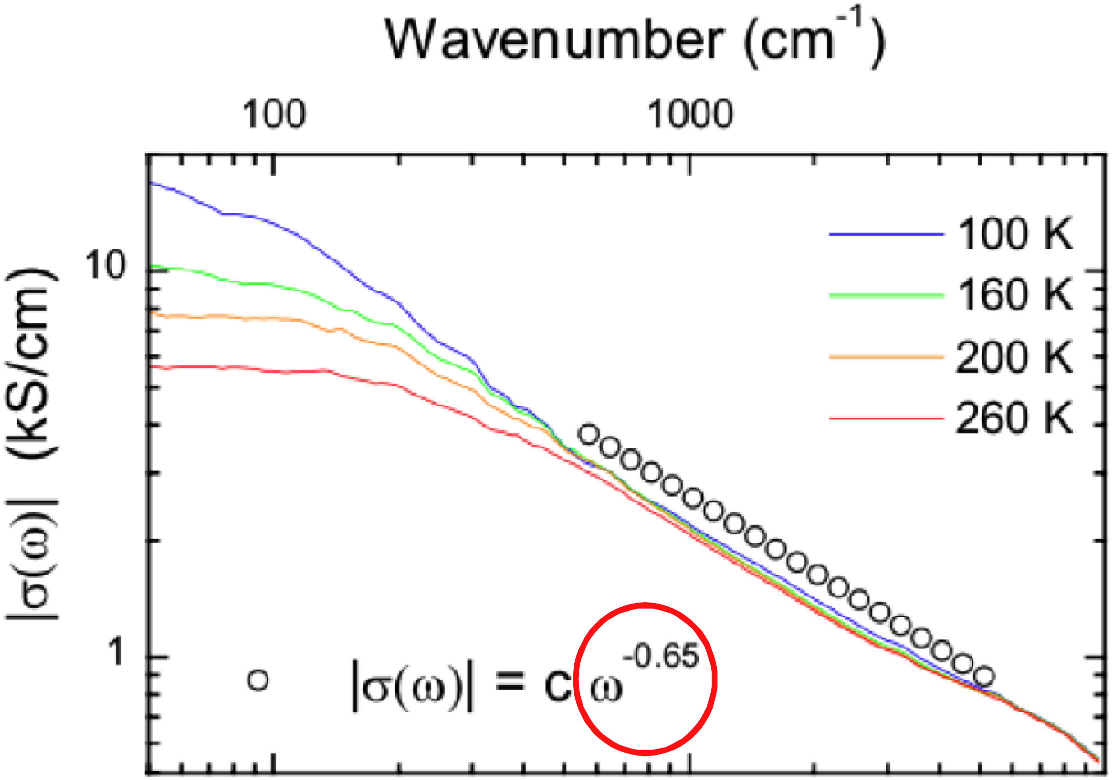
Intermediate frequency shows scaling regime:

$$|\sigma| = \frac{B}{\omega^{2/3}} + C$$



Lines show 4 different temperatures:
.033 < T/ε < .055

Comparison with the cuprates
(van der Marel, et al 2003)



$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

What happens in the superconducting regime?

We now add a charged scalar field to our action:

$$S = \int d^4x \sqrt{-g} \left[R + \frac{6}{L^2} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2|(\partial - ieA)\Phi|^2 + \frac{4|\Phi|^2}{L^2} \right]$$

Gubser (2008) argued that at low temperatures, charged black holes would have nonzero ū .

Hartnoll, Herzog, GH (2008) showed this was dual to a superconductor (in homogeneous case).

The scalar field has mass $m^2 = -2/L^2$, since for this choice, its asymptotic behavior is simple:

$$\Phi = z\phi_1 + z^2\phi_2 + \mathcal{O}(z^3)$$

This is dual to a dimension 2 charged scalar operator O with source ϕ_1 and $\langle O \rangle = \phi_2$. We set $\phi_1 = 0$.

For electrically charged solutions with only A_t nonzero, the phase of ū must be constant.

We keep the same boundary conditions on A_t as before:

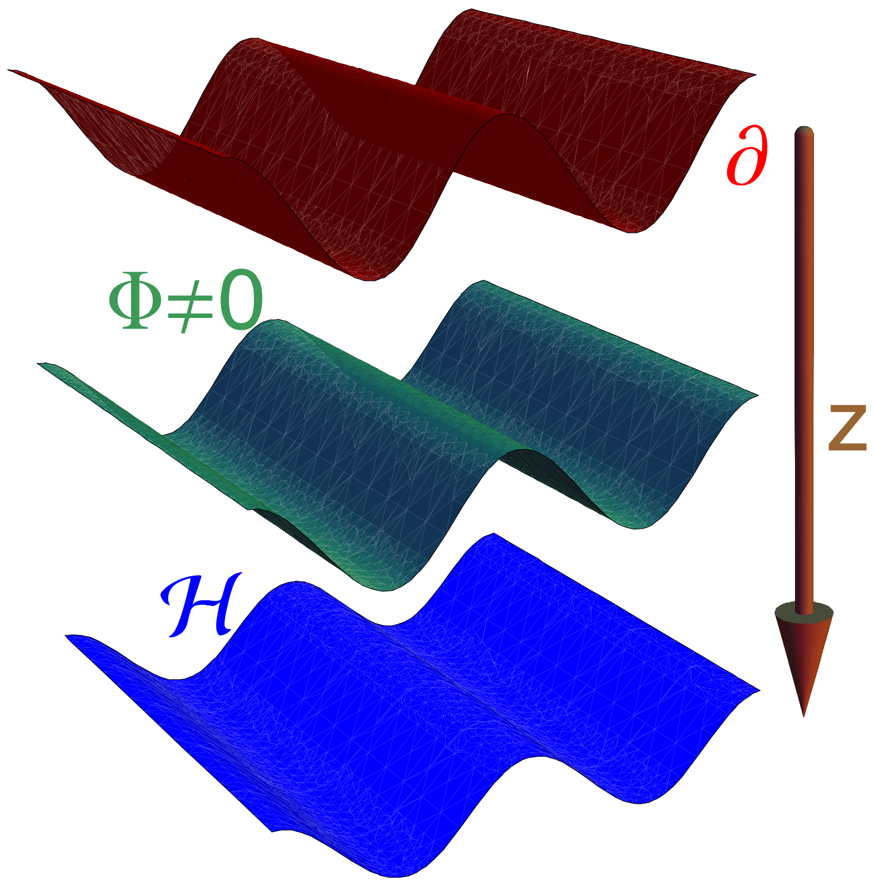
$$\mu(x) = \bar{\mu}\,[1 + A_0 \cos(k_0 x)]$$

Start with previous rippled charged black holes with $\bar{u} = 0$ and lower T. When do they become unstable?

Onset of instability corresponds to a static normalizable mode of the scalar field. This can be used to find T_c.

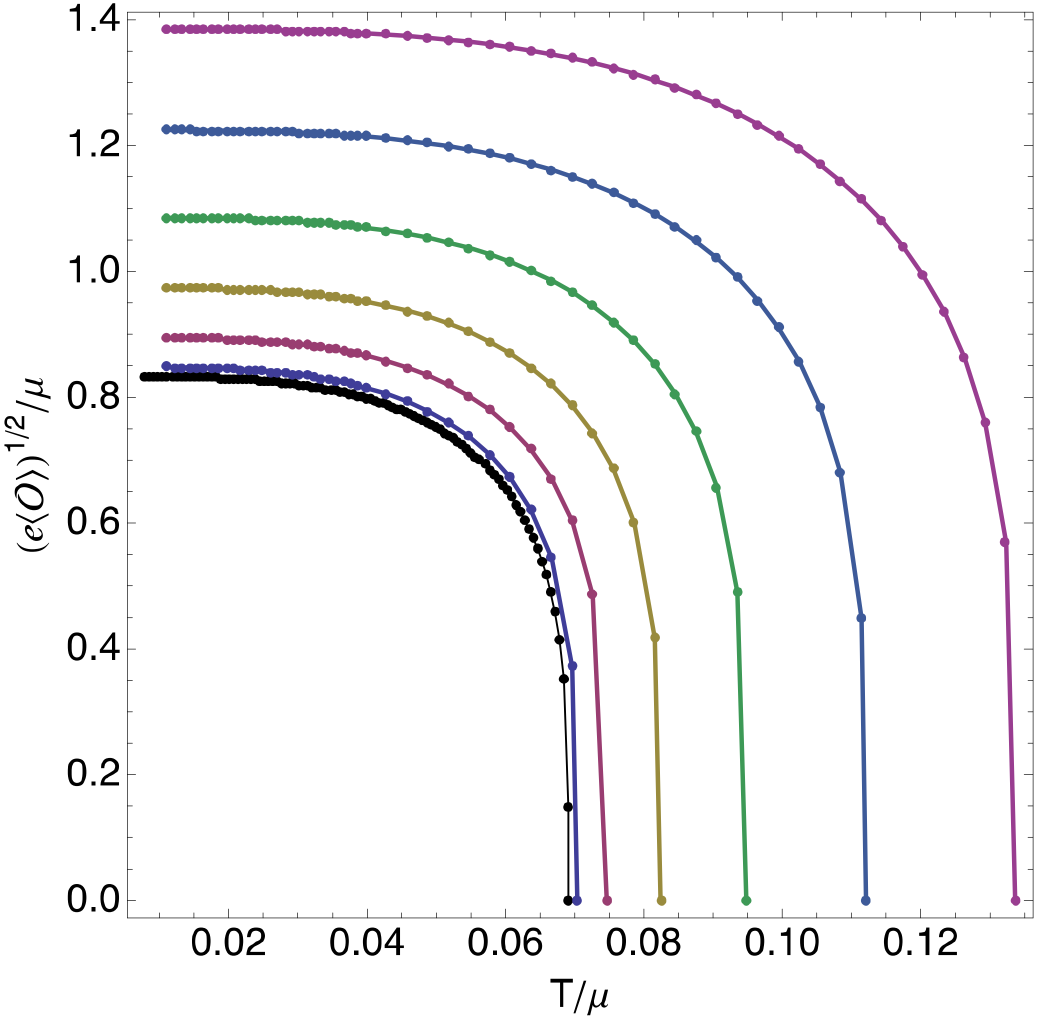
Having found T_c, we now find solutions for $T < T_c$ numerically.

These are hairy, rippled, charged black holes.



From the asymptotic behavior of ū we read off the condensate as a function of temperature.

Condensate as a function of temperature



Lattice amplitude grows from 0 (inner line) to 2.4 (outer line).

We again perturb these black holes as before and compute the conductivity as a function of frequency.

Find that curves at small ɤ are well fit by adding a pole to the Drude formula

$$\sigma(\omega) = i\frac{\rho_s}{\omega} + \frac{\rho_n \tau}{1 - i\omega\tau}$$

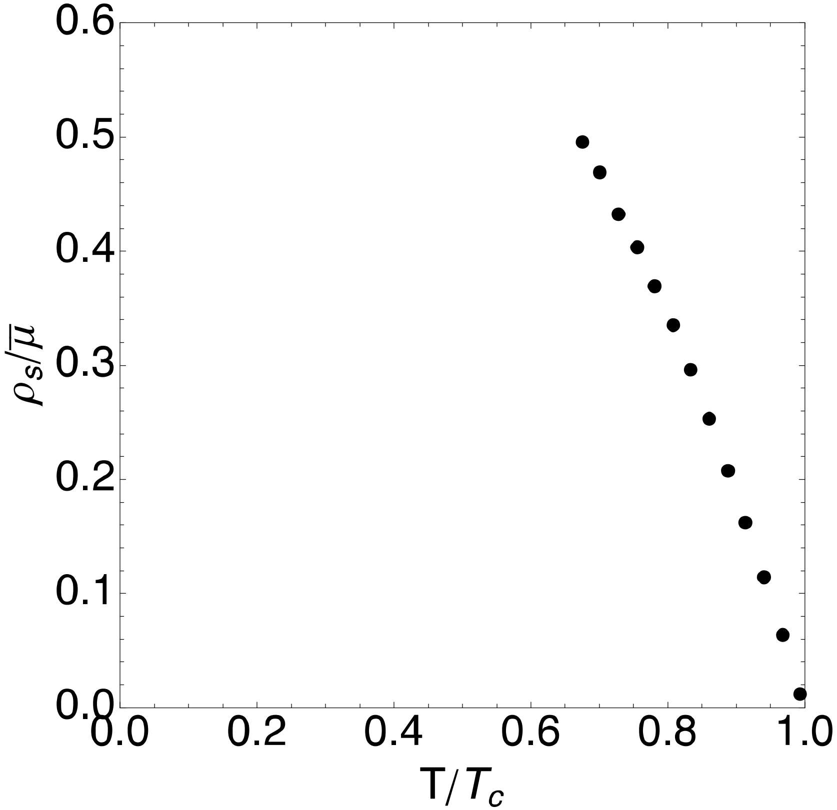
Superfluid component

Normal component

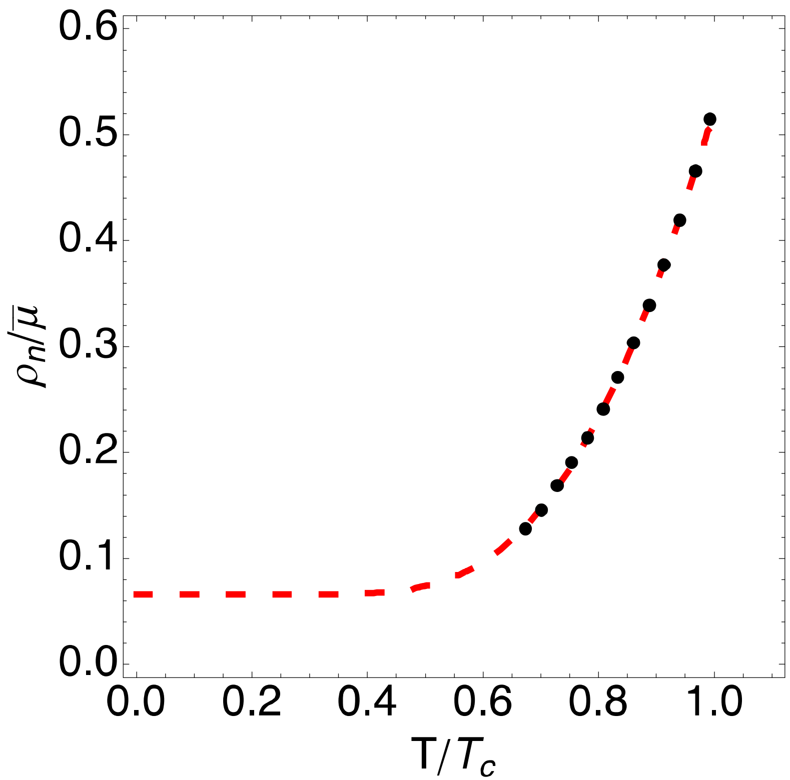
The lattice does not destroy superconductivity

(Siopsis et al, 2012; Iizuka and Maeda, 2012)

Fit to: $\sigma(\omega) = i\frac{\rho_s}{\omega} + \frac{\rho_n \tau}{1 - i\omega\tau}$



superfluid density



normal fluid density

The dashed red line through ρ_n is a fit to:

$$\rho_n = a + b e^{-\Delta/T}$$

with $\Delta = 4\ T_c$.

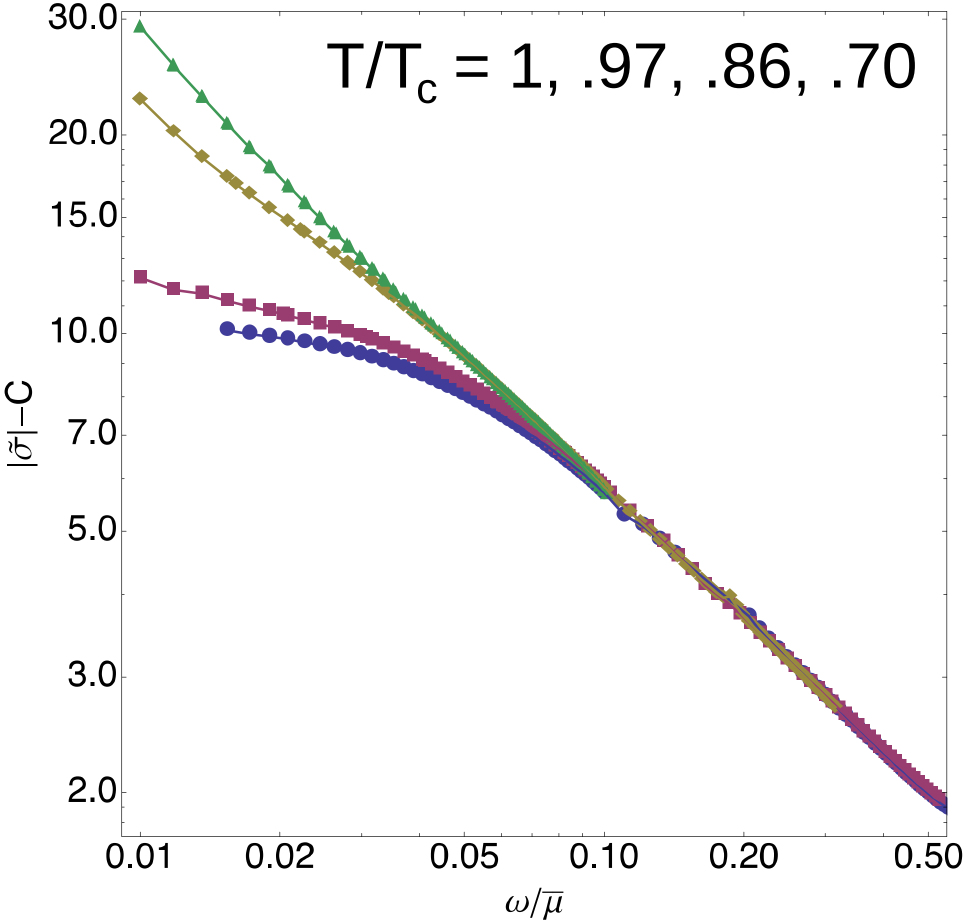
This is like BCS with thermally excited quasiparticles but:

(1) The gap Δ is much larger, and comparable to what is seen in the cuprates.

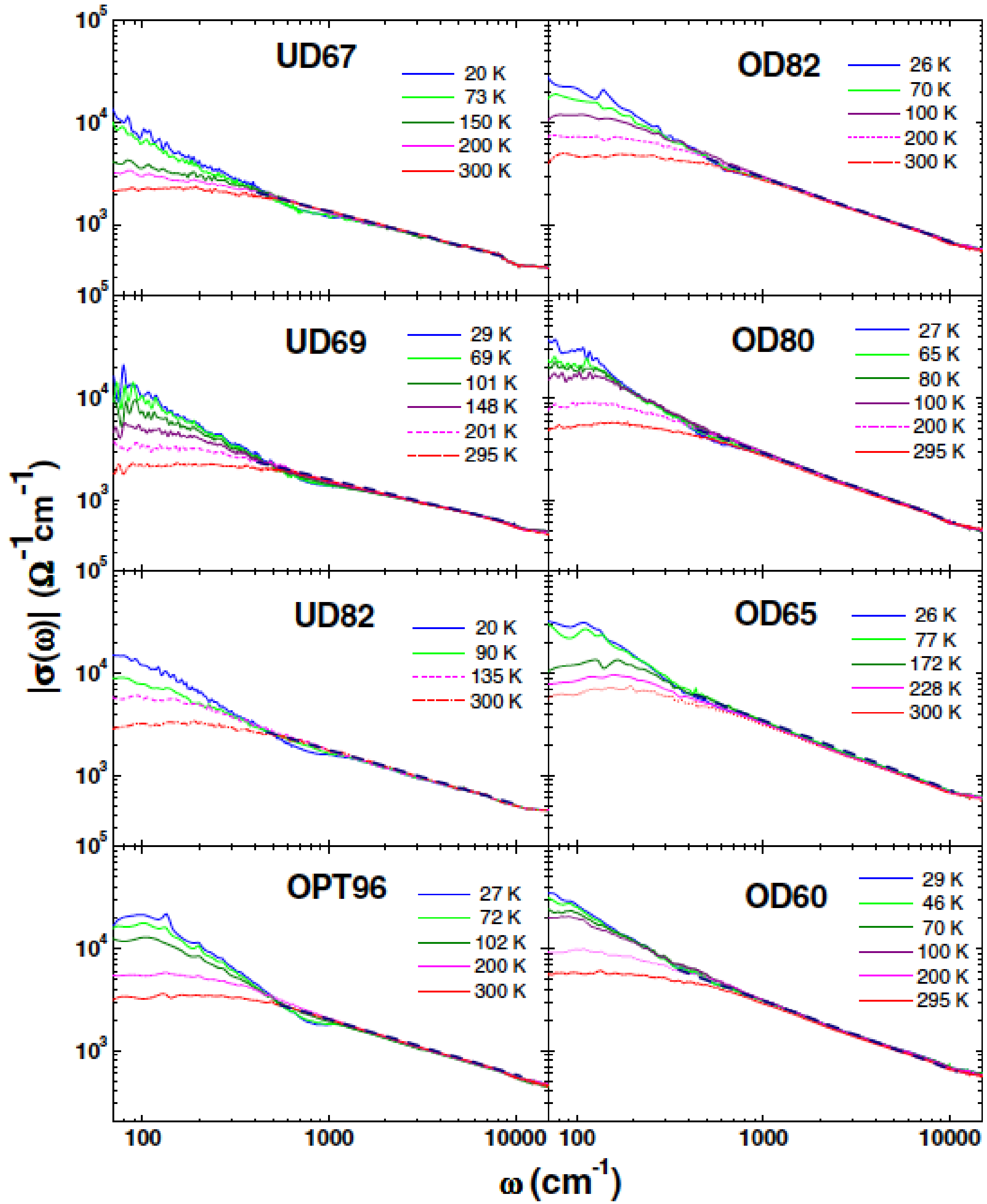
(2) Some of the normal component remains even at T = 0 (this is also true of the cuprates).

Intermediate frequency conductivity again shows the same power law:

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$



Coefficient B and exponent 2/3 are independent of T and identical to normal phase.



8 samples of BSCCO with different doping.

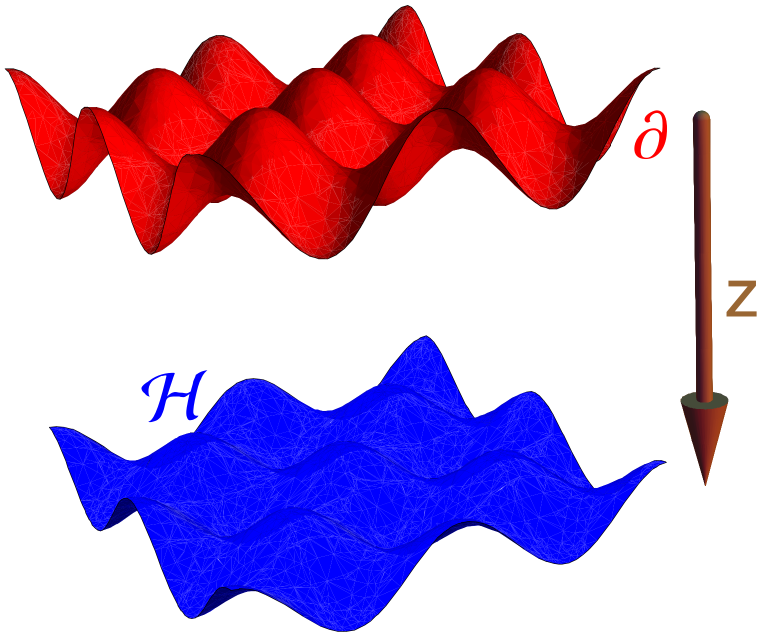
Each plot includes $T < T_c$ as well as $T > T_c$.

No change in the power law.

(Data from Timusk et al, 2007.)

Preliminary results on a full 2D lattice ($T > T_c$) show very similar results to 1D lattice.

The optical conductivity in each lattice direction is nearly identical to the 1D results.



Our simple gravity model reproduces many properties of cuprates:

- Å Drude peak at low frequency
- Å Power law fall-off $ϒ^{-2/3}$ at intermediate ϒ
- Å Gap $2φ = 8\ T_c$
- Å Normal component doesnôt vanish at T = 0

But key differences remain

Å Our superconductor is s-wave, not d-wave

Å Our power law has a constant off-set C

Å é