Monte Carlo approach to string/M-theory

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Based on papers with
Anagnostopoulos, Hyakutake, Ishiki, Kanamori, Mannelli, Matsuo, Matsuura, Miwa, Nishimura, Sekino, Sugino, Takeuchi, Yoneya

STRINGS 2014 @ IAS Princeton

I stole the title from this paper :)

Cornell University Library

arXiv.org > hep-th > arXiv:hep-th/9803117

High Energy Physics – Theory

Monte Carlo Approach to M-Theory

Werner Krauth, Hermann Nicolai, Matthias Staudacher

(Submitted on 13 Mar 1998 (v1), last revised 1 Apr 1998 (this version, v3))

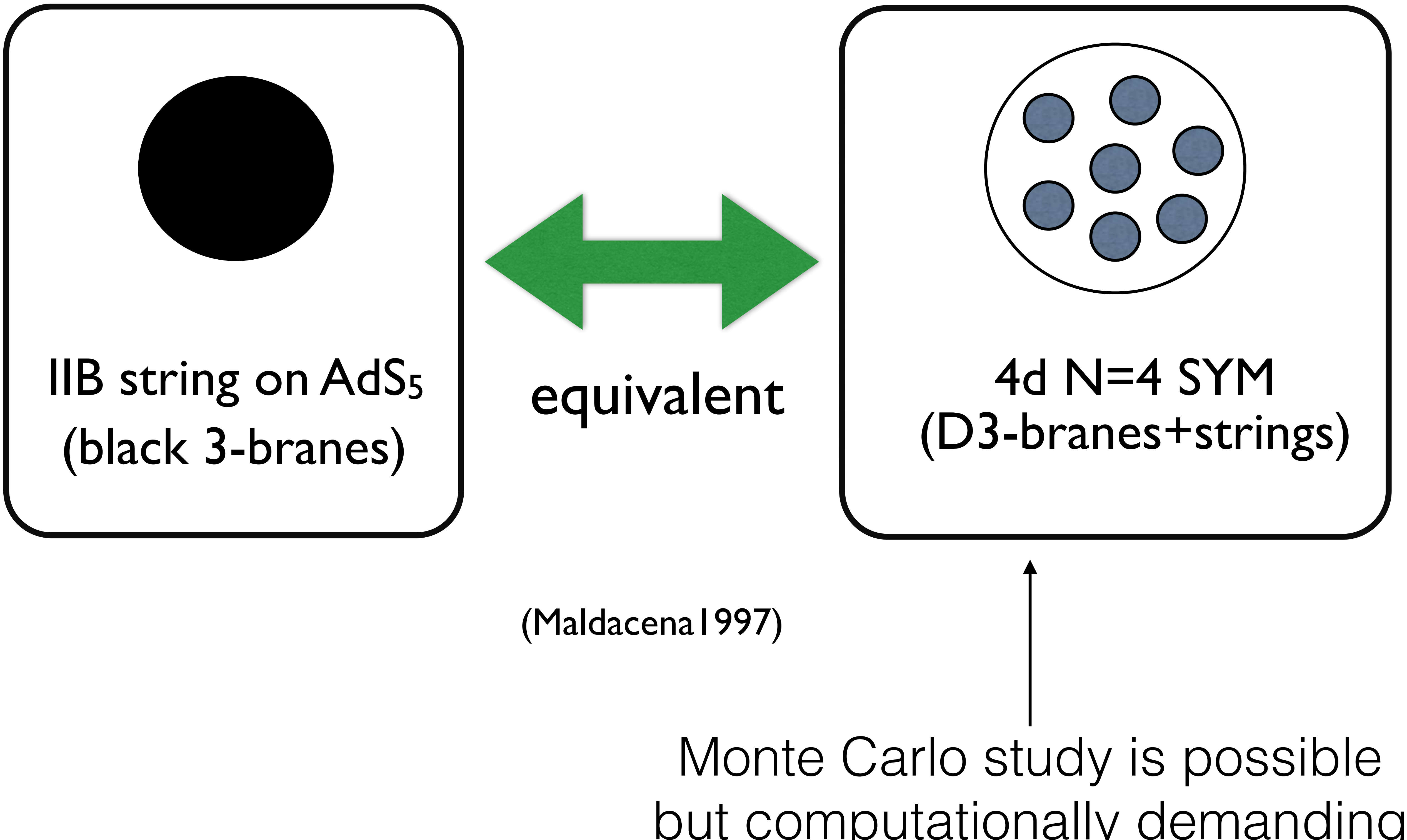
Motivation

Combine the gauge/gravity duality and numerical techniques (e.g. lattice gauge theory) in order to study *quantum gravity*.

Super Yang-Mills → Quantum Gravity

(Monte Carlo simulation of SYM)

some recent papers with *analytic* methods:
Bhattacharyya-Marino-Sen,
Arabi Ardehali-Liu-Szepietowski
Dabholkar-Drukker-Gomes, …


IIB string on AdS5
(black 3-branes)
equivalent
4d N=4 SYM
(D3-branes+strings)
(Maldacena1997)
Monte Carlo study is possible
but computationally demanding

Which SYM can be simulated?

Possible/Impossible

(without fine tuning; not necessarily lattice)

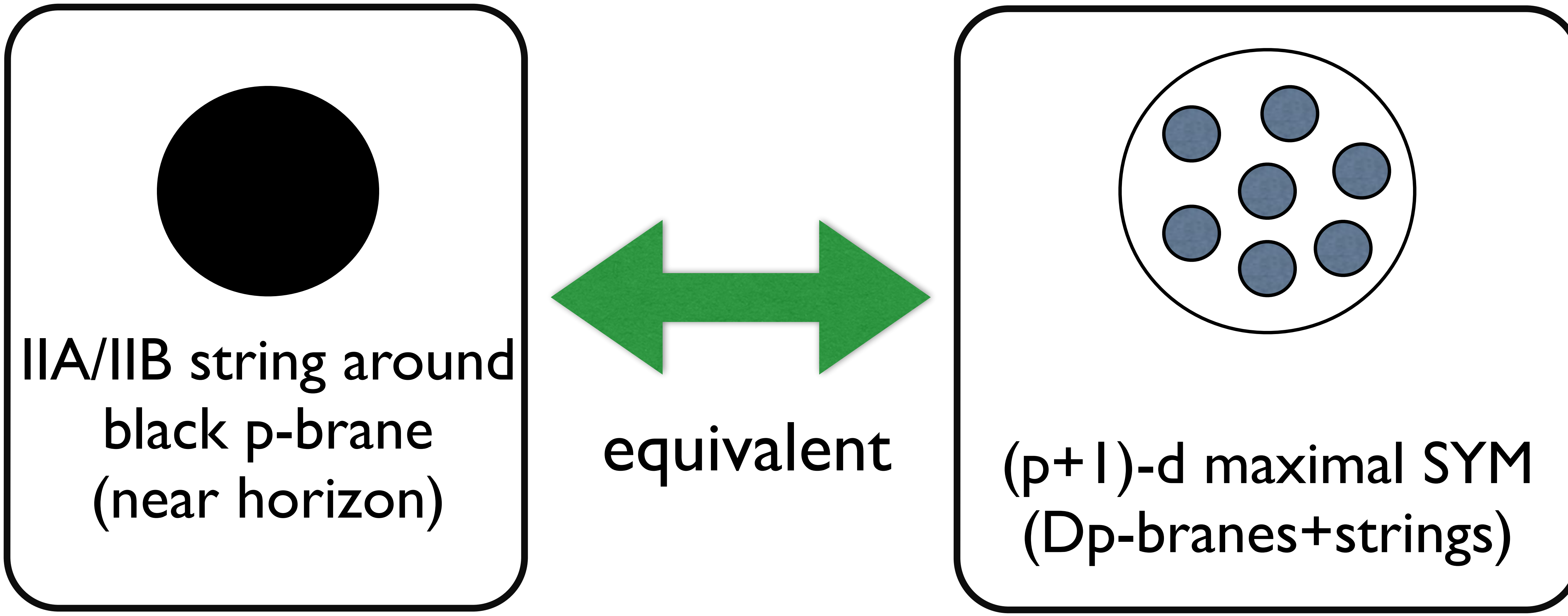
smaller simulation cost

(0+1)-d
(1+1)-d
} any number of SUSY, various matter contents

(2+1)-d maximal SUSY less SUSY matter fields

(3+1)-d without matter (pure $\mathcal{N}=1$)
SUSY QCD (matter fields)
maximal SUSY

larger simulation cost

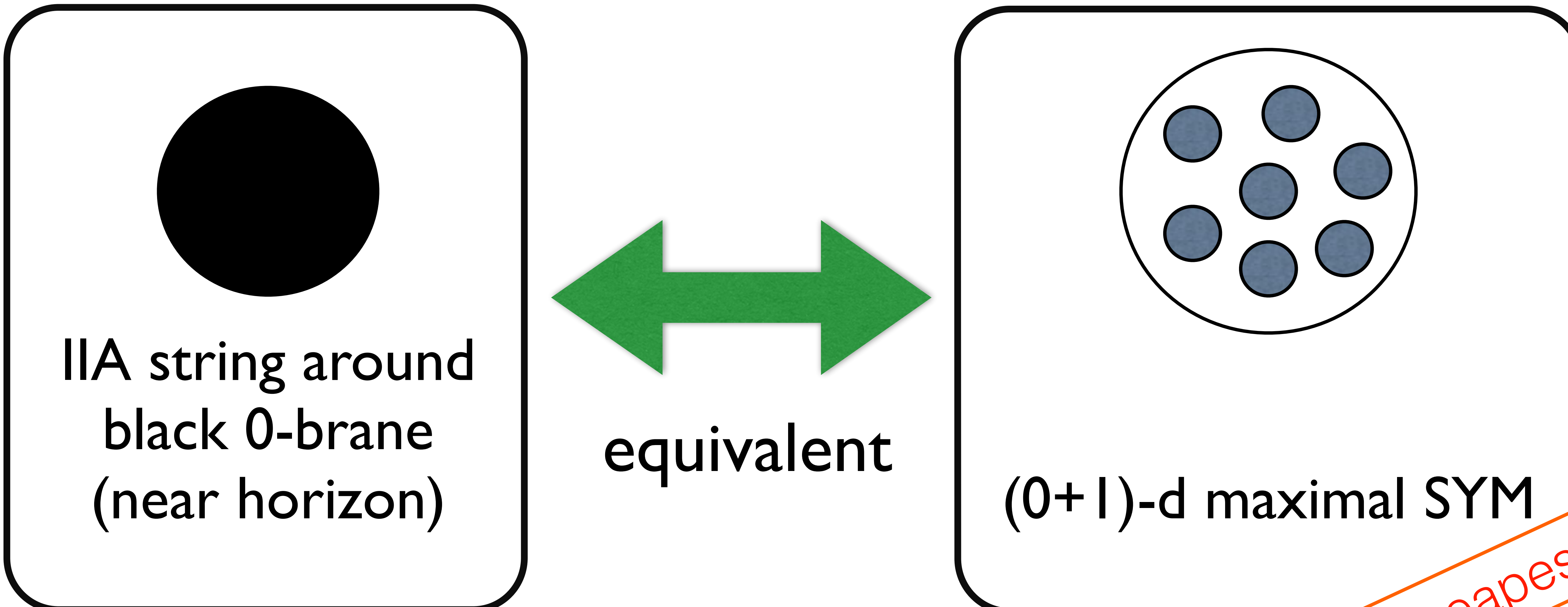


(Maldacena1997, Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

smaller p is easier to simulate on computer.

we study this case

Black hole = matrix model



numerically cheapest

simulation cost $\sim N^6T^{-3}$

high temperature is cheap, low temperature is expensive.

SYM ⟷ STRING

SYM		STRING
$1/\lambda$	⟷	α'/R_{BH}^2
$g_{YM}^2 \sim 1/N$		g_s

$\lambda=\infty$, $N=\infty$ corresponds to supergravity.

$1/\lambda$ and $1/N$ corrections are interesting.

But first of all, we have to test this conjecture.

D0-brane quantum mechanics

$$S = \frac{N}{\lambda} \int dt\, Tr \Big\{ \frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i [X_i, \psi] \Big\}$$

- Matrix model of M-theory (Banks-Fishler-Shenker-Susskind, 1996 de Wit-Hoppe-Nicolai, 1988)
- gauge/gravity duality → dual to black 0-brane

It should reproduce thermodynamics of black 0-brane.

effective dimensionless temperature $T_{eff} = \lambda^{-1/3} T$

strong coupling = low temperature → more simulation cost

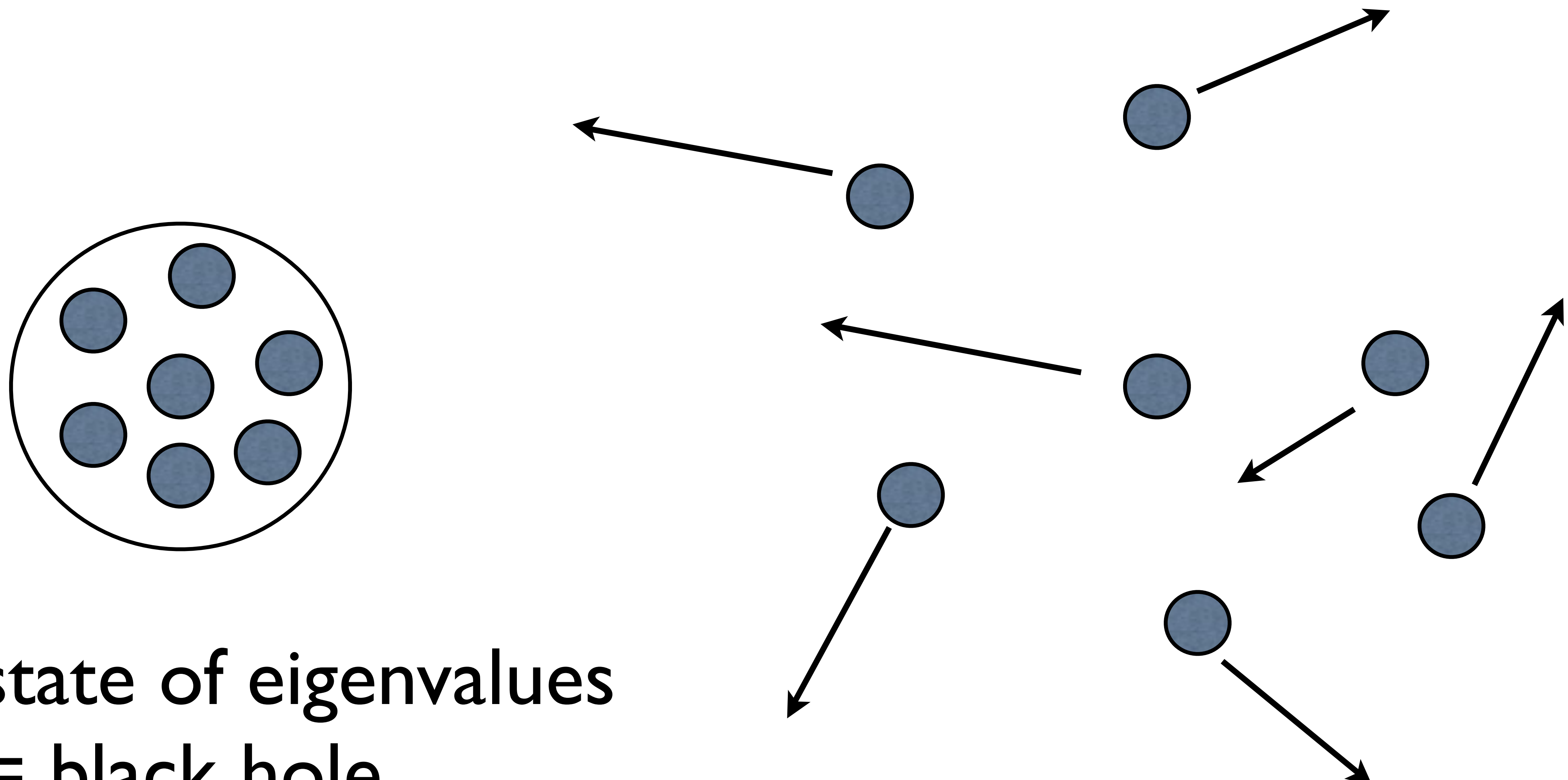
problem with flat direction

$$S = \frac{N}{\lambda} \int dt \; Tr \Big\{ \frac{1}{2} (D_t X_i)^2 - \frac{1}{4} [X_i, X_j]^2 + \frac{1}{2} \bar{\psi} D_t \psi - \frac{1}{2} \bar{\psi} \gamma^i [X_i, \psi] \Big\}$$

There is a flat direction even at quantum level.

$$[X_i, X_j] = 0$$

‘eigenvalues’ = position of D0-branes



bound state of eigenvalues
= black hole

flat direction
$\sim$ gas of D0-branes

One has to restrict the path integral
in order to extract the black hole.

Confirmation at *classical* string level

$(N=\infty,\ g_s=0)$

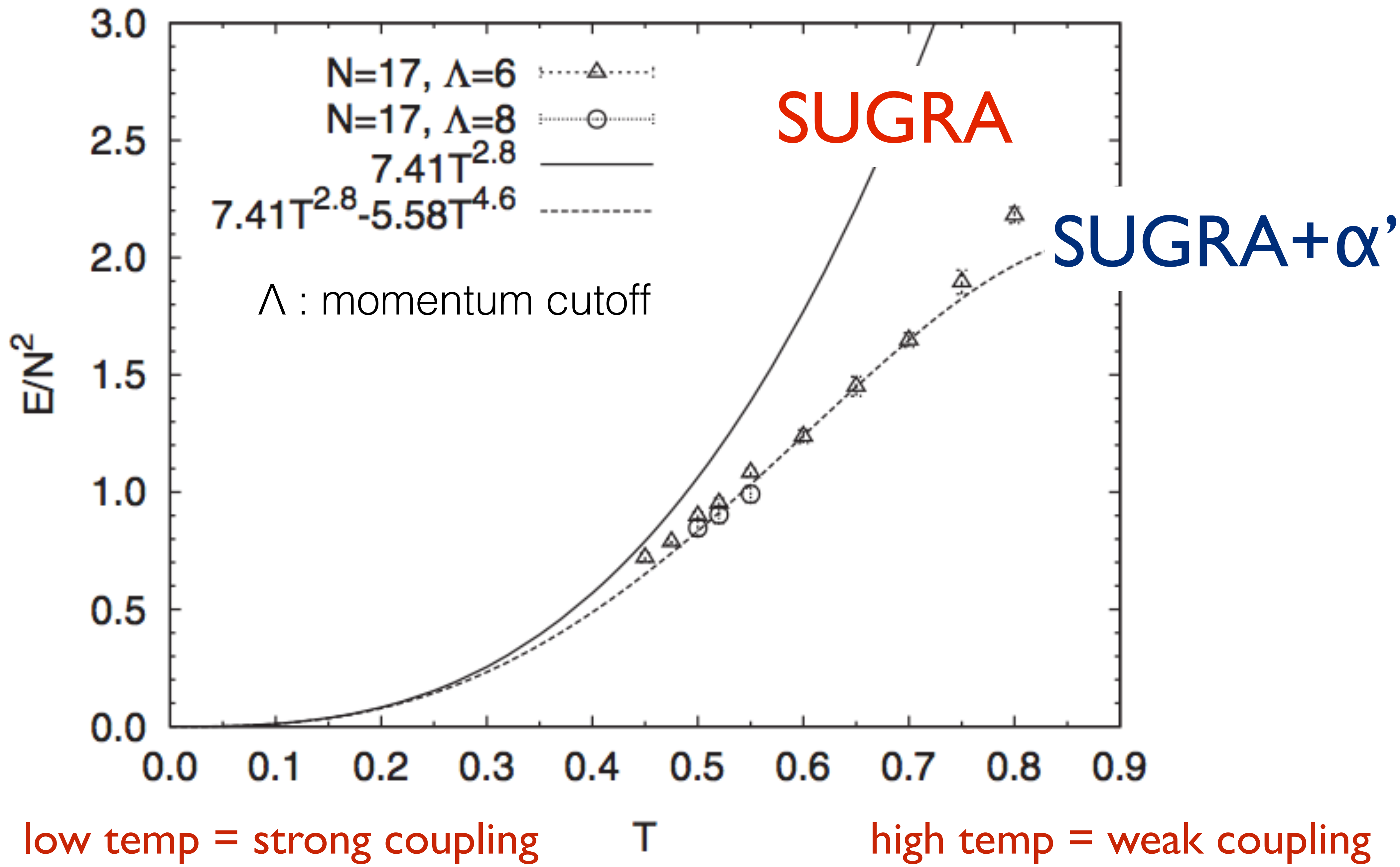
How to tame the flat direction

In string theory, this BH is stable at $g_s=0$.

→ In the gauge theory, bound state should become stabler as N becomes larger

We can confirm this expectation numerically.

solution: take N large enough.



Anagnostopoulos-M.H.-Nishimura-Takeuchi, PRL 2008
M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

($\lambda^{-1/3}T$: dimensionless effective temperature)

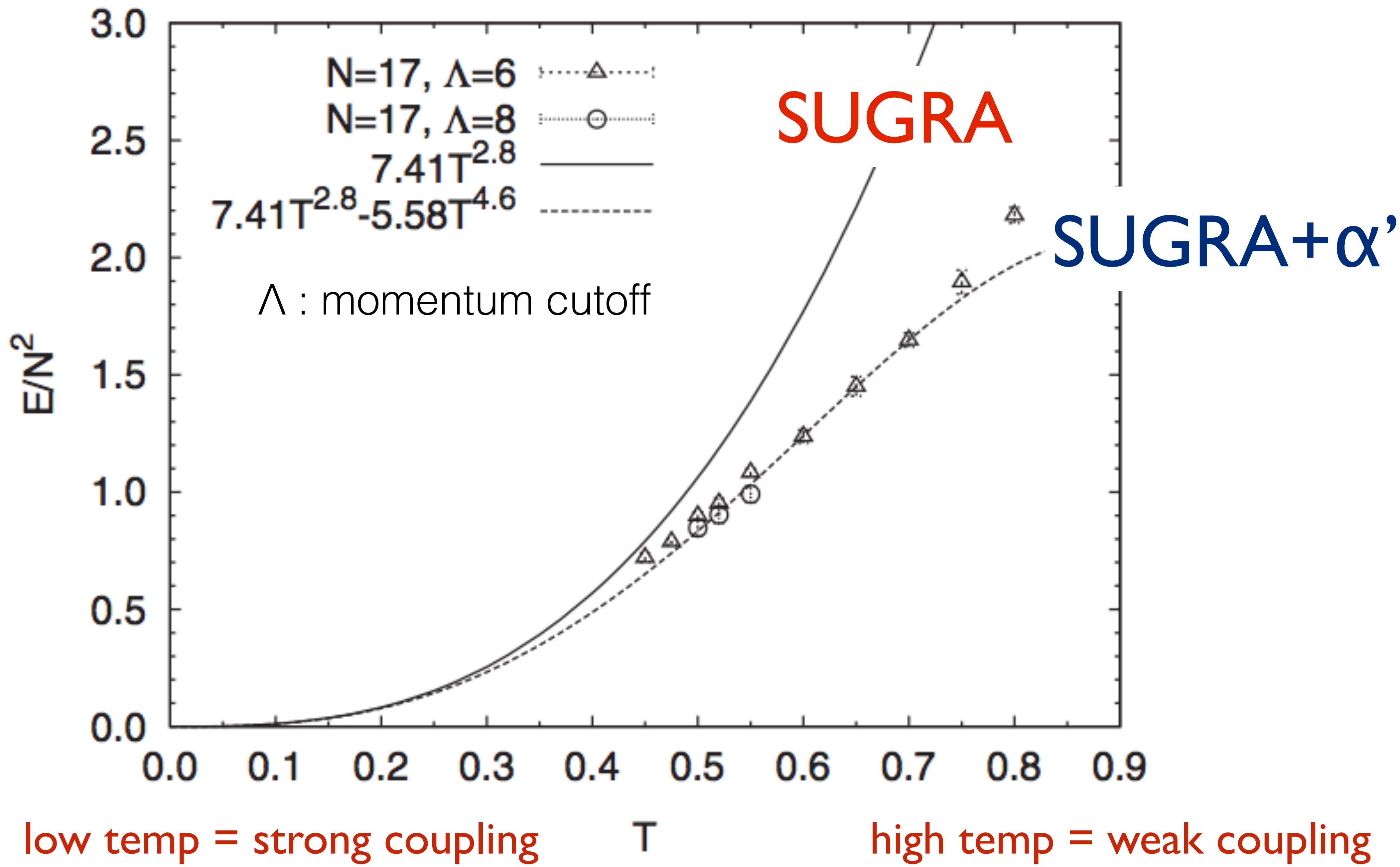
(see also papers by Catterall-Wiseman and by Kadoh)

α' correction

- deviation from the strong coupling (low temperature) corresponds to the α' correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2 = 7.41T^{2.8} - 5.58T^{4.6}$

(4.6 = 2.8 + 1.8)
prediction by string

'prediction' by SYM simulation



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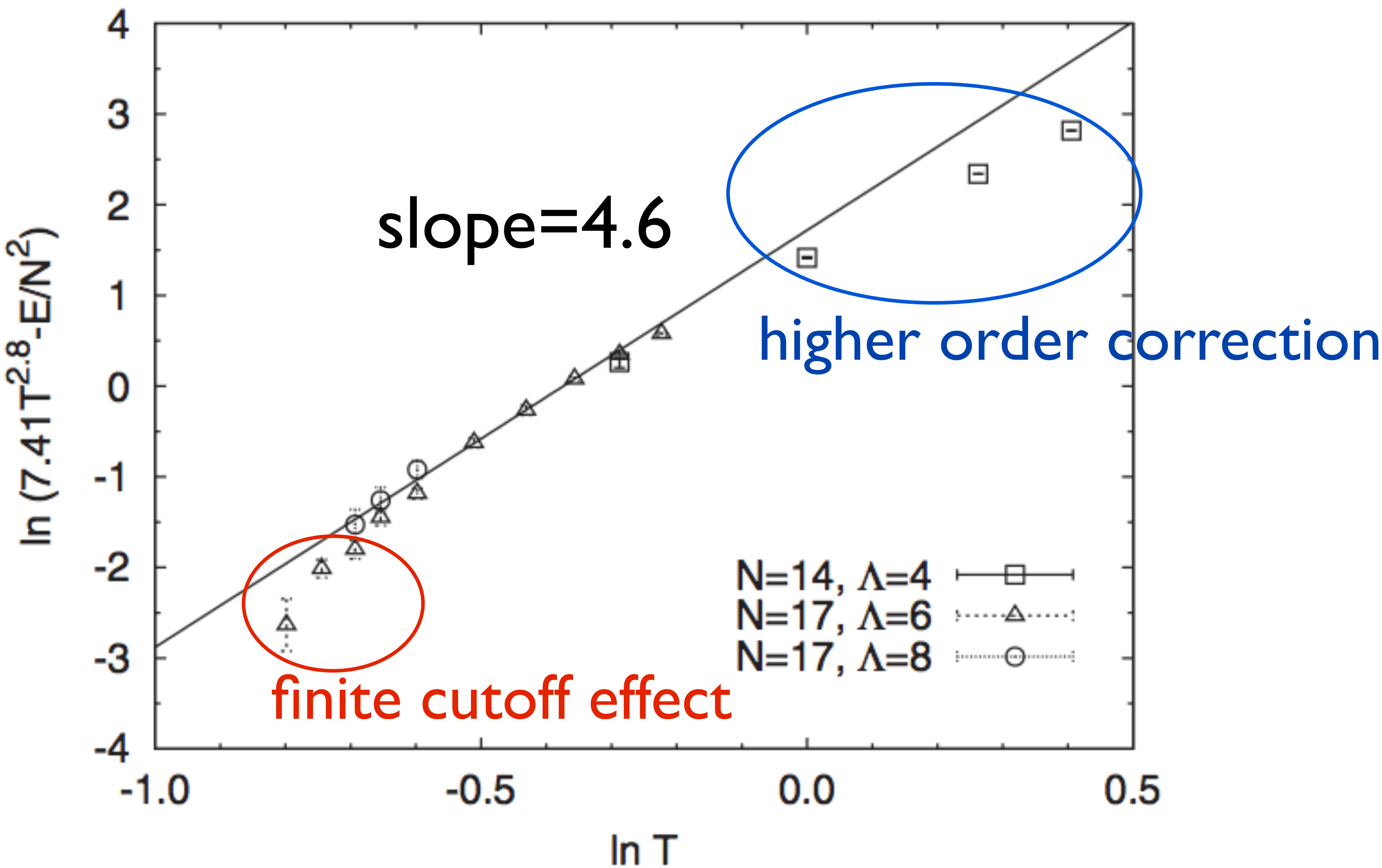
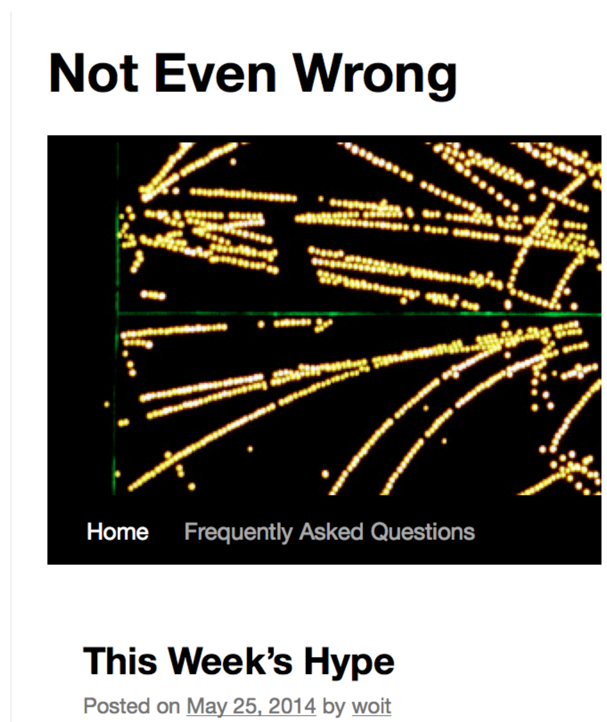


M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

Confirmation at *<u>quantum</u>* string level

(finite-N)



Peter Woit's "This week's Hype"
on May 25, 2014

g_s correction in the gravity side (Y. Hyakutake, PTEP 2013)

$$E/N^2 = 7.41T^{2.8} - 5.58T^{4.6} + \ldots.$$
$$+(1/N^2)(-5.77T^{0.4} + aT^{2.2} + \ldots)$$
$$+(1/N^4)(bT^{-2.6} + cT^{-2.0} + \ldots)$$
$$+\ldots..$$

- We study T∼0.1, so that unknown part is negligible.

How to tame the flat direction

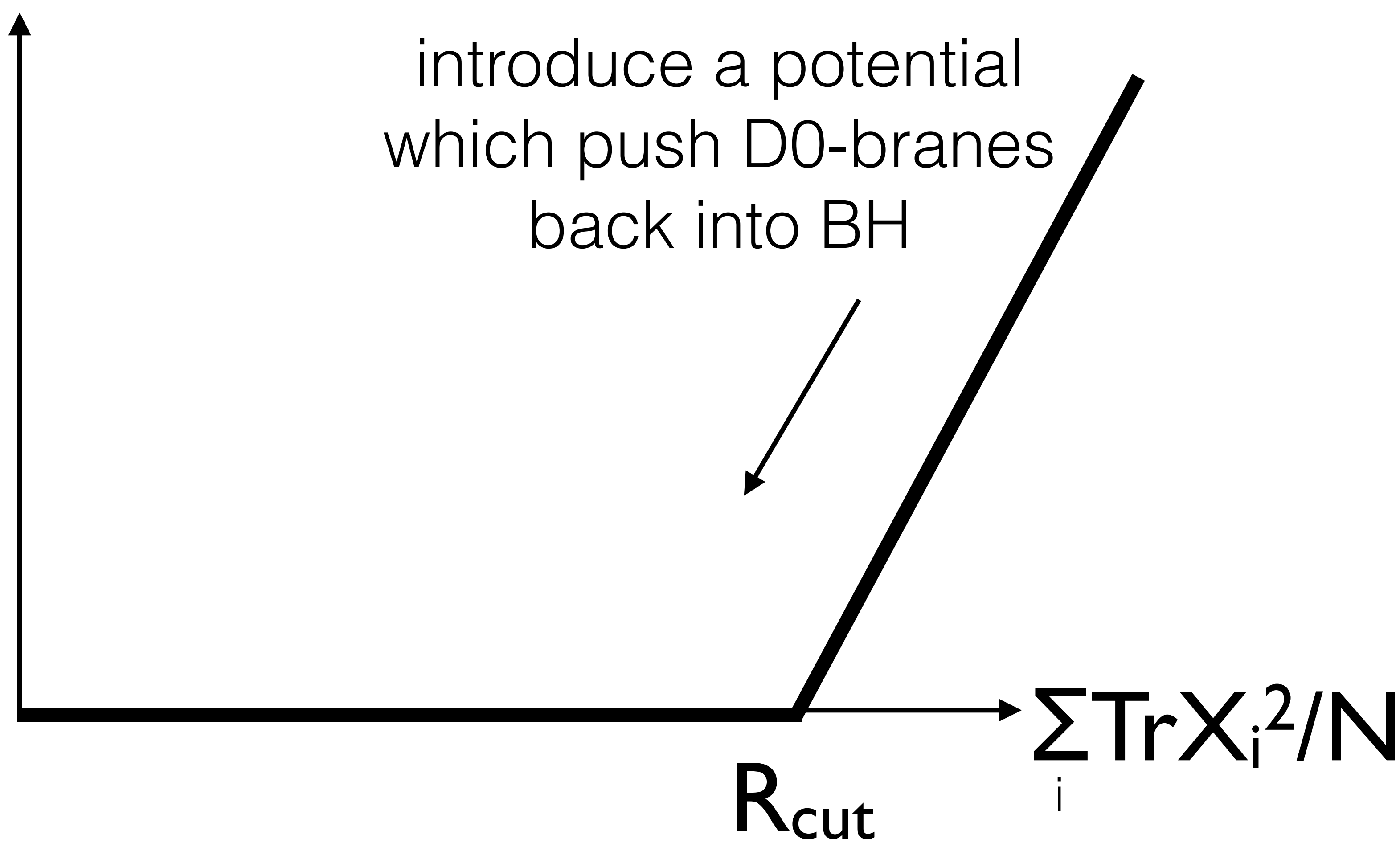
We have to consider small values of N.

→ FLAT DIRECTION IS BACK!

It is unavoidable, because we want to study an *unstable* object — evaporating BH.

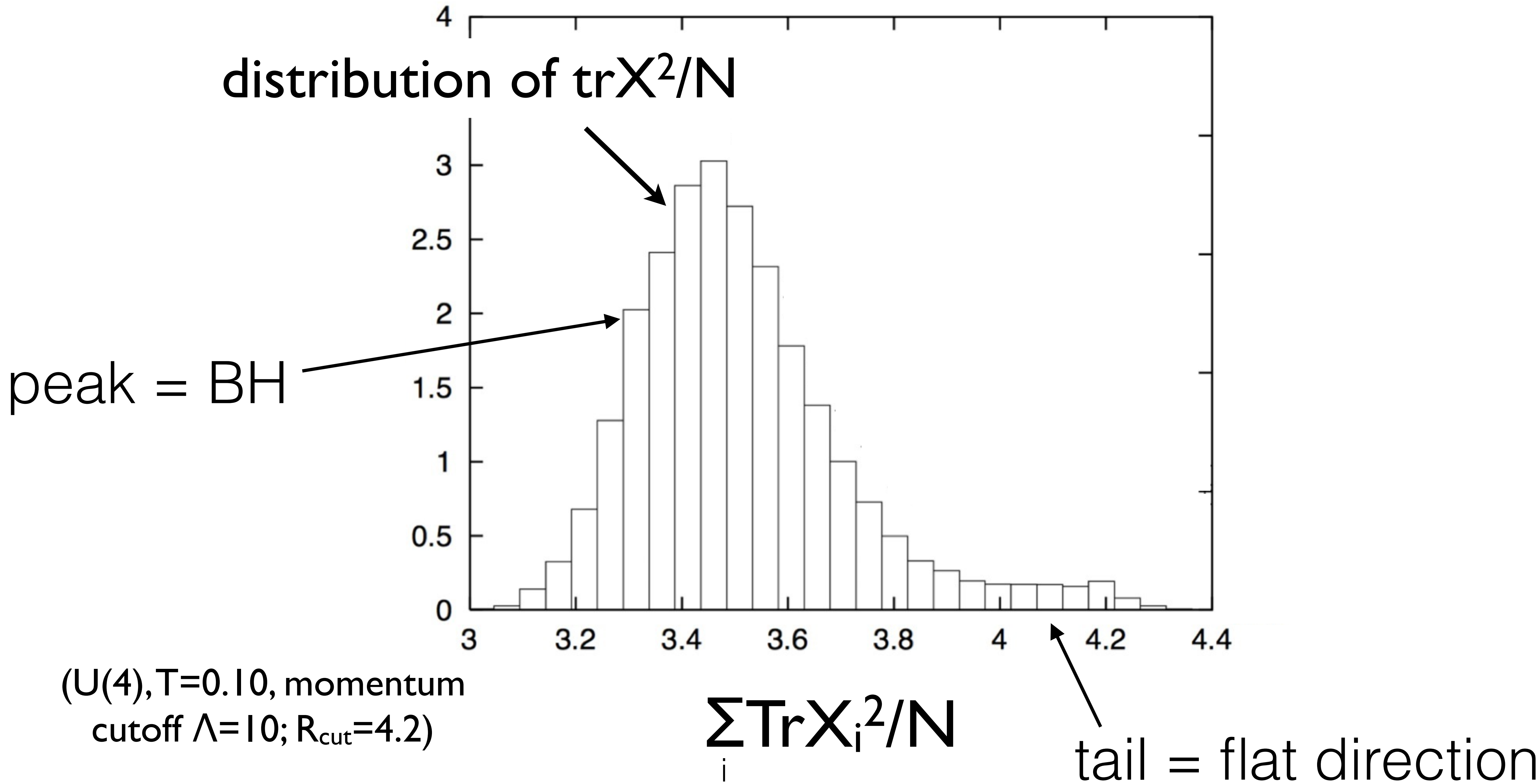
A practical solution (1)

Put the BH in a box.



add potential $\gamma \int dt\, |\mathrm{Tr}X^2/N - R_{cut}|$ at $\mathrm{Tr}X^2/N > R_{cut}$

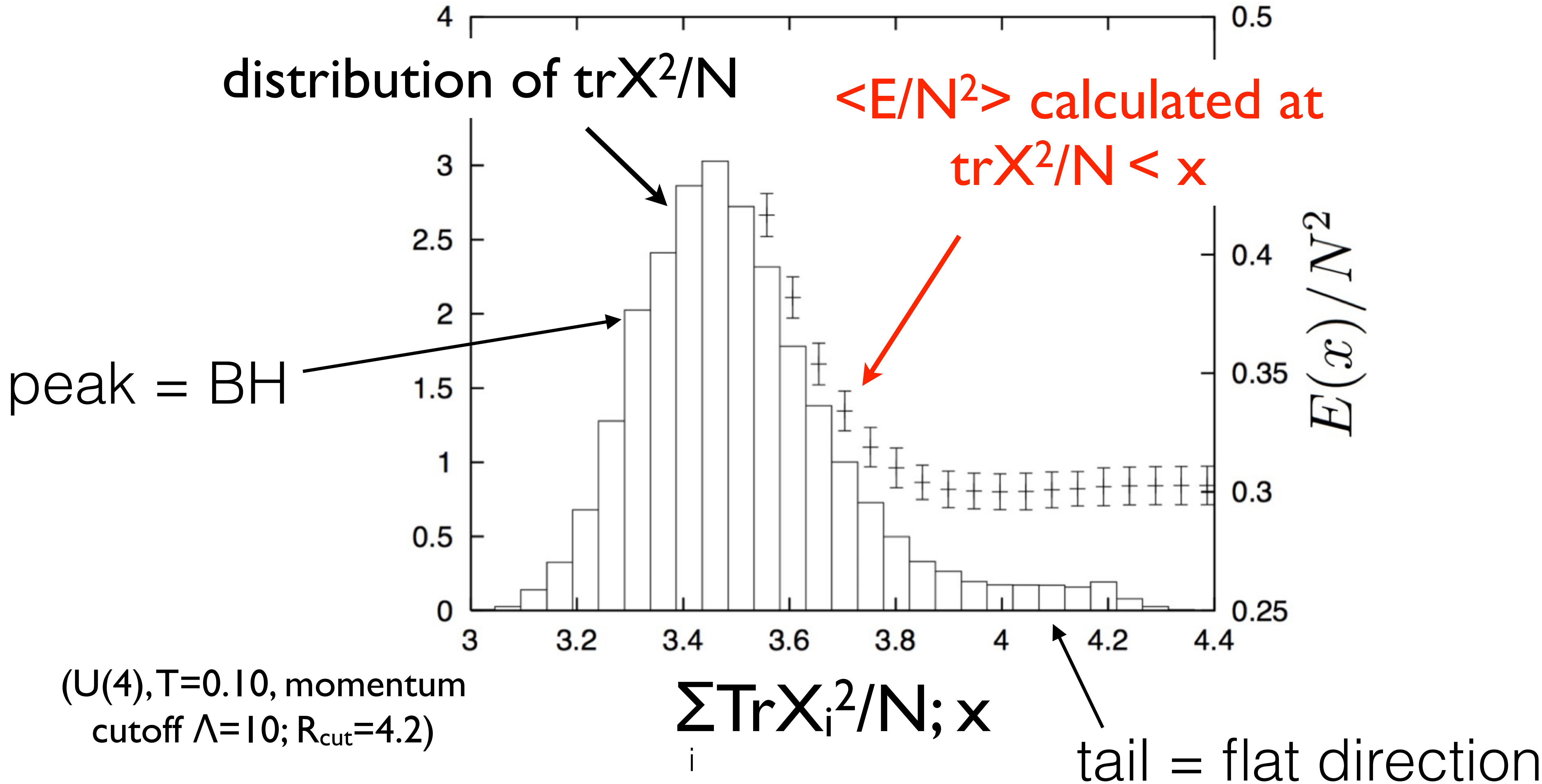
A practical solution (2)



(U(4), T=0.10, momentum cutoff Λ=10; R_{cut}=4.2)

Where is the border of BH?

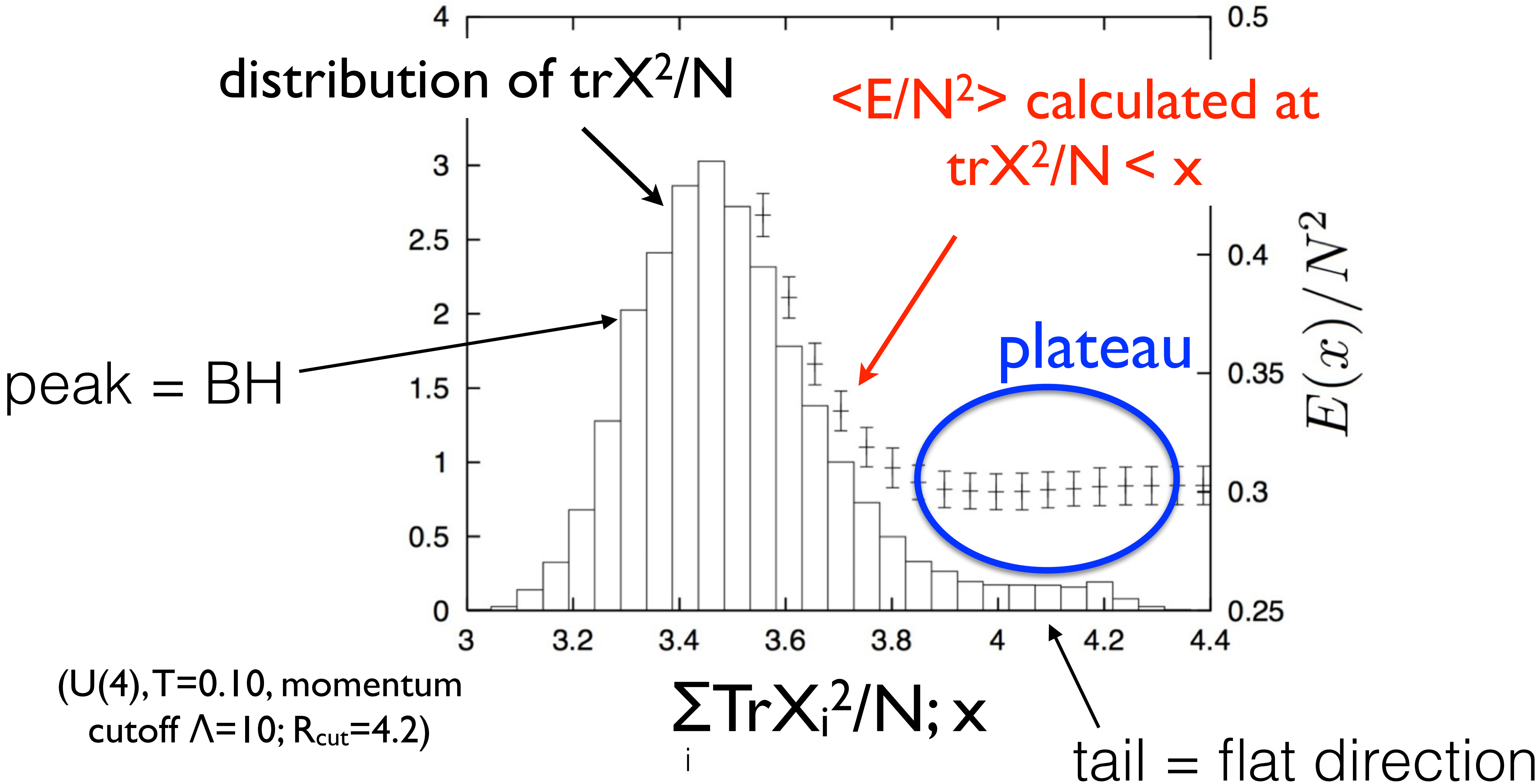
A practical solution (3)



(U(4), T=0.10, momentum cutoff Λ=10; R_{cut}=4.2)

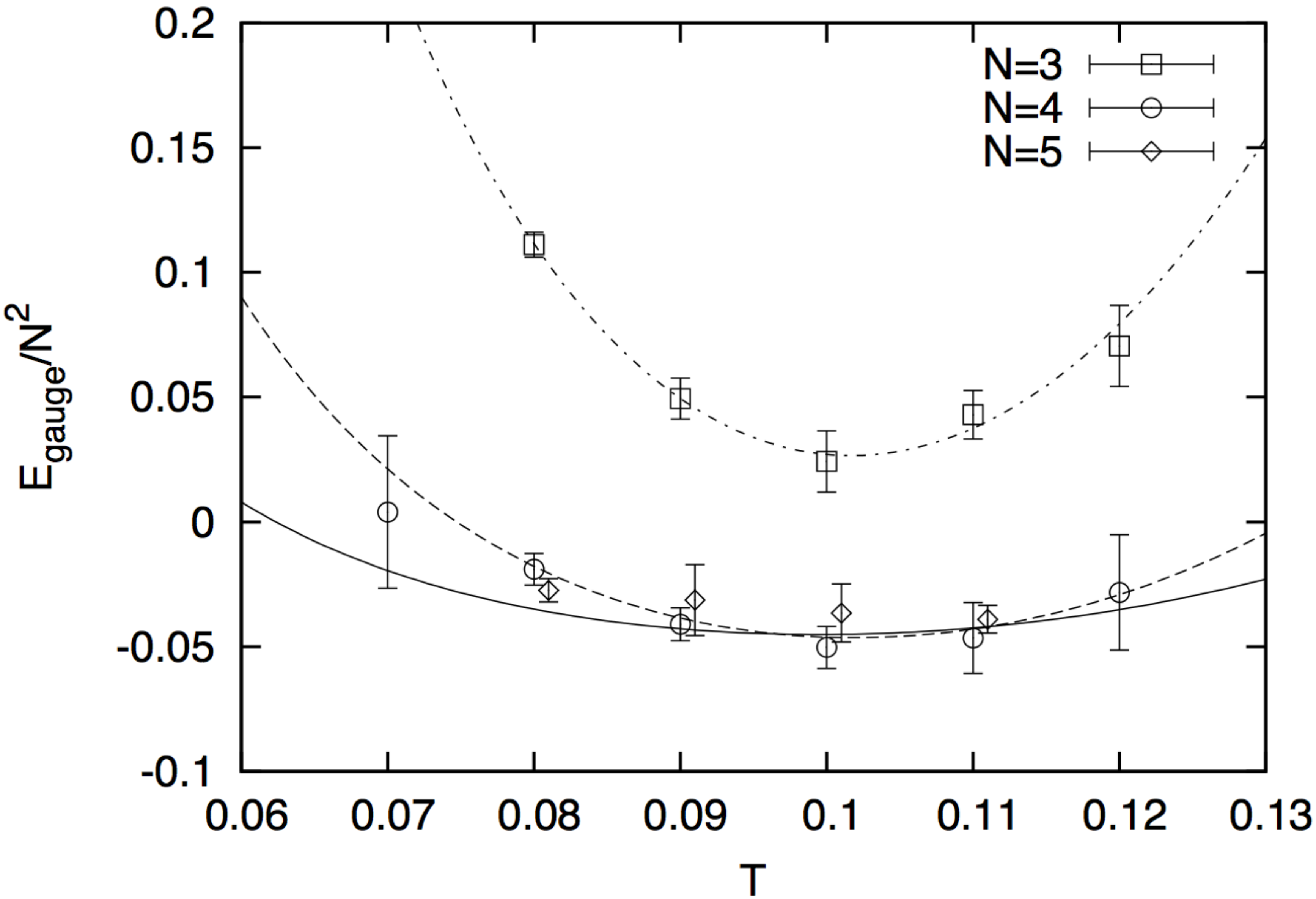
Where is the border of BH?

A practical solution (4)



(U(4), T=0.10, momentum cutoff Λ=10; R_{cut}=4.2)

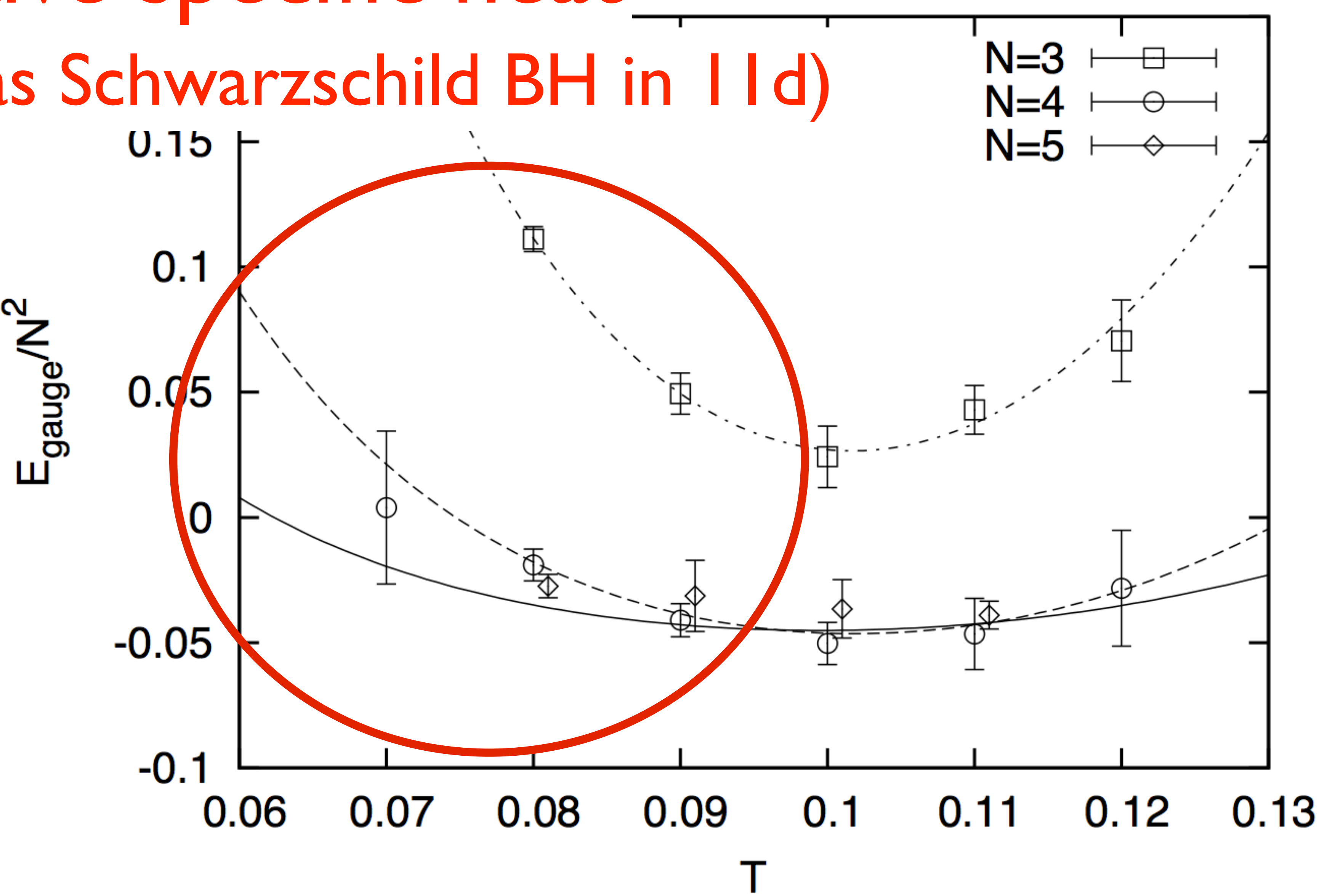
value @ plateau = energy of BH



M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

Negative specific heat

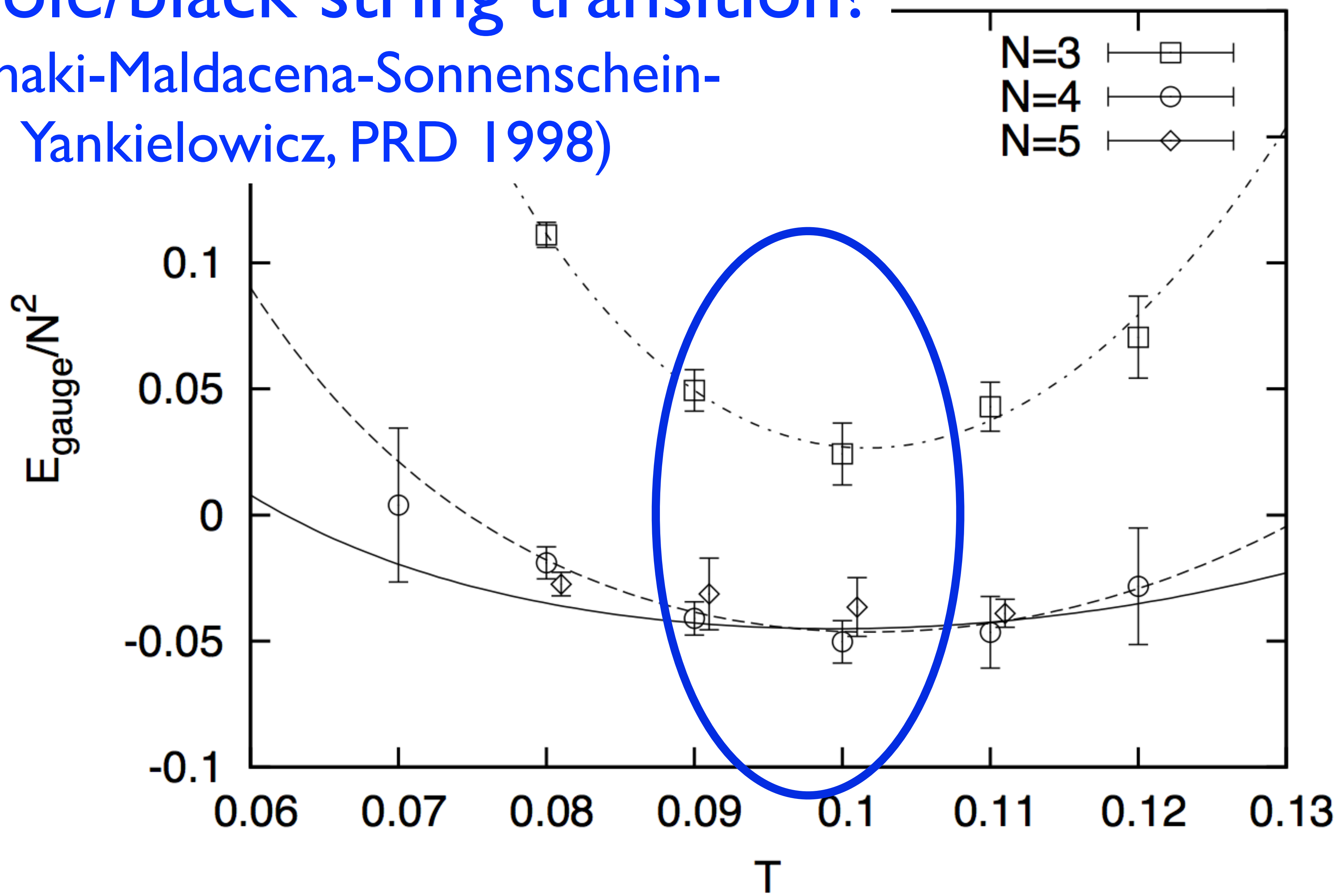
(the same as Schwarzschild BH in 11d)

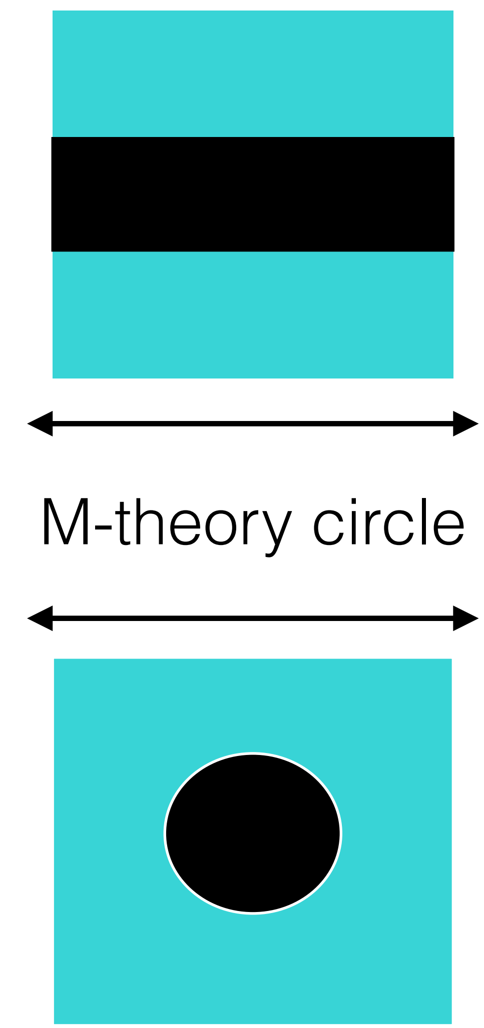


M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

Black hole/black string transition?

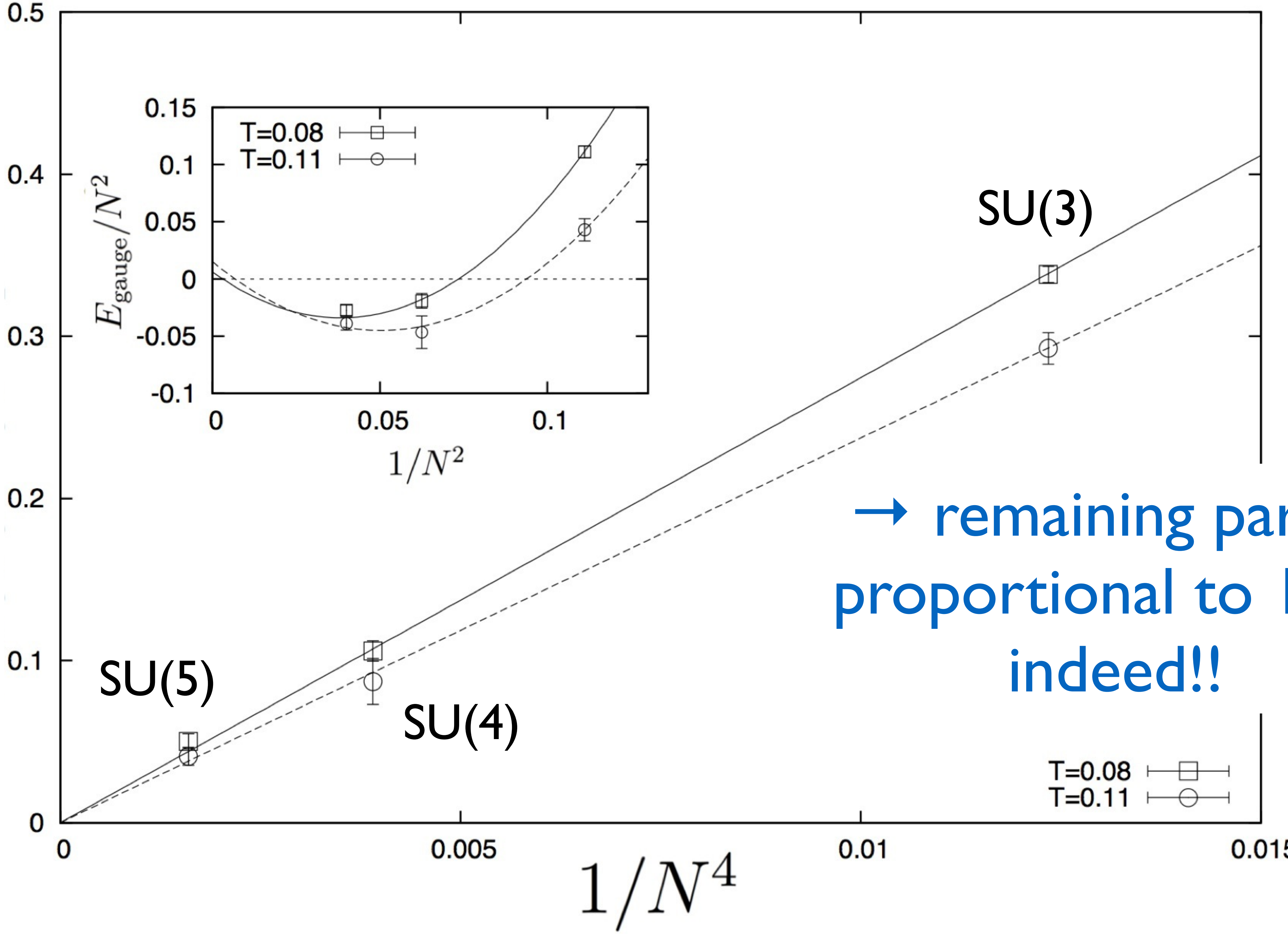
(Itzhaki-Maldacena-Sonnenschein-Yankielowicz, PRD 1998)





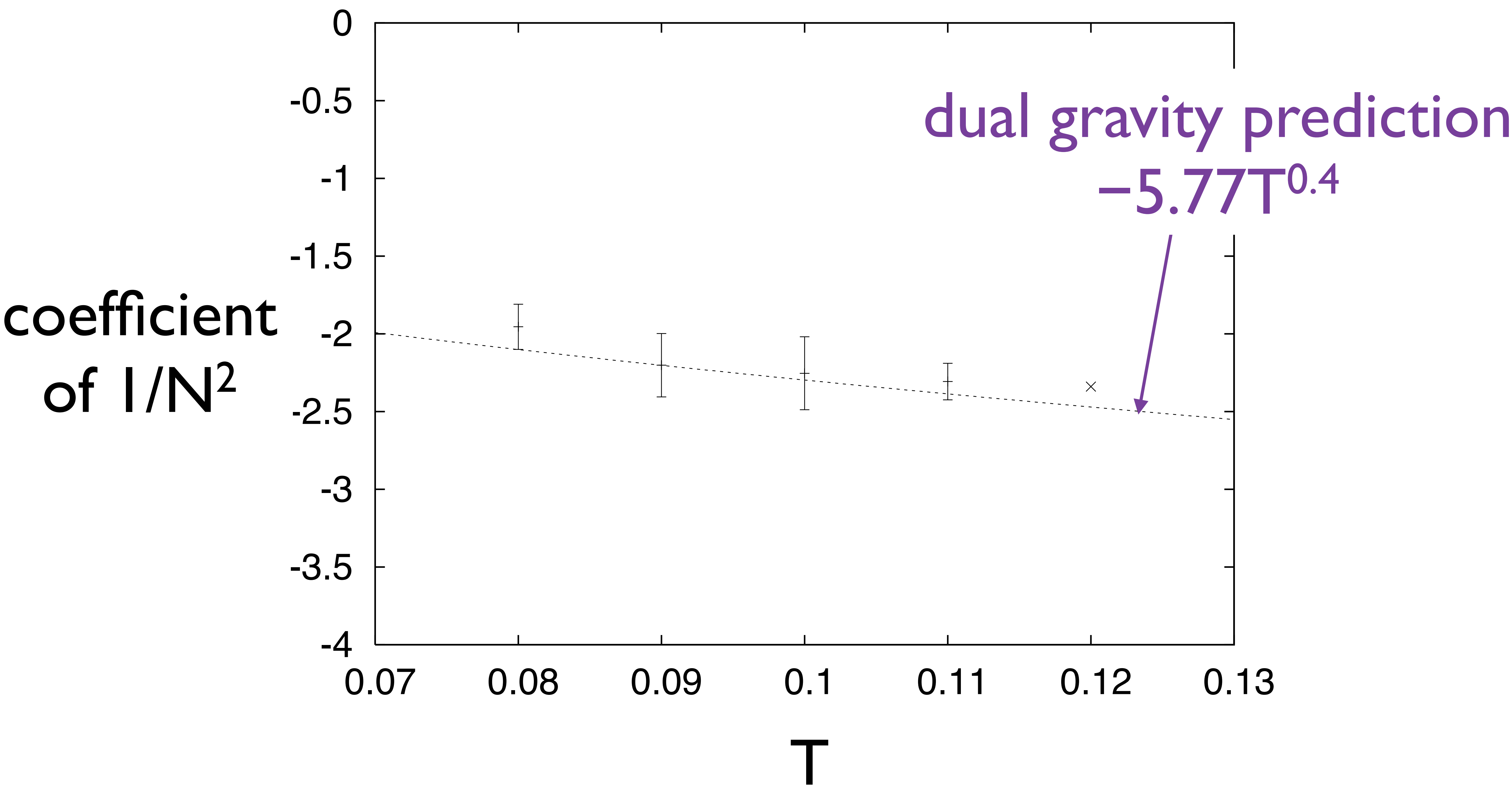
M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

$E/N^2 - (7.41T^{2.8} - 5.77T^{0.4}/N^2)$ vs. $1/N^4$



→ remaining part is proportional to $1/N^4$ indeed!!

M.H.-Hyakutake-Ishiki-Nishimura, Science 2014



M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

simulating other theories

Which SYM can be simulated?

Possible/Impossible

(without fine tuning; not necessarily lattice)

smaller simulation cost

(0+1)-d

(1+1)-d

any number of SUSY, various matter contents

(2+1)-d maximal SUSY

less SUSY

matter fields

(3+1)-d

without matter (pure $\mathcal{N}=1$)

SUSY QCD (matter fields)

maximal SUSY

larger simulation cost

Other simulations

(0+1)-d

- Independent tests @ large-N (Catterall-Wiseman 2008-2010; Kadoh 2013)
- Polyakov loop (M.H.-Miwa-Nishimura-Takeuchi 2008)
- Two-point function

  massless modes (M.H.-Nishimura-Sekino-Yoneya 2009,2011)
  massive modes (Azeyanagi-M.H.-Nishimura-Sekino-Yoneya, in progress)

(1+1)-d

- Black hole/black string transition (Catterall-Joseph-Wiseman 2010)
- 2d N=(2,2) SYM (Kanamori-Suzuki 2007–2008, M.H.-Kanamori 2009-2010, Kanamori 2010, Catterall 2011)

Other simulations

(2+1)-d

- It can be realized by considering k-coincident fuzzy sphere in the plane wave matrix model (Berenstein-Maldacena-Nastase matrix model)
  (Maldacena-Sheikh Jabbari-van Raamsdonk 2003)

Space is embedded in matrices;
large-N = “big lattice”

No numerical simulation so far :(

Other simulations

(3+1)-d

- lattice simulation, U(2) (the fine tuning and U(3), U(4) are ongoing) (Catterall, Damgaard, DeGrand, Galvez, Giedt, Mehta, Schaich, 2012–present)
- Large-N volume reduction (Eguchi-Kawai reduction) (Honda, Ishikim, Kim, Nishimura, Tsuchiya, 2008 – present)
- 4d N=1 pure SYM (Brower, Catterall, Fleming, Giedt, Vranas, 2008; Endress, 2008–2009; Bergner, Giudice, Montvay, Munster, Piemonte, Sandbrink, …, 1995–present; Fukaya, Hashimoto, Kim, Matsufuru, Nishimura, Onogi, 2011)

Simulation codes

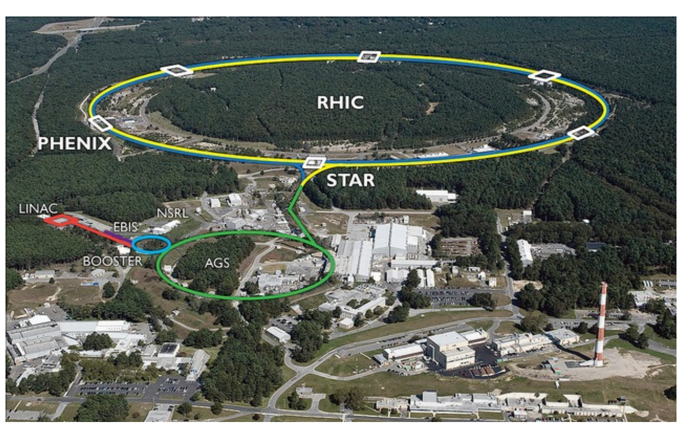
- We are planning to provide an open source simulation code for (0+1)-d SYM
- Old version for (0+1)-d SYM is available upon request. Please email me.
- Lattice simulation code for (3+1)-d SYM by Catterall et al. can be downloaded from https://www.assembla.com/code/smilc/subversion/nodes

my slide in
a talk for
lattice/nuclear
theorists
@Stony Brook

conclusion

Maldacena's conjecture is correct
at finite temperature,
including 1/λ and 1/N corrections,
at least to the next-to-leading order.

so, lattice/nuclear theorists can study
quantum gravity, by studying field theory.
You can do something string theorists cannot do.



RHIC is a machine for quantum gravity!


Occupy Princeton

conclusion (for string theorists)

Maldacena's conjecture is correct
at finite temperature,
including $1/\lambda$ and $1/N$ corrections,
at least to the next-to-leading order.

Let's find good problems in SYM,
which nuclear/lattice theorists can solve,
and at the same time,
tells us about quantum gravity.

(Even qualitative argument in pure YM
would be a good starting point.)

Your ideas will be appreciated!

backup slides

black p-brane solution

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[- \left(1 - \frac{U_0^{7-p}}{U^{7-p}} \right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}} \left(1 - \frac{U_0^{7-p}}{U^{7-p}} \right)} dU^2 + g_{YM}\sqrt{d_p N} U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

>> 1

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2} \right),$$

<< 1

SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \quad (p=0)$$

higher dimensions require more computational cost

$$\int [dA][d\psi] e^{-S_B[A]-S_F[A,\psi]} = \int [dA] \det D[A] \cdot e^{-S_B[A]}$$

※ Pfaffian for Majorana fermions

Dirac operator (adjoint repr.) : $N^2L^{p+1} \times N^2L^{p+1}$

cost for calculating determinant is

$(N^2L^{p+1})^3 = N^6L^{3(p+1)}$

(0+1)-d is the best starting point

Wilson's lattice gauge theory

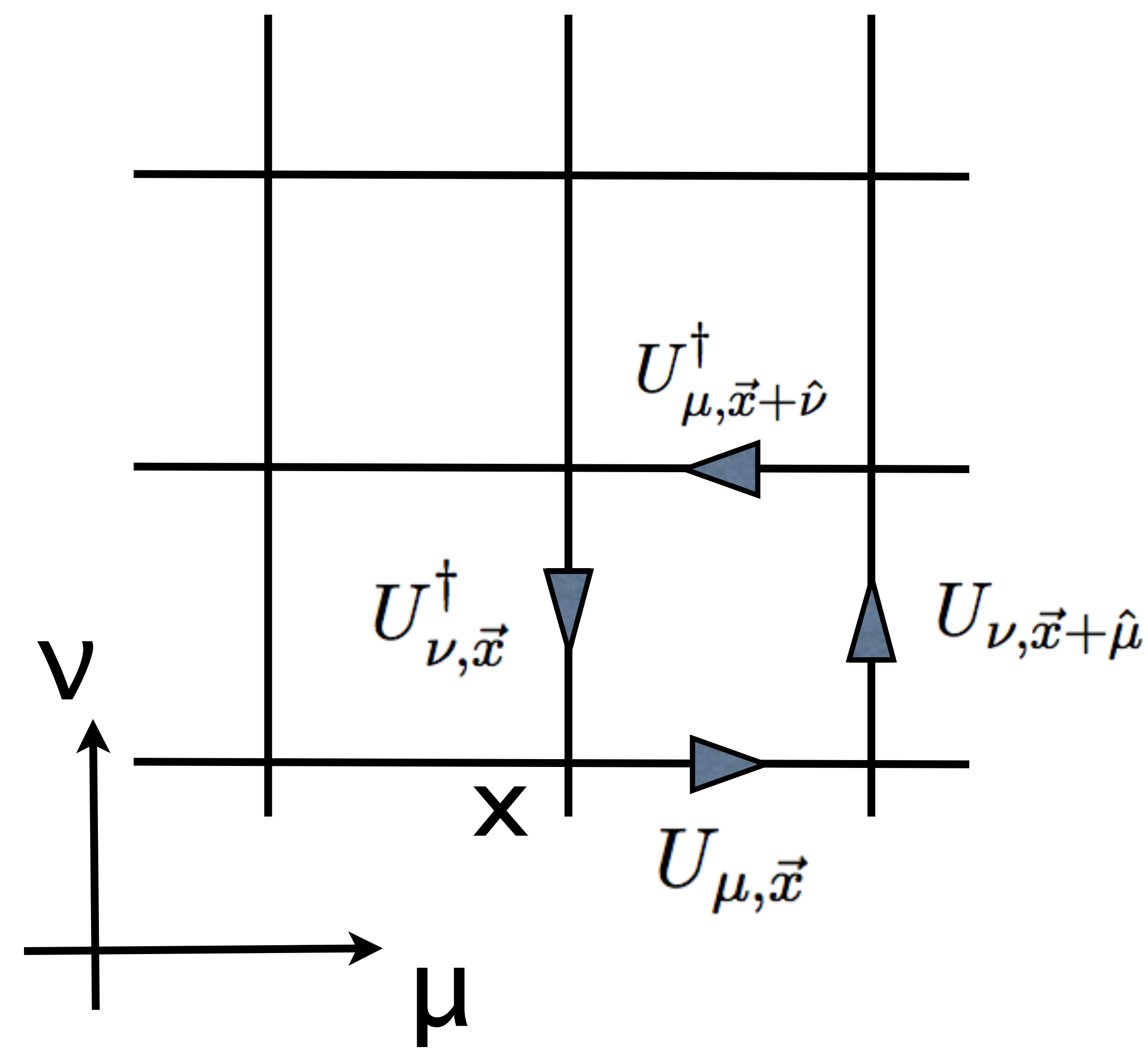
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr \left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}} \right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_\mu(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

‘Exact’ symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

These symmetries exist *at discretized level*.

Continuum limit $a \to 0$ respects exact symmetries at discretized level.

Exact symmetries at discretized level

↓

gauge invariance, translational invariance, rotationally invariant,... in the continuum limit.

What happens if the gauge symmetry is explicitly (not spontaneously) broken, (e.g. the sharp momentum cutoff prescription)?

- We are interested in low-energy, long-distance physics (compared to the lattice spacing a).
- So let us integrate out high frequency modes.

Then...

gauge symmetry breaking radiative corrections can appear.

To kill them, one has to add counterterms to lattice action, whose coefficients must be fine-tuned!

'fine tuning problem'

This is the reason why we *must* preserve symmetries exactly.