

Wormholes, Emergent Gauge Fields, and the Weak Gravity Conjecture

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Some Electromagnetic Conjectures

In this talk, I will discuss three conjectural restrictions on low-energy electromagnetism that follow from quantum gravity:

- (1) There must exist charged states.
- (2) There must exist states with all charges allowed by charge quantization ($Q = nq$, $n \in \mathbb{Z}$).
- (3) There must exist a particle of charge $Q = nq$ and mass m , with $m < m_P$, such that

$$m\sqrt{G} \leq nq.$$

Conjecture (3) is a version of the “Weak Gravity Conjecture”

[Arkani-Hamed/Motl/Nicolis/Vafa](#).

The condition $m < m_P$ ensures that the particle is not a black hole, and thus requires a field to exist in the low-energy effective action.

None of these conjectures can be derived in perturbation theory, but all three seem to be true in string theory.

The existing arguments for (1) – (3) are not completely compelling:

- If there are no charges, how do you make a charged black hole?
- Even if there are charged black holes, why can't they have only even charges?
- The original argument of AMNV for (3) was that non-BPS charged black holes should be able to decay, but why?
- In fact AMNV only argued for a weaker conjecture, that there exists a state of charge Q and mass m obeying $m\sqrt{G} \leq Q$, and this version *can* be satisfied by black holes. So what is the conjecture anyway?

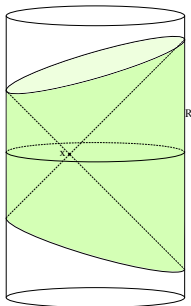
[Heidenreich/Reece/Rudelius](#)

In the remainder of this talk, I will present a paradox in AdS/CFT whose resolution seems to require all three conjectures.

My arguments for (1) and (2) will be quite general, while (3) will depend more on the details of the resolution. [Harlow 1510.07911](#), see also [Jafferis/Guica 1511.05627](#),

[Harlow/Ooguri 16xx](#)

Recently there has been a lot of progress in understanding how to “back off” of the extrapolate dictionary and directly describe bulk operators in CFT language (see Xi Dong’s talk):

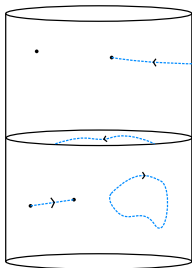


We write

Banks/Douglas/Horowitz/Martinec; Hamilton/Kabat/Lifshitz/Lowe; Heemskerck/Marolf/Polchinski/Sully

$$\phi(x) = \int_R dX K(x; X) \mathcal{O}(X) + O(1/N).$$

This construction also works when there are gauge fields in the bulk:



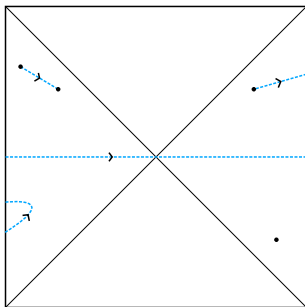
Gauge-invariant operators in the bulk can be represented in the CFT in terms of the boundary current dual to A_μ :

$$J_\mu(t, \Omega) = (d - 3) \lim_{r \rightarrow \infty} r^{d-3} A_\mu(r, t, \Omega).$$

Note that a global symmetry in the CFT becomes a gauge symmetry in the bulk, consistent with the nonexistence of bulk global symmetries (for discrete symmetries see [Harlow/Ooguri 16xx](#)).

A Reconstruction Paradox

Today the main subject of interest will be reconstruction of gauge-invariant operators in a different background, the AdS-Schwarzschild geometry:



There is now a new kind of gauge-invariant operator: a Wilson line stretching from one boundary to another.

This operator *cannot* be reconstructed using the techniques I've just mentioned!

The problem is that the CFT Hilbert space factorizes:

$$\mathcal{H}_{CFT} = \mathcal{H}_L \otimes \mathcal{H}_R$$

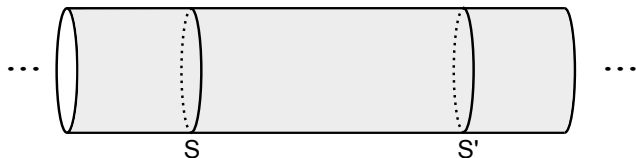
$$|TFD\rangle_{CFT} = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta E_i/2} |i^*\rangle_L |i\rangle_R.$$

This means that *any* operator is a sum of tensor product operators between the two sides, but if we try to cut the Wilson line in the bulk we find pieces that are not gauge invariant:

$$W_n \equiv e^{in \int_L^R A} = e^{in \int_L^0 A} e^{in \int_0^R A} \quad (n \in \mathbb{Z}).$$

This prevents any attempt to make the two parts separately out of J_μ in the left and right CFT's respectively.

We can get more intuition for the problem by considering pure QED on a spatial $\mathbb{R} \times S^2$:



The Gauss constraint tells us that

$$\int_S E = \int_{S'} E,$$

so two macroscopically separated operators are equal in all states in the physical Hilbert space!

We thus can never have a factorization of the Hilbert space into “left” and “right” pieces: I call this apparent conflict with the CFT the *factorization problem*.

We can begin to get a sense of how the factorization problem might be resolved by considering the algebra of W_n with the asymptotic charges:

$$[Q_L - Q_R, W_n] = 2nW_n.$$

So in other words, W_n is a creation operator for $2n$ units of $Q_L - Q_R$: it makes the wormhole a “little bit Reissner-Nordstrom”.

In particular we have

$$\langle TFD | W_n^\dagger (Q_L - Q_R) W_n | TFD \rangle = 2n,$$

so using the factorization of the Hilbert space we immediately see that

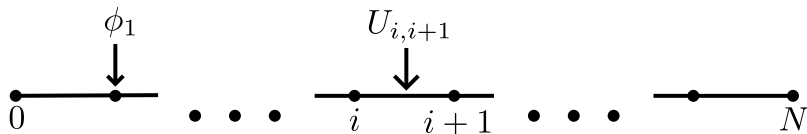
$$Q \neq 0$$

for the CFT on one \mathbb{S}^{d-1} . This establishes conjecture (1).

- Once dynamical charges exist they obstruct the QED argument above, since the Gauss constraint now has something nontrivial on the right hand side.
- Indeed we will now see that including dynamical charges turns the factorization problem into a UV issue.
- To make this precise, I will now describe the factorization problem in $1 + 1$ -dimensional lattice gauge theory. [Donnelly, Casini/Huerta/Rosabal, Radicevic](#)

Factorization in Lattice Gauge Theory

1 + 1 dimensional scalar electrodynamics on the lattice:



We have gauge transformations:

$$\phi'_i = V_i \phi_i$$

$$U'_{i,i+1} = V_{i+1} U_{i,i+1} V_i^\dagger.$$

I'll choose boundary conditions:

$$V_0 = V_N = 1$$

$$\phi_0 = \phi_N = 0.$$

Interesting gauge-invariant operators include

$$\begin{aligned}
 W &\equiv U_{0,1} U_{1,2} \dots U_{N-1,N} \\
 \overrightarrow{\phi}_i &\equiv \phi_i U_{i,i+1} U_{i+1,i+2} \dots U_{N-1,N} \\
 \overleftarrow{\phi}_i &\equiv U_{0,1}^\dagger U_{1,2}^\dagger \dots U_{i-1,i}^\dagger \phi_i
 \end{aligned}$$

Now say we want to cut the Hilbert space between site ℓ and site $\ell + 1$. We can solve the constraint to write an arbitrary gauge-invariant state as

$$\langle U, \phi, \phi^\dagger | \psi \rangle = \Psi \left[W, \overleftarrow{\phi}_i, \overleftarrow{\phi}_i^\dagger, \overrightarrow{\phi}_j, \overrightarrow{\phi}_j^\dagger \right] \quad i \leq \ell, \quad j \geq \ell + 1.$$

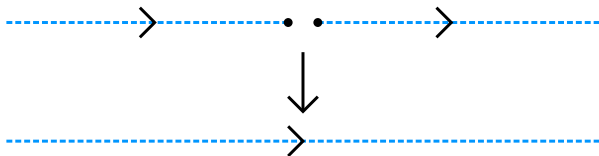
This almost factorizes, but not quite: we still have W , which cannot be generated by any pair of algebras localized on the left and right respectively.

Introducing charges has not solved the factorization problem - but I claim the situation has improved.

The reason is that

$$W' \equiv \overleftarrow{\phi}_\ell^\dagger \overrightarrow{\phi}_{\ell+1}$$

is an operator that behaves identically to W in low-energy correlation functions.

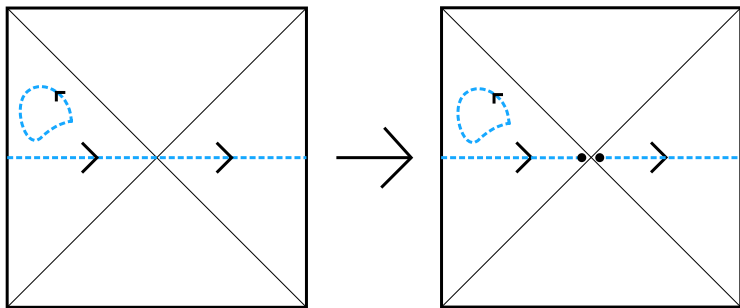


We can justify this using a type of “gauge-covariant operator product expansion” [see also Gadde](#):

$$\phi^\dagger(x)\phi(y) = e^{i \int_x^y A} (G(x, y) + \text{less singular terms}).$$

Representing W in the CFT

We thus arrive at a simple proposal for how to represent W in the CFT:



The left and right pieces are separately gauge-invariant, and they indeed can be represented in the left and right CFTs using the standard method.

Two immediate comments:

- In order to cut a Wilson line in the fundamental representation, we need a bulk operator of charge one. There must thus be a *state* of charge one, so we have derived conjecture (2).
- The charges could be quite heavy, say at the GUT scale, and we would still need to know about them to represent a rather low-energy operator in the CFTs!

The latter point is not an observable violation of bulk effective field theory, but it *is* a violation of the notion of “effective conformal field theory”.

[Fitzpatrick/Katz/Poland/Simmons-Duffin](#)

I view it as a particularly sharp manifestation of the UV sensitivity of IR questions when black holes are present.

Let's assess what we have really accomplished here:

- You give me a low-energy correlation function involving W , and I give you a CFT representation of W that works in that correlation function.
- But now say you change your mind, and want to add an extra operator near the center: you can catch me lying!
- I can fix it by bringing the two operators even closer together, but you can catch me again with another operator.

To avoid this, I need to pre-emptively bring the operators together until one of two things happens:

- I bring them to within the Planck scale. I have now postponed the factorization problem into the realm of ignorance: perhaps this already counts as a solution?
- I bring them together to within some larger length scale, where the physics nonetheless changes qualitatively and is no longer described by Maxwell dynamics.

In other words, the gauge field must be emergent!

A Case Study: the $\mathbb{C}\mathbb{P}^{N-1}$ model in d dimensions

To see how emergence can help us, it will be very helpful to study an explicit example: the $\mathbb{C}\mathbb{P}^{N-1}$ model. [D'Adda/Luscher/Di Vecchia, Witten](#)

- N complex scalar fields z_a , $a = 1, \dots, N$.
- Impose a constraint $\sum_a z_a^* z_a \equiv z^\dagger z = 1$.
- Impose a gauge symmetry $z'_a = e^{i\theta(x)} z_a$.
- Impose an $SU(N)$ flavor symmetry rotating the z s.

These constraints together imply that the Hilbert space factorizes site-by-site into a copy of $\mathbb{C}\mathbb{P}^{N-1}$ at each point in space.

Nonetheless there is a phase of this model which flows to scalar electrodynamics in the IR!

Reminder: this is a model for the bulk.

The natural Lagrangian is a non-linear σ model:

$$\mathcal{L} = -\frac{N}{g^2} (D_\mu z)^\dagger (D^\mu z)$$

$$D_\mu \equiv \partial_\mu - iA_\mu$$

$$A_\mu = \frac{1}{2i} \left(z^\dagger \partial_\mu z - \partial_\mu z^\dagger z \right).$$

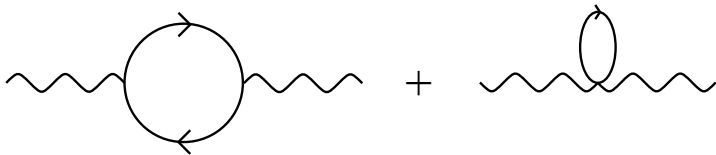
In any spacetime dimension, this model is solvable at large N .

The idea is to introduce a Lagrange multiplier σ , and view A_μ as an auxiliary field:

$$\mathcal{L} = -\frac{N}{g^2} \left[(D_\mu z)^\dagger (D^\mu z) + \sigma (z^\dagger z - 1) \right].$$

We can then integrate out the z s to get an effective action for A_μ and σ , which at large N will be exact. By adjusting g we can arrange that $0 < \sigma_0 \ll \Lambda$, with $SU(N)$ unbroken.

We are especially interested in the piece of the effective action that is quadratic in A_μ :



Evaluating these diagrams (and tuning g to be in the right phase for $d > 2$) we find an effective Maxwell term

$$S_{\text{eff}} \supset -\frac{1}{4q^2} \int d^d x F_{\mu\nu} F^{\mu\nu},$$

with:

$$\frac{1}{q^2} = \begin{cases} \frac{N}{6\pi\sigma_0} & d = 2 \\ \frac{N}{12\pi\sqrt{\sigma_0}} & d = 3 \\ \frac{N}{12\pi^2} \log\left(\frac{\Lambda}{\sqrt{\sigma_0}}\right) & d = 4 \\ N\Lambda^{d-4} & d > 4. \end{cases}$$

How is the existence of an IR Coulomb phase consistent with the microscopic factorization of the Hilbert space?

Above we saw that

$$A_\mu = \frac{1}{2i} \left(z^\dagger \partial_\mu z - \partial_\mu z^\dagger z \right).$$

The lattice version of this equation is

$$U_{x\delta} = \frac{z_x^\dagger z_{x+\delta}}{|z_x^\dagger z_{x+\delta}|},$$

which you can check indeed transforms as a Wilson link.

But this is precisely the gauge-covariant OPE! It is apparently exact at the lattice scale.

On to the Weak Gravity Conjecture

In the \mathbb{CP}^{N-1} model, the mass $m = \sqrt{\sigma_0}$ and coupling q of the charges in the IR are computable quantities. We can use this to test the weak gravity conjecture

$$m\sqrt{G} \leq q.$$

To do this, we clearly need to couple the model to gravity. In order to solve the factorization problem, we need to take the cutoff Λ of the model to be less than the Planck scale, and to have the strongest test of the conjecture we should take them to be comparable.

In doing the test, we need remember that the presence of N light fields renormalizes Newton's constant G , essentially by the same diagrams we studied before:

$$\frac{1}{G} \sim N\Lambda^{d-2}.$$

Here Λ is the scale where gravity becomes strongly coupled, which we have taken to be the same as the \mathbb{CP}^{N-1} model cutoff, so for $d > 4$ we have

$$\frac{1}{q^2} \sim N\Lambda^{d-4}.$$

Combining these equations we have $q^2/G \sim \Lambda^2$, so we then have

$$m^2 \ll \Lambda^2 = \frac{q^2}{G}.$$

The \mathbb{CP}^{N-1} model automatically obeys the weak gravity conjecture!

Why did this happen? The logic is actually quite simple:

- In order to solve the factorization problem, we had to have an energy scale, at most the Planck scale, where the Maxwell term in the effective action has zero coefficient. This corresponds to infinitely strong gauge coupling.
- If we want to have a weak gauge coupling at low energies, then we need to generate a large coefficient for this Maxwell term as we run the renormalization group. We do this by integrating out loops of charged fields.
- But say all of the charge fields have masses up at the Planck scale: then they will gap out immediately, and will not be able to generate any significant Maxwell term.

Thus we arrive at the basic point of the WGC: a weak gauge coupling requires a light charged field.

In fact this argument generalizes to multiple $U(1)$'s, where it reproduces a rather nontrivial generalization of the WGC due to Cheung and Remmen.

What have we learned?

- The fact that the CFT Hilbert factorizes despite the geometric connection through the bulk gives us powerful information about short-distance physics in the bulk. In particular it provides support to conjectures (1)-(3).
- That we can describe the Wilson line W despite this factorization is an important check of the idea that the wormhole exists, since if there were no bridge we would not expect it to. (see also [Engelhardt/Freivogel/Iqbal](#))
- The charges from which the gauge field emerges give a UV-regularization of the electromagnetic “edge modes” / “soft hair” of [Donnelly/Wall](#), [Hawking/Perry/Strominger](#), whose gravitational cousins appeared recently in a CFT explanation of the Ryu-Takayanagi formula [Harlow](#).
- Indeed there is a gravitational factorization problem too! Its solution will similarly require UV information about quantum gravity in the bulk. But what is the bulk description of the degrees of freedom from which the graviton emerges?



谢谢大家!