Recent progress at the holography/condensed matter interface

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Plan of talk

- **Effective field theory of (low T) transport.**
- **Application to Fermi liquids.**
- **Application to Holographic liquids**
  (i) Semi-local quantum criticality.
  (ii) The DBI action.
- **Holographic insulators.**

Charge transport at strong coupling

- **Most computations of conductivities etc. use the Boltzmann equation:**

$$-\vec{E} \cdot \frac{\partial f_k}{\partial \vec{k}} = -I_{\rm ei}[f_k] - I_{\rm ee}[f_k]$$

- **Assumes long lived 'quasiparticles', not useful at strong coupling.**

- **First objective: <u>effective field theory framework</u> for strongly coupled transport.**

- **Theorem (1960s, easy):**
  If there exists a conserved quantity P that overlaps with the electrical current operator J, i.e.

$$\chi_{PJ} \neq 0$$

Then the d.c. conductivity is infinite:

$$\sigma \sim \frac{\chi_{PJ}^2}{\chi_{PP}} \delta(\omega)$$

- **Example: absence of lattice and impurities ⇒ momentum conserved**

- **Consequence:**
  Suppose conservation of P is violated only by an irrelevant operator O in the low energy effective theory.

  Then the d.c. conductivity is large:

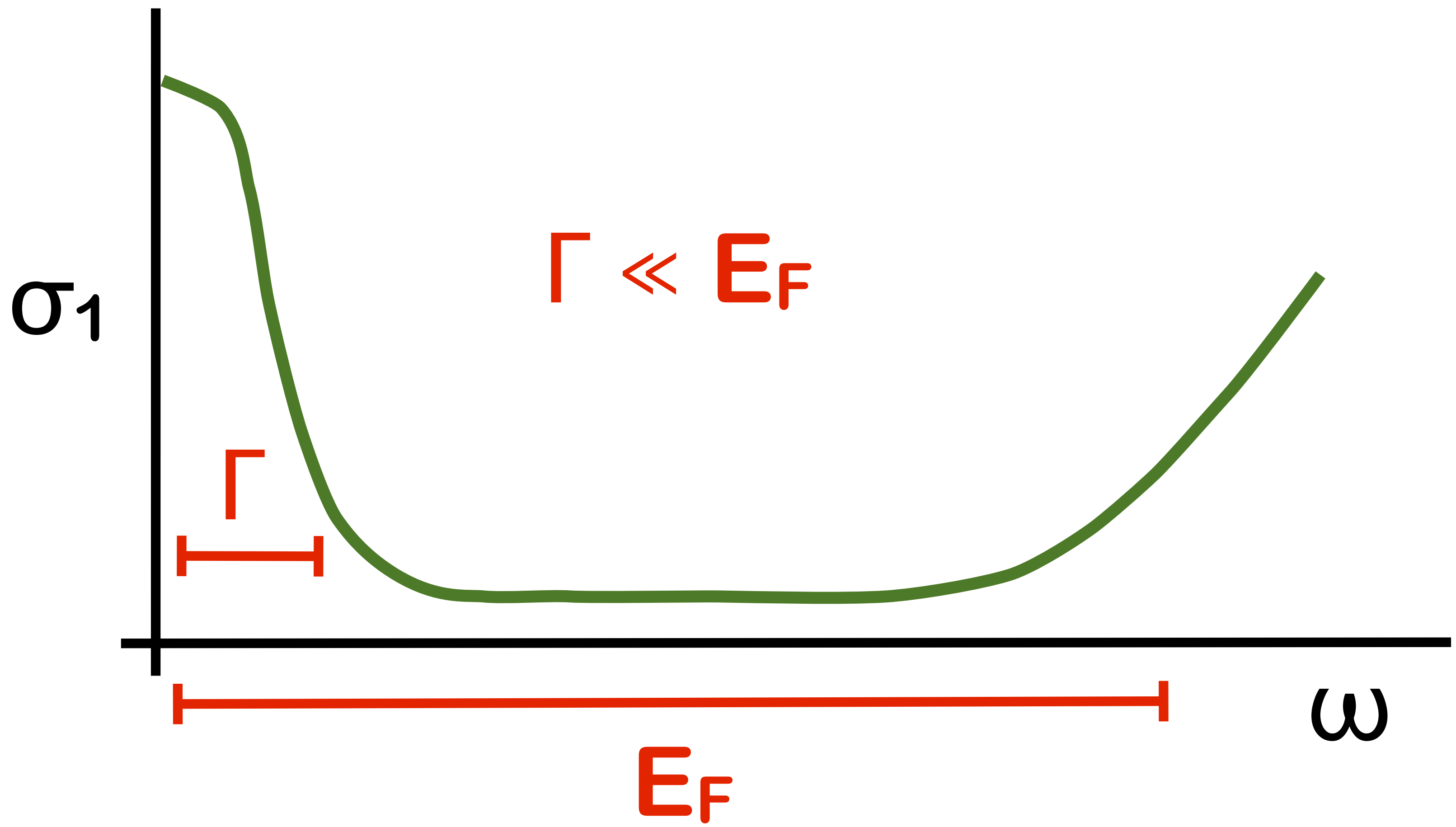
$$\sigma = \frac{\chi_{PJ}^2}{\chi_{PP}} \frac{1}{\Gamma}$$

Relaxation rate (→ Γ)

- **At low temperatures, dominant T dependence is from Γ. Thus, resistivity:**

$$\rho \sim \Gamma$$

- **The small scale Γ furthermore gives a Drude peak:**



There is a sum rule that: $$\int_0^\infty \mathrm{Re}\,\sigma(\omega)d\omega$$

is given by fixed UV data.

- **Remnant of UV lattice in IR is a momentum-carrying operator O(k_L).**

- **If O is irrelevant, Γ can be computed <u>perturbatively</u> in the IR coupling g of O:**

$$\Gamma = \frac{g^2 k_L^2}{\chi_{PP}} \lim_{\omega \to 0} \frac{\operatorname{Im} G^R_{\mathcal{O}\mathcal{O}}(\omega, k_L)}{\omega}\bigg|_{g=0}$$

Hartnoll-Hofman @ 1201.3917

(case of impurities: Hartnoll-Kovtun-Muller-Sachdev @ 0706.3215)

- **Results quoted so far are all derived using the ''memory matrix formalism''.**

- **This formalism builds around almost-conserved quantities, and is the correct way to think about charge transport in strongly correlated metallic systems.**

  Suggested reading:

Hartnoll-Hofman @ 1201.3917

Mahajan-Barkeshli-Hartnoll @ 1304.4249

Andrei-Shimshoni-Rosch @ cond-mat/0307578

Fermi liquids:
The physics is at nonzero momentum

- Famously (1930s!): a clean Fermi liquid has a low T electrical resistivity

$$\rho \sim T^2$$

- An effective field theory derivation of this result reveals nontrivial physics.

- **Recall the formula for relaxation rate:**

$$\Gamma = \frac{g^2 k_L^2}{\chi_{PP}} \lim_{\omega \to 0} \left. \frac{\operatorname{Im} G^R_{\mathcal{O}\mathcal{O}}(\omega, k_L)}{\omega} \right|_{g=0}$$

- **Significant relaxation <u>requires low energy spectral weight</u> (i.e. on shell excitations) <u>at nonzero momentum</u> k_L.**

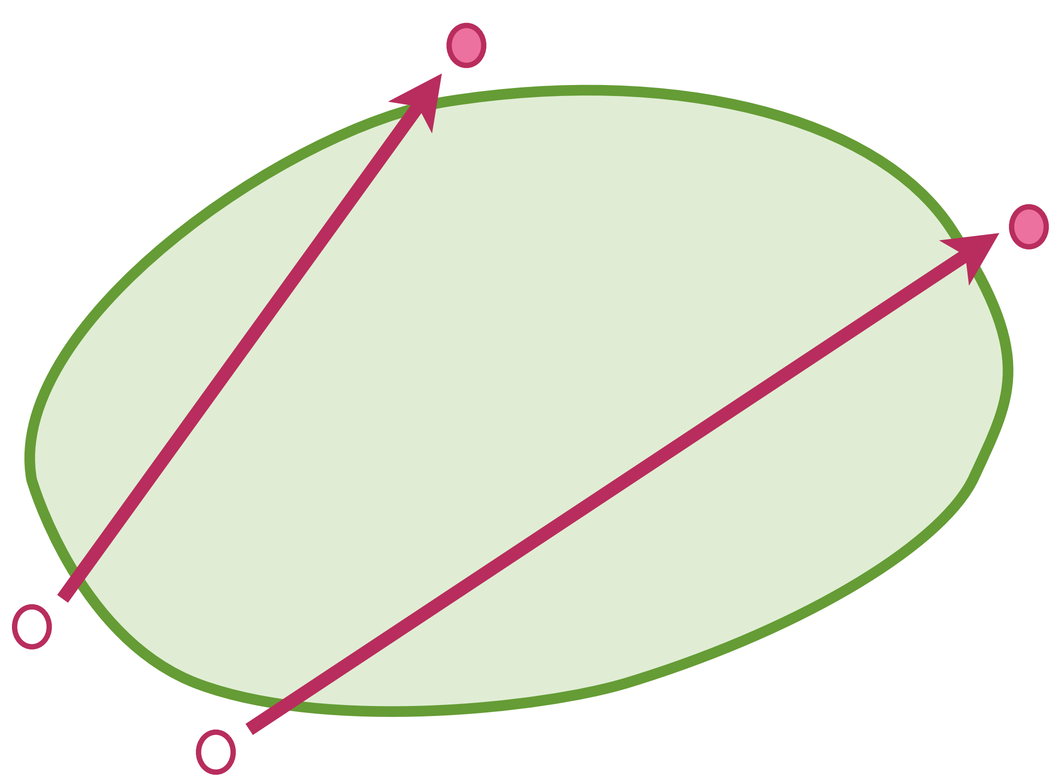
- **Clearly, such excitations do not exist in e.g. a Lorentz invariant theory:**

$$\omega \sim k \qquad \Rightarrow \qquad \Gamma \sim e^{-k_L/T}$$

- **In a Fermi Liquid, low energy excitations live on the Fermi surface.**

- **Leading irrelevant operator with finite momentum is the <u>umklapp operator</u>:**

$$\mathcal{O}(k_L) = \int \left(\prod_{i=1}^{4} d\omega_i d^2 k_i \right) \psi^\dagger(k_1)\psi^\dagger(k_2)\psi(k_3)\psi(k_4)\delta(k_1+k_2-k_3-k_4-k_L)$$



- **Using the RG flow for Fermi surfaces of Polchinski (hep-th/9210046), the umklapp operator O(w,k) has scaling dimension Δ=1. It is irrelevant.**

- **Dimensional analysis then gives**

$$\rho \sim \Gamma \sim T^2$$

[Hartnoll-Hofman (1201.3917)]

- **Lesson: resistivity of a Fermi liquid depends upon the <u>interplay of two momentum scales: k_L and k_F</u>.**

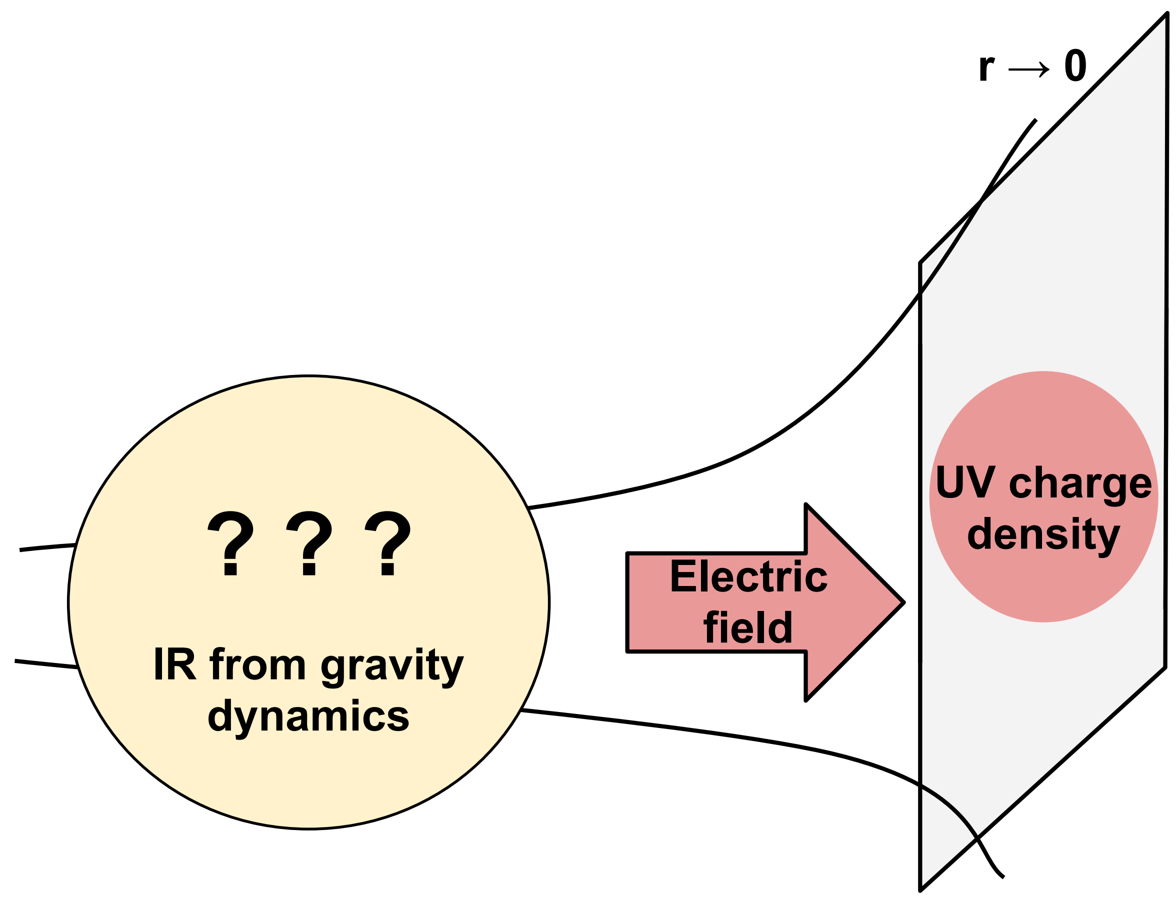
Holographic liquids:
New physics in momentum space

- A Fermi surface is (in the first instance) a weakly coupled notion, and depends on Pauli exclusion operating on a Fock space of fermionic states.
- Holographic theories find other ways to push low energy excitations out to nonzero momentum. Strongly coupled cousin of Pauli exclusion?

(Anantua-Hartnoll-Martin-Ramirez @ 1210.1590)

Holography at nonzero density 101

- Density $\Rightarrow$ Electric flux at boundary.



- IR physics determined by near horizon geometry.

I: Semi-local criticality

- Term introduced by Iqbal-Liu-Mezei (1105.4621) to describe the physics of $AdS_2 \times R^d$ near horizon geometries.

- The notion can be extended to a broader class of geometries:

$$ds^2 \sim \frac{1}{r^\eta}\left(\frac{-dt^2 + dr^2}{r^2} + dx^2 + dy^2\right)$$

- Entropy density: $s \sim T^\eta$.

- **Geometries arise as $z \to \infty$ limit of hyperscaling-violating geometries.**

Gouteraux-Kiritsis (1107.2116), Hartnoll-Shaghoulian (1203.4236), early appearance in Gubser-Rocha (0911.2898).

- **Admit a scaling action in which time scales but space does not ($z=\infty$).**

- **In particular, charge density correlators have the form:**

$$G^R_{J^t J^t}(\omega, k) \sim \omega^{1+2\Delta(k)}$$

(Hartnoll-Shaghoulian @ 1203.4236)

- **Semi-local criticality leads to power law low energy spectral weight at nonzero momentum.**

- **Via formula for Γ, power law resistivity:**

$$\rho \sim \lim_{\omega \to 0} \frac{\mathrm{Im}\, G^R_{J^t J^t}(\omega, k_L)}{\omega} \sim T^{2\Delta(k_L)}$$

Hartnoll-Hofman (1201.3917),
Anantua-Hartnoll-Martin-Ramirez (1210.1590)

Verified with numerical lattice by Horowitz-Santos-Tong (1204.0519)

- **Questions: Strong coupling generalization of Fermi surface? Realize in field theory?**

II: The DBI action

- **DBI action: nonlinear dynamics of bulk Maxwell field, Dp/Dq system in probe limit.**

Karch-Randall (hep-th/0105132), DeWolfe-Freedman-Ooguri (hep-th/0111135), Erdmenger-Guralnik-Kirsch (hep-th/0203020), ...

- **At nonzero density, nonlinearities of DBI action ⇒ momentum dependence drops out of equation describing fluctuation of Maxwell field in far IR!**

(Hartnoll-Polchinski-Silverstein-Tong @ 0912.1061, Kulaxizi-Parnachev @ 0811.2262)

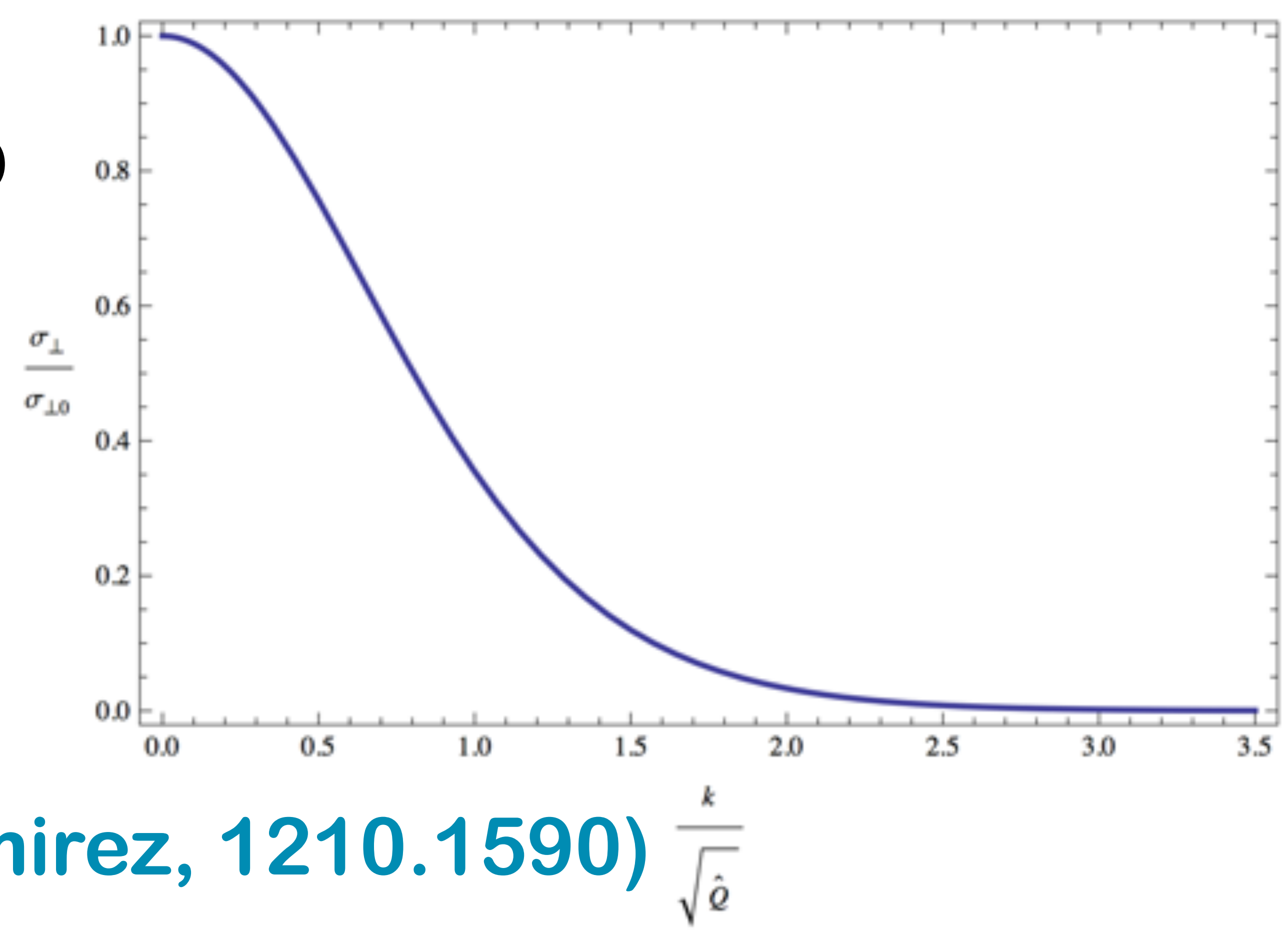
- **E.g. At T = 0, eqn. for fluctuations:**

$$\delta A_\perp'' + \frac{2Q^2 r^3}{1+Q^2 r^4}\delta A_\perp' + \left(\omega^2 - \frac{k^2}{1+Q^2 r^4}\right)\delta A_\perp = 0$$

- **In Maxwell limit, lose Q^2r^4 terms. With these terms, k drops out as r → ∞ (IR).**

- **Allows nonzero**

$$\lim_{\omega\to 0}\frac{\operatorname{Im} G^R(\omega,k)}{\omega}$$



(Anantua-Hartnoll-Martin-Ramirez, 1210.1590)

- This spectral weight cannot be used to compute a resistivity.

- In the probe limit, dominant mechanism of momentum relaxation is the O(N) fundamental d.o.f. dumping momentum into the $O(N^2)$ adjoint d.o.f. (Karch-O'Bannon @ 0705.3870).

- Question: Does this feature of DBI dynamics survive away from the probe limit? Or does one go over to the semi-local criticality case?

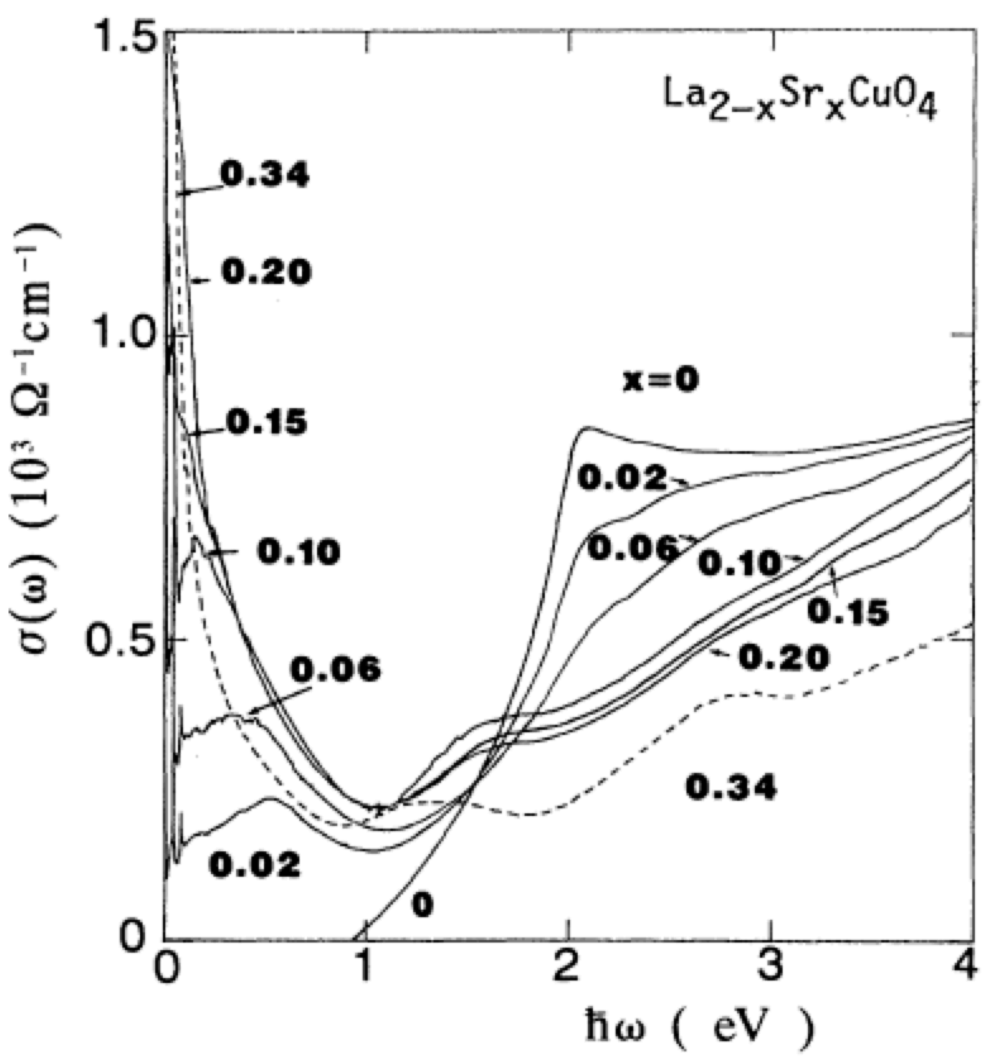
Holographic insulators
Momentum space becomes relevant

- So far, have described good metals: momentum non-conservation described by irrelevant operators in IR. Physics captured by (i) formula for Γ and (ii) knowledge of IR kinematics.
- If the lattice operators becomes relevant in the IR, we might expect to obtain insulators or perhaps incoherent metals.

(Donos-Hartnoll @ 1212.2998)

Localization transitions 101

- **Metal-insulator transitions are dramatic phenomena: re-arrangement of degrees of freedom from <u>itinerant</u> to <u>localized</u>.**

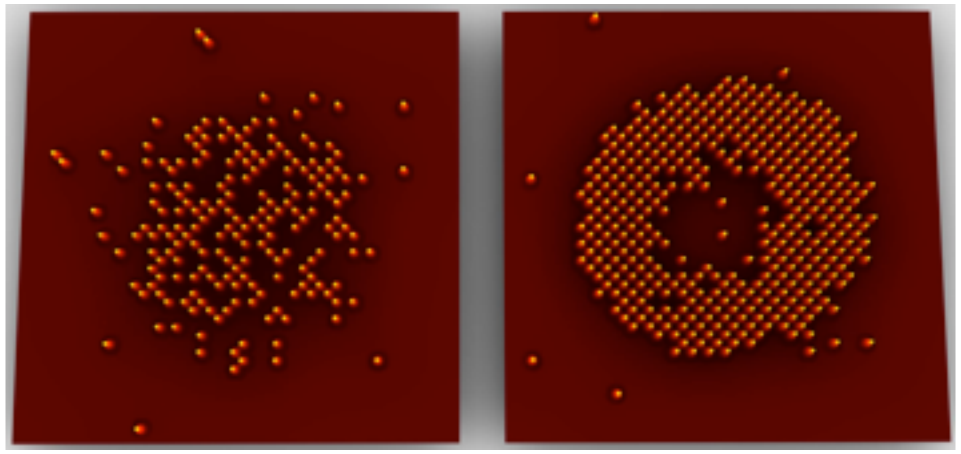


- **Spectral weight transfer from the Drude peak to the UV scale.**

Theories of localization

review: Dobrosavljevic @ 1112.6166

- Band insulators
- Anderson localization (impurities):
  Free electrons in random potential have localized wavefunctions.
- Mott transition (charge commensurability):
  Electrons 'jam' at half filling.
  Low energy excitations particle-hole symmetric: $\chi_{PJ} = 0$



Holographic insultor

- **Objective: realize a new type of localization. Main input from holography is that operators get O(1) anomalous dimensions.**

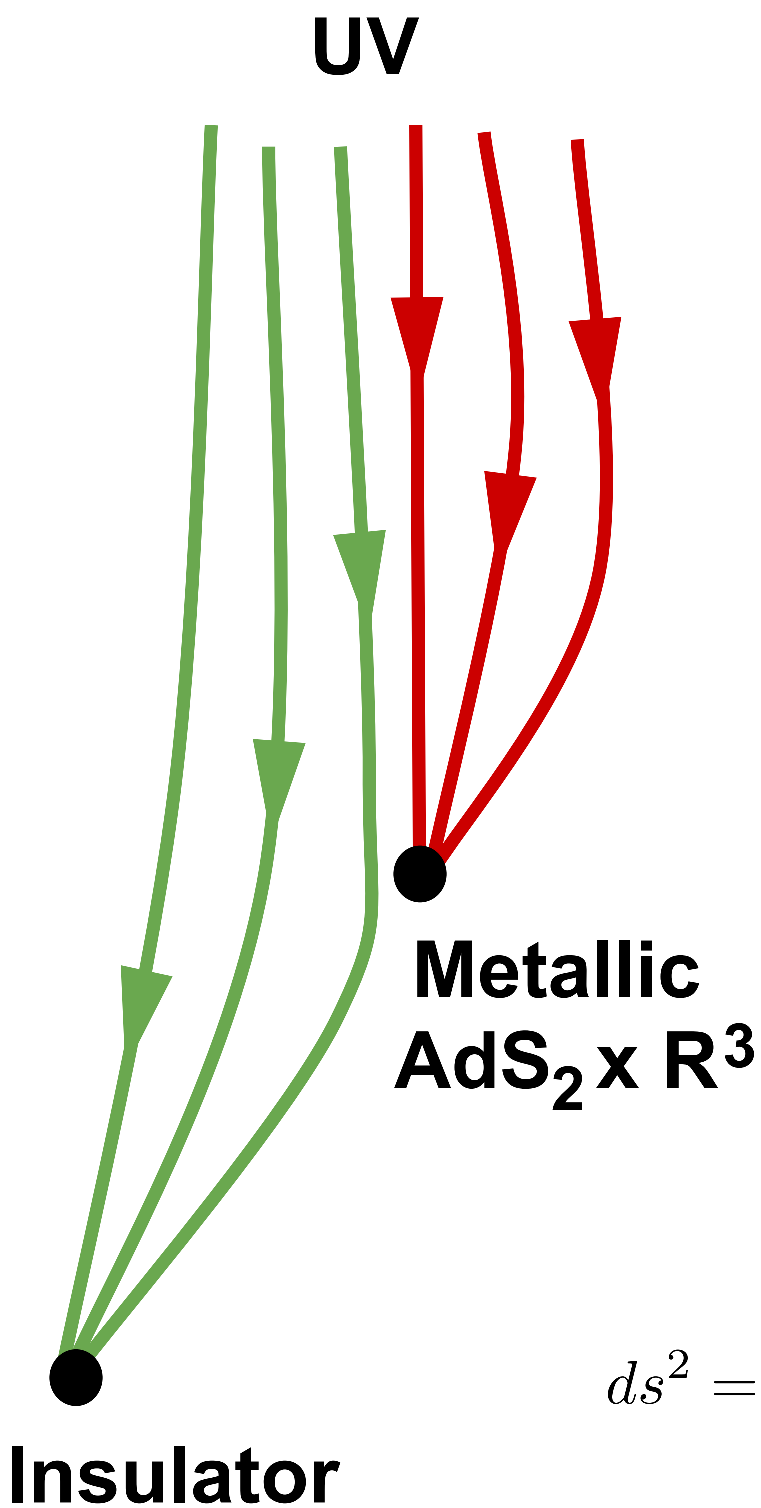
- **Bulk action:**

$$S=\int d^5x\sqrt{-g}\left(R+12-\frac{1}{4}F_{ab}F^{ab}-\frac{1}{4}W_{ab}W^{ab}-\frac{m^2}{2}B_aB^a\right)-\frac{\kappa}{2}\int B\wedge F\wedge W\,.$$

- **Used a helical lattice to avoid solving PDEs:**

$$B^{(0)}=\lambda\,\omega_2 \qquad\qquad \omega_2+i\omega_3=e^{ipx_1}\,(dx_2+idx_3)$$

cf. Ooguri-Park (1007.3737), Donos-Gauntlett (1109.3866), Iizuka-Kachru-Kundu-Narayan-Sircar-Trivedi (1201.4861)



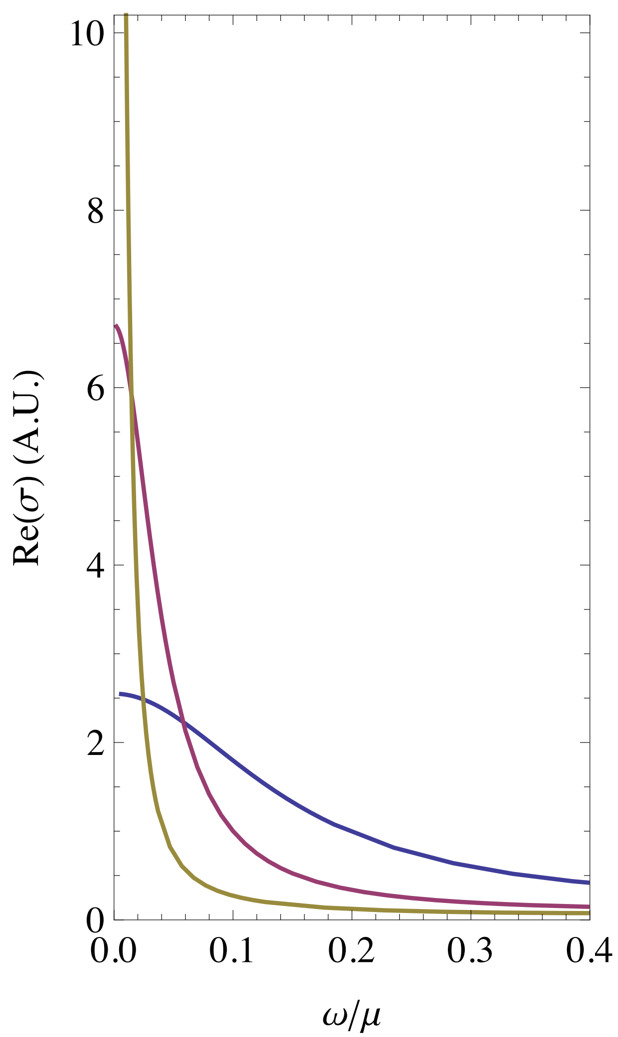
- **As a function of UV parameters, IR geometry undergoes a phase transition when lattice becomes relevant.**

$$ds^2 = -cr^2 dt^2 + \frac{dr^2}{cr^2} + \frac{dx_1^2}{r^{1/3}} + r^{2/3}\omega_2^2 + r^{1/3}\omega_3^2\,, \quad A = 0\,, \quad B = b\,\omega_2\,.$$

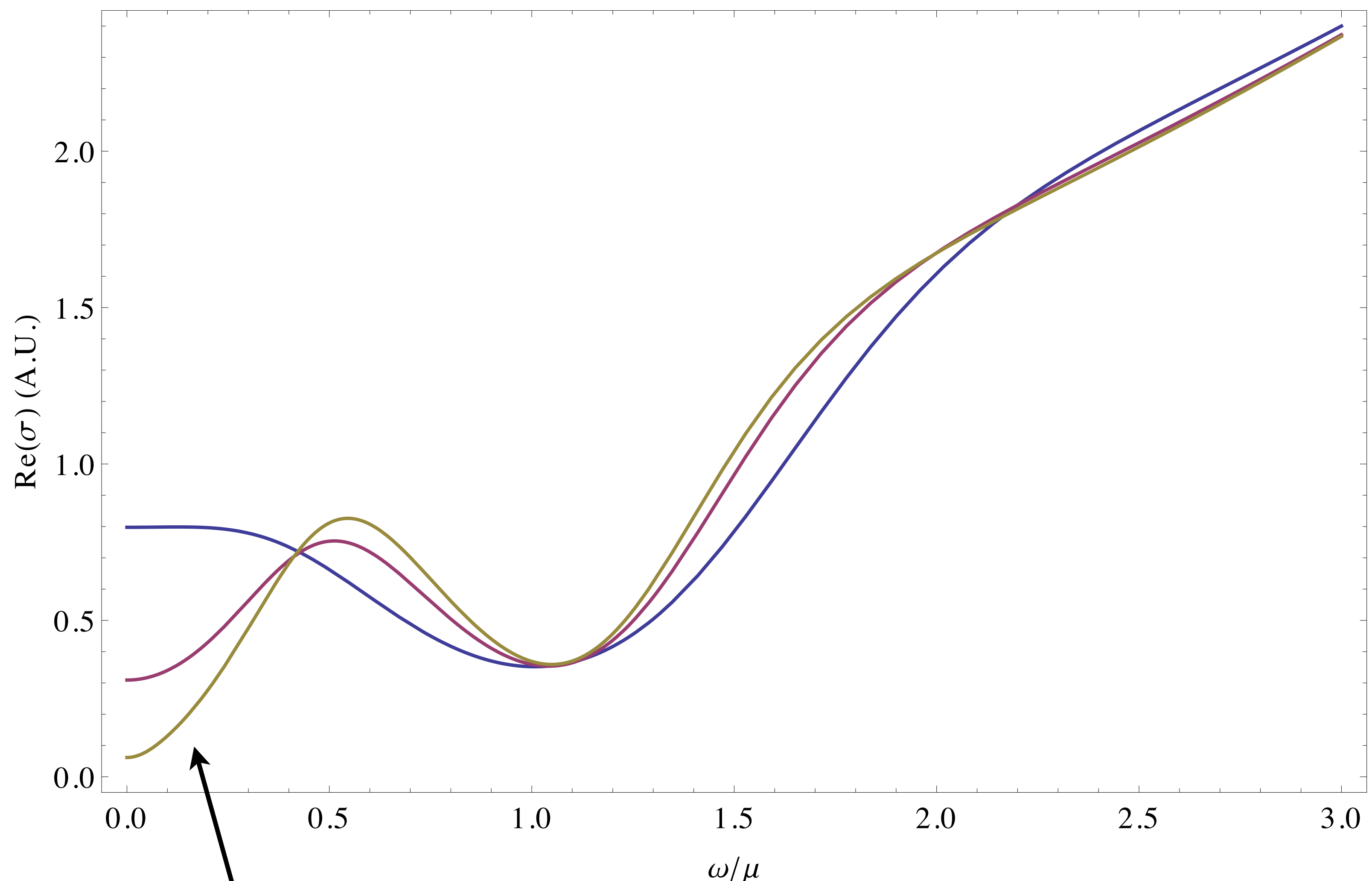
Zero temperature IR geometry

- **Spectral weight transfer!**

(Donos-Hartnoll @ 1212.2998)



Metal



Insulator

$$\sigma(\omega) \sim \omega^{4/3} \quad \text{at } T = 0$$

Conclusions

- **Strongly coupled metallic transport should be organized around <u>almost conserved quantities</u>.**
- **Holographic theories at finite charge density push <u>low energy excitations out to finite momentum</u> in novel ways.**
- **Holography enables the realization of a <u>new, strong coupling, mechanism of charge localization</u> -- lattice scattering becomes relevant in the IR.**