Logarithmic Corrections to Entropy of Extremal Black Hole

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Proposal

Extremal black hole has AdS_2 factor in it's near horizon geometry.

Quantum degeneracy associated to the horizon of extremal black hole with charge $\vec{q}$ is given by,

$$d_{hor}(\vec{q}) = \mathcal{Z}^{finite}_{AdS_2 \times K}(\vec{q}). \tag{1}$$

Thus the quantum corrected entropy is,

$$S_{BH} = \ln d_{hor}(\vec{q}). \tag{2}$$

One can test this proposal by comparing with known examples.

e.g. $\frac{1}{4}$th BPS black hole in $\mathcal{N} = 4$ and $\frac{1}{8}$th BPS black hole in $\mathcal{N} = 8$.

Use this proposal to predict the quantum entropy in unknown cases.

e.g. $\frac{1}{2}$-BPS black hole in $\mathcal{N} = 2$.

Both examples provide consistency checks of the proposal.

Saddle points

Large charge limit corresponds to semiclassical analysis.

Hence we need to know saddle points of path integral.

In 4 dim. leading contribution comes from $AdS_2 \times S^2$,

$$ds^2 = a^2 \left(d\eta^2 + \sinh^2\eta d\theta^2\right) + a^2 \left(d\psi^2 + \sin^2\psi d\phi^2\right). \qquad (3)$$

It's classical contribution is,

$$d_{hor} \sim Exp\left[\frac{A_H}{4}\right], \qquad A_H \sim a^2. \qquad (4)$$

There are other saddle points.

Asymptotic analysis of $\mathcal{N} = 4$ and 8 suggest saddle–points which are obtained by taking the $\mathbb{Z}_N$ orbifold of $AdS_2 \times S^2$,

$$(\theta, \phi) \equiv \left(\theta + \frac{2\pi}{N}, \phi - \frac{2\pi}{N}\right). \tag{5}$$

It's classical contribution is,

$$d_{hor/N} \sim Exp\left[\frac{A_H}{4N}\right]. \tag{6}$$

same as microscopic answer.

Logarithmic corrections

We want to go beyond the classical contribution.

We compute one loop partition function of the supergravity fields.

We look for logarithmic corrections.

These come from two derivative action of massless fields at one loop and indep. of massive fields.

Integrated Heat Kernel and Determinant

One loop determinant is given in terms of integrated heat kernel of the operator.

$$\text{Small t-expansion:} \qquad K(t) = \frac{b_0}{t^2} + \frac{b_1}{t} + b_2 + .. \tag{7}$$

Logarithmic correction is,

$$\ln(\mathcal{Z})_{log} \sim \frac{1}{2}\left(b_2 + \sum_{\phi} n^0_{\phi}\left(1 - \beta_{\phi}\right)\right)\ln\left(A_H\right). \tag{8}$$

n^0_{ϕ} is the number of zero modes and β_{ϕ} is the scaling dimension of field.

Zero modes (n_ϕ^0):

Fields	Unquotient	Quotient	β_ϕ
Vector	1	1	1
Gravity	6	2	2
gravitino	4	2	3

Results

Contribution of these saddle points:

$\frac{1}{4}$- BPS black hole in $\mathcal{N} = 4$ supergravity:

$$d_{hor/N}(\vec{q}) = Exp\left[\frac{A_H}{4N}\right] \times \mathcal{O}(1) \tag{9}$$

$\frac{1}{8}$- BPS black hole in $\mathcal{N} = 8$ supergravity:

$$d_{hor/N}(\vec{q}) = Exp\left[\frac{A_H}{4N} - 4\ln A_H\right] \times \mathcal{O}(1) \tag{10}$$

$\frac{1}{2}$- BPS black hole in $\mathcal{N} = 2$ supergravity:

$$d_{hor/N}(\vec{q}) = Exp\left[\frac{A_H}{4N} + \left(2 - \frac{N\chi}{24}\right)\ln A_H\right] \times \mathcal{O}(1) \qquad (11)$$

Here ,

$$\chi = 2(n_V - n_H + 1) \qquad (12)$$

Implications and understanding of the result need more work.

Thank You.