QUANTUM SPECTRAL CURVE AND ADS/ CFT SPECTRAL PROBLEM

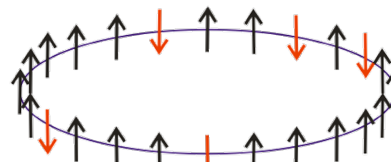
Nikolay Gromov

Based on

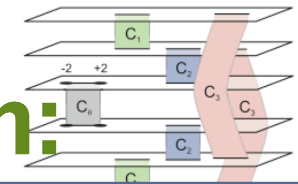
N. G., V. Kazakov, S. Leurent, D. Volin 1305.1939 , 1405.4857
N. G., F. Levkovich-Maslyuk, G. Sizov, S. Valatka 1402.0871
A. Cavaglia , D. Fioravanti, N. G., R. Tateo 1403.1859
N. G., G. Sizov 1403.1894
M. Alfimov,N. G., V. Kazakov to appear

Strings 2014

Integrability in gauge theory



Plan:



Review of the QSC construction

Examples:
1) near BPS 2) BFKL limit

Generalization to ABJM

Origins of YM integrability: Lipatov's BFKL Hamiltonian

P
int
N=4 SYM

on AdS5×S5

1993

2002

2003-2008

Lipatov
Faddeev,Korchemsky

M
Beisert,K

an,
Schafer-Nameki
Beisert,Kazakov,Sakai,Zarembo
NG,Vieira

Motivation from classics

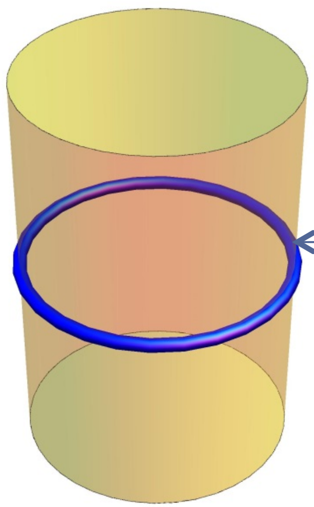
[Bena, Polchinski, Roiban]

$$S = g \int \mathrm{str}(J^{(2)} \wedge * J^{(2)} - J^{(1)} \wedge J^{(3)})$$

$PSU(2,2|4)$ current

$$J = -g^{-1} dg = J^{(1)} + J^{(2)} + J^{(3)} + J^{(0)}$$

EOM equivalent to $\partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu] = 0 \quad \forall u \in \mathbb{C}$ where $\mathcal{A}(u) = J^{(0)} + \frac{u}{\sqrt{u^2 - 4g^2}} J^{(2)} - \frac{2g}{\sqrt{u^2-4g^2}} * J^{(2)} + \ldots$



$$\Omega(u,\tau) = \mathrm{Pexp} \oint \mathcal{A}_\sigma d\sigma$$

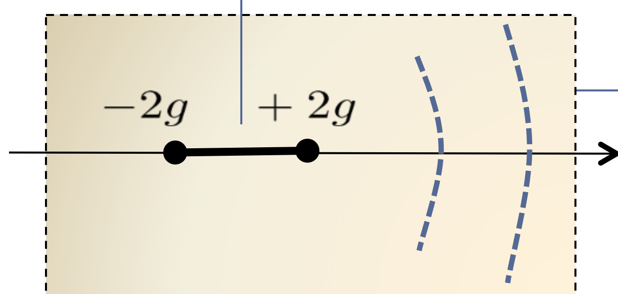
on EOM $\partial_\tau \mathrm{tr}\Omega(u,\tau) = 0$

Eigenvalues of the monodromy matrix:

$$(e^{ip_1}, e^{ip_2}, e^{ip_3}, e^{ip_4} | e^{iq_1}, e^{iq_2}, e^{iq_3}, e^{iq_4})$$

S^5 $\quad$ AdS_5

Analytic properties:



State-dependent cuts

[Dorey, Vicedo] $\oint p(u) du = \mathbb{Z}$

From weak coupling

[Beisert, Sctaudacher]

$$\mathcal{O}_i(x) = \mathrm{tr} D_+^{n_1} Z D_+^{n_2} Z D_+^{n_3} Z D_+^{n_4} Z D_+^{n_5} Z$$

Can be mapped to a spin chain state: $|n_1, n_2, n_3, n_4, n_5\rangle$

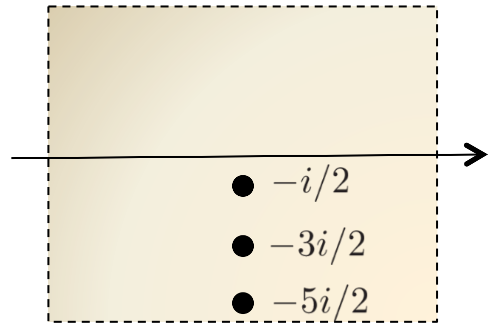
The one-loop dilatation operator coincides with $sl(2)$ Heisenberg spin chain Hamiltonian

Sklyanin separation of variables allows to factorize the wave function

$\Psi = \prod_i^L Q(v_i)$ where $T(u)Q(u) + (u - i/2)^L Q(u-i) + (u+i/2)^L Q(u+i) = 0$

In the simplest case $T(u) = -2u^2 + S^2 + S + \frac{1}{2}$

Two solutions: polynomial $Q_1 \sim u^S$ singular solution $Q_2 \sim u^{-1-S}$



Generalization to finite coupling

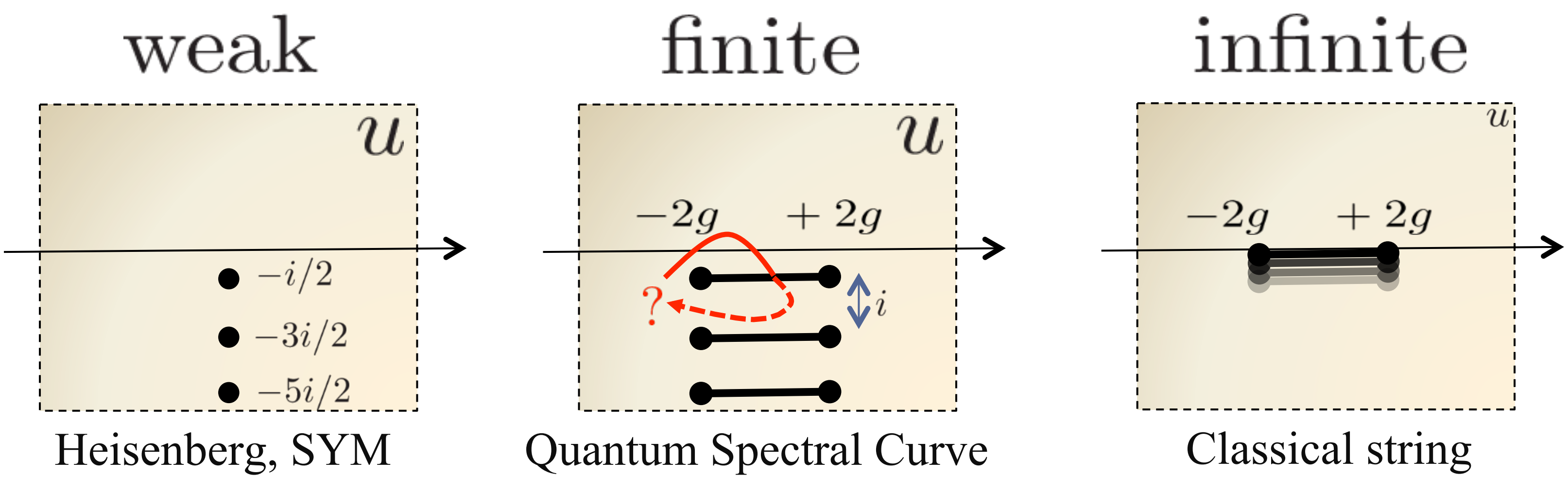
[N.G., Kazakov, Leuren, Volin]

$$g \to \text{finite}$$

1) We start exploring all DOS of the string $\quad sl(2) \to psu(2,2|4)$

$$(Q_1,\ Q_2) \quad \to \quad (\underbrace{\mathbf{P}_1,\ \mathbf{P}_2,\ \mathbf{P}_3,\ \mathbf{P}_4}_{S^5}|\underbrace{\mathbf{Q}_1,\ \mathbf{Q}_2,\ \mathbf{Q}_3,\ \mathbf{Q}_4}_{AdS_5})$$

2) Poles open into cuts

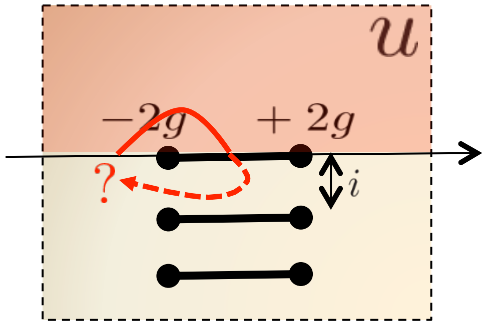


3) Need to know monodromies, when going under the cuts

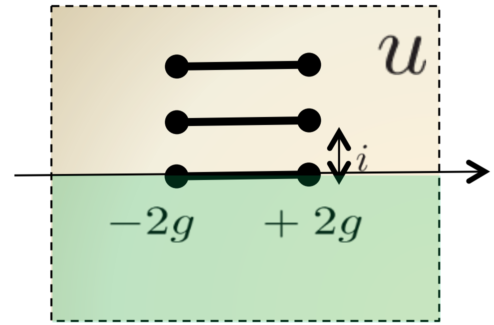
"Miraculous" simplification

[N.G., Kazakov, Leuren, Volin]

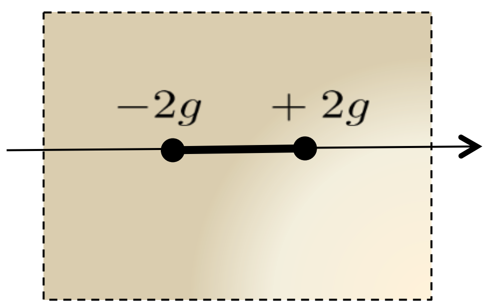
$\mathbf{Q}_a$



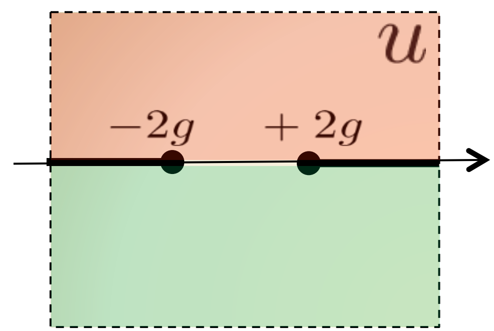
$? = \tilde{\mathbf{Q}}_a$



$\mathbf{P}_a$



$\mathbf{Q}_a$



$\mathbf{P}_a \simeq u^{\mathrm{R-charge}} \quad , \quad u \to \infty$

Charges in S^5 are integer

$\mathbf{Q}_a \sim u^{\text{conformal charge}}$

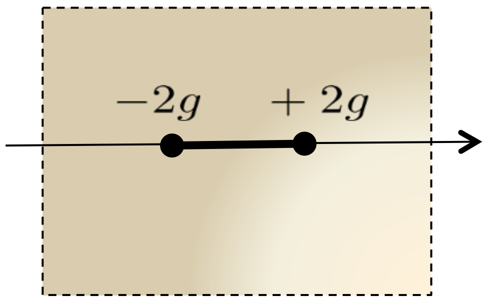
Charges in AdS_5 contain anom.dimension

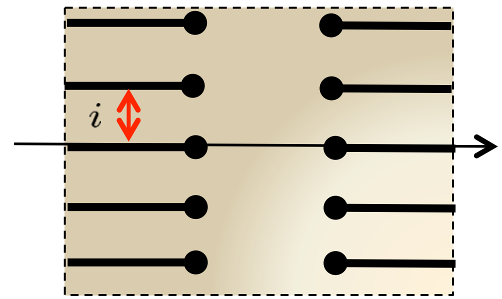
$\mathbf{P}\mu$ - system

The system reduced to 4+6 functions:

[N.G., Kazakov, Leuren, Volin]

$\mathbf{P}_a$ $\quad\quad$ $\mu_{a,b} = -\mu_{b,a}$ $\quad\quad$ $a, b = 1, \ldots, 4$



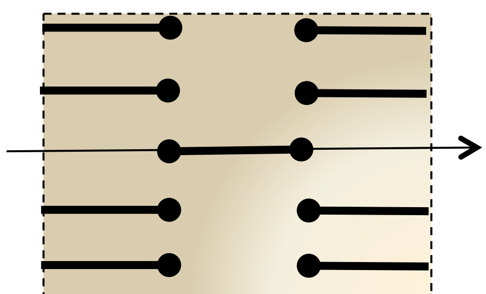


$$\mu_{a,b}(u+i) = \mu_{a,b}(u)$$

Analytical continuation to the next sheet:

$$\tilde{\mathbf{P}}_a = \mu_{ab}\mathbf{P}^b$$

$$\tilde{\mu}_{a,b} = \mu_{a,b} + \mathbf{P}_a\tilde{\mathbf{P}}_b - \mathbf{P}_b\tilde{\mathbf{P}}_a$$



$\mathbf{P}\mu$-system is a closed system of equtions!

Quadratic branch cuts:

$$\tilde{\tilde{\mathbf{P}}}_a = \mathbf{P}_a \Rightarrow \mu_{ab}\mu^{bc} = \delta_a^c$$

$$\mathbf{P}_a \simeq (\text{conformal charges})\, u^{\text{R}-\text{charge}} \;,\quad u \to \infty$$

Examples: near-BPS expansion

Near BPS limit: small S

[NG. Sizov, Valatka, Levkovich-Maslyuk]

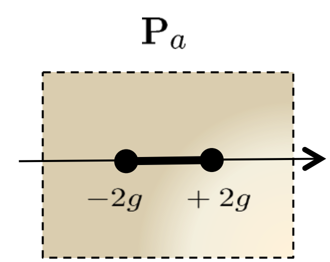
In the BPS limit: $\mathbf{P}_b \to 0$, $\mu_{ab}-$entire periodic function

For $\mathrm{tr} D^S Z^2$ in the small S limit:

$$\mathbf{P}_a(u-i0) = \mu_{ab}\mathbf{P}^b(u+i0)$$ - simple Riemann-Hilbert problem

$$\Downarrow$$

$$\begin{aligned} \mathbf{P}_3(u-i0) + \mathbf{P}_3(u+i0) &= 0 \\ \mathbf{P}_4(u-i0) - \mathbf{P}_4(u+i0) &= \mathbf{P}_3(u+i0)\sinh(2\pi u) \end{aligned}$$



Solution:

$$\mathbf{P}_3 = \sqrt{u^2 - 4g^2}$$

$$\mathbf{P}_4 = \int_{-2g}^{2g} \frac{\sqrt{v^2-4g^2}\sinh(2\pi v)}{v-u} \propto \frac{I_1(4\pi g) - I_3(4\pi g)}{u^2} + \mathcal{O}\left(\frac{1}{u^4}\right)$$

Result: $$\Delta = 2 + S + 2\pi g \frac{I_3(4\pi g)}{I_2(4\pi g)}$$

[Basso] [Zarembo; Pestun]

Similar to the localization results!

More orders in small S

Not hard to iterate the procedure and go further away from BPS.

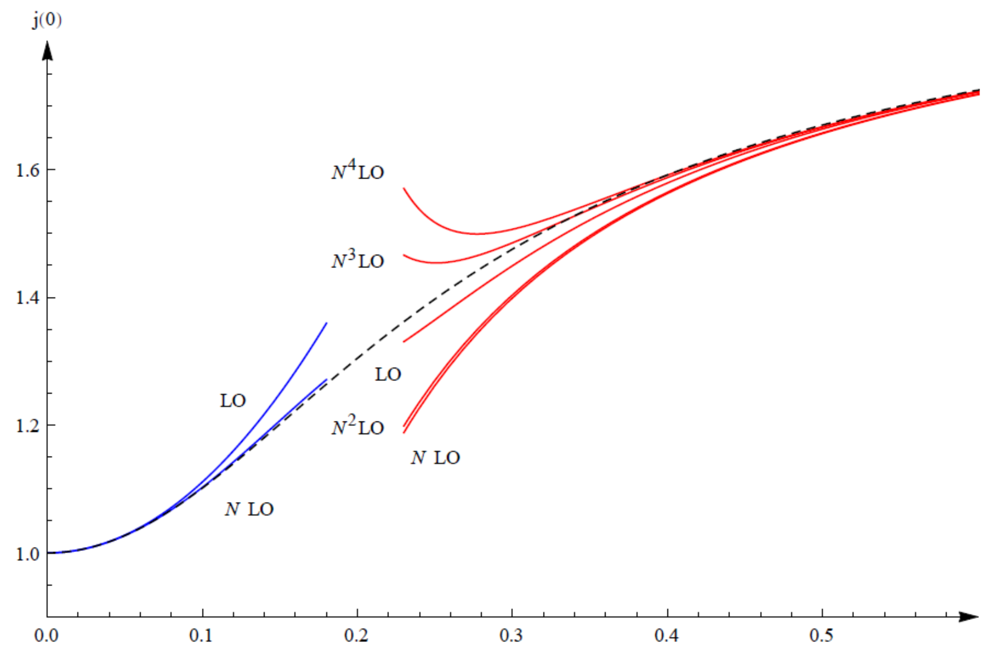
Extrapolating results to finite spin

[Basso][NG. Sizov, Valatka, Levkovich-Maslyuk]

$$\Delta_{Konishi} = 2\lambda^{1/4} + \frac{2}{\lambda^{1/4}} + \frac{-3\zeta(3)+\frac{1}{2}}{\lambda^{3/4}} + \frac{\frac{15\,\zeta(5)}{2} + 6\,\zeta(3) - \frac{1}{2}}{\lambda^{5/4}}$$

Gubser, Klebanov, Polyakov `98

Gromov, Serban, Shenderovich, Volin`11; Roiban, Tseytlin`11; Vallilo, Mazzucato`11 Plefka, Frolov`13



We also extract pomeron intercept:

Costa, Goncalves, Penedones` 12

Kotikov, Lipatov`13

Gubser, Klebanov, Polyakov `98

$$j(0) = 2 + S(0) = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\,\lambda^{3/2}} + (6\zeta(3)+2) + \left(18\,\zeta(3) + \frac{361}{64}\right)\frac{1}{\lambda^{5/2}} + \left(39\,\zeta(3) + \frac{447}{32}\right)\frac{1}{\lambda^{3}}$$

BFKL regime

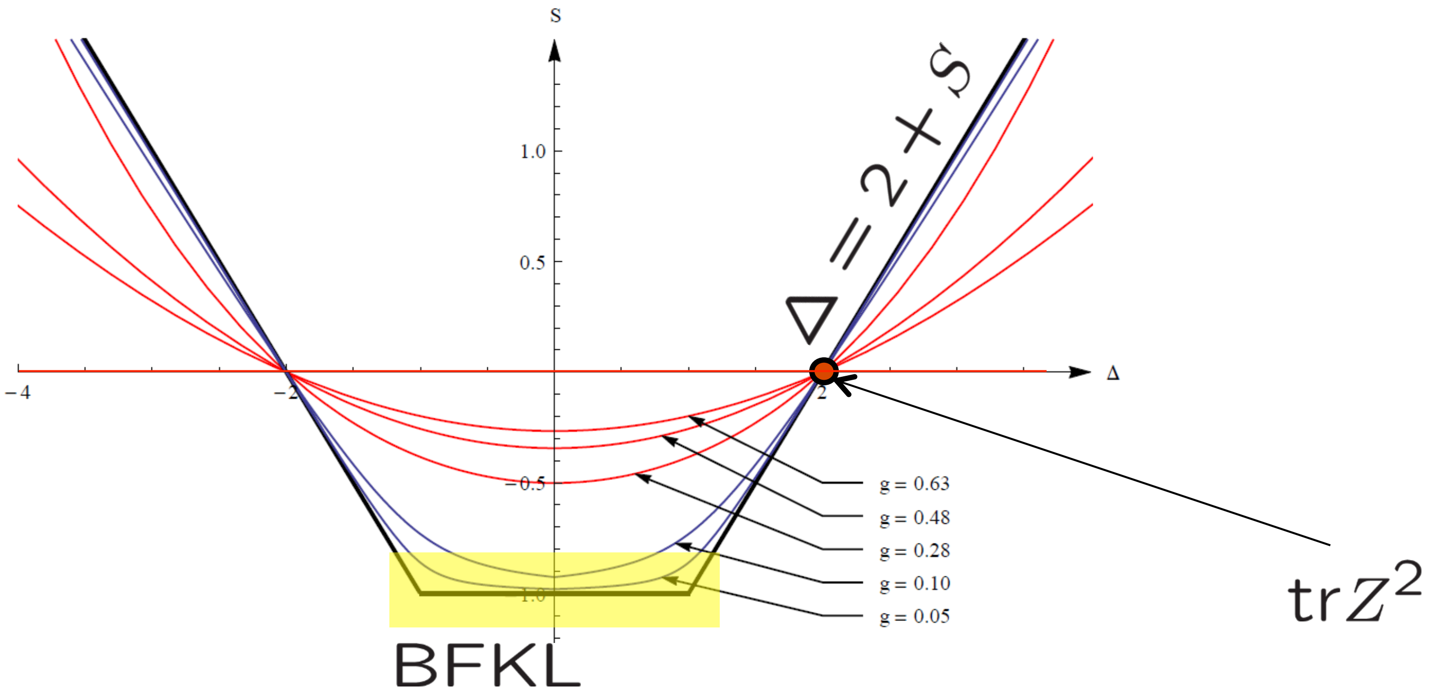
BFKL regime

Important class of single trace operators:

$$\mathrm{tr} D^S Z^2 + \text{permutations}$$

Spectrum for different spins:

[Brower, Polchinski, Strassler, -Itan `06]



BFKL regime:

$S \to -1 \quad , \quad g \to 0$ So that: $\dfrac{g^2}{S+1} \simeq 1$ Resumming to all loops terms $\left(\dfrac{g^2}{S+1}\right)^n$

In this regime SYM is undistinguishable from the real QCD

BFKL limit of $\mathbf{P}\mu$-system

Small coupling $\Rightarrow$ no branch cuts

[Alfimov, N.G., Kazakov to appear]

$$\mu_{ab} = \text{Polynom} + \text{Polynom}\, e^{+2\pi u} + \text{Polynom}\, e^{-2\pi u}$$

S= -1 is when for the first time this ansatz is consistent for non-integer Δ

Plugging it into $\mathbf{P}\mu$ - system we get:

$$\mathbf{P}_1 = \frac{1}{u} \qquad \mathbf{P}_2 = \frac{1}{u^2} \qquad \mathbf{P}_3 = -u\frac{i(\Delta^2-1)(\Delta^2-25)}{96} - \frac{i(\Delta^2-1)^2}{96u} \qquad \mathbf{P}_4 = -\frac{i(\Delta^2-1)(\Delta^2-9)}{32}$$

The problem is essentially about gluons, i.e. it is more natural to pass to AdS

$$\mathbf{Q}_i(u+i) + \mathbf{Q}_i(u-i) - \left(2 + \frac{1-\Delta^2}{4u^2}\right)\mathbf{Q}_i(u) = 0$$

Can be solved explicitly

[Kotikov, Lipatov]

$$\mathbf{Q}_1 = 2iu\; {}_3F_2\left(iu+1, \frac{1}{2}-\frac{\Delta}{2}, \frac{1}{2}+\frac{\Delta}{2}; 1, 2; 1\right) \Rightarrow \frac{S(\Delta)+1}{g^2} = -\Psi\left(\frac{1}{2}-\frac{\Delta}{2}\right) - \Psi\left(\frac{1}{2}+\frac{\Delta}{2}\right) - 2\gamma_E$$

Enters into the Q-function of Lipatov, de Vega; Korchemsky, Faddeev!

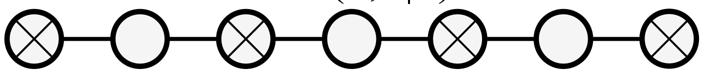
ABJM Theory

Spectral curve for ABJM

[A. Cavaglia , D. Fioravanti, N. G., R. Tateo]

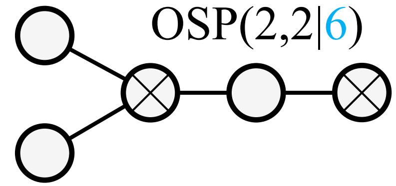
SYM

PSU(2,2|4)



$\mathbf{P}_a, \quad \mu_{ab}, \quad a,b = 1,\ldots,4$

ABJM



$\mathbf{P}_A, \quad \mu_{AB}, \quad A,B = 1,\ldots,6$

Constrains

SYM: $\mathrm{Pf}\ \mu_{ab} = 1$

ABJM: $\mu_{AB} =$ biliniar combinations of

$\nu_1, \nu_2, \nu_3, \nu_4$

define

$$\mathbf{P}_{ab} = \begin{pmatrix} 0 & \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \\ -\mathbf{P}_1 & 0 & \mathbf{P}_6 & \mathbf{P}_4 \\ -\mathbf{P}_2 & -\mathbf{P}_6 & 0 & \mathbf{P}_5 \\ -\mathbf{P}_3 & -\mathbf{P}_4 & -\mathbf{P}_5 & 0 \end{pmatrix}$$

Discontinuities

SYM:

$$\tilde{\mathbf{P}}_a = \mu_{ab}\mathbf{P}^a$$

$$\tilde{\mu}_{ab} = \mu_{ab} + \mathbf{P}_a\tilde{\mathbf{P}}_b - \mathbf{P}_b\tilde{\mathbf{P}}_a$$

ABJM:

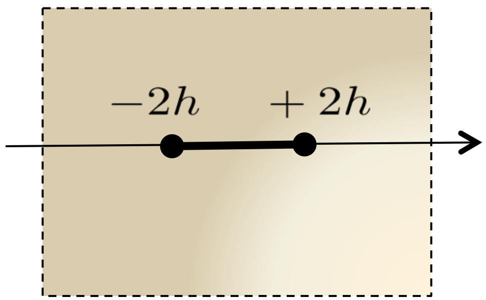
$$\tilde{\nu}_a = \mathbf{P}_{ab}\,\nu^a$$

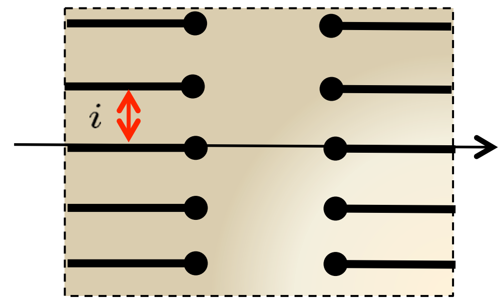
$$\tilde{\mathbf{P}}_{ab} = \mathbf{P}_{ab} + \nu_a\tilde{\nu}_b - \nu_b\tilde{\nu}_a$$

Spectral curve for ABJM

Algebraically $\mathbf{P}$ and μ interchanged their roles, but not analytically

SYM: $\mathbf{P}_a$ μ_{ab} i-periodic





ABJM: $\mathbf{P}_{ab}$ ν_a i-(anti)periodic

Another important difference is the position of the branch points:

SYM: $\pm 2g(\lambda) = \pm\frac{\sqrt{\lambda}}{2\pi}$ ABJM: $\pm 2h(\lambda) = ?$

$h(\lambda)$ enters into many important quantities: cusp dimension, magnon dispertion

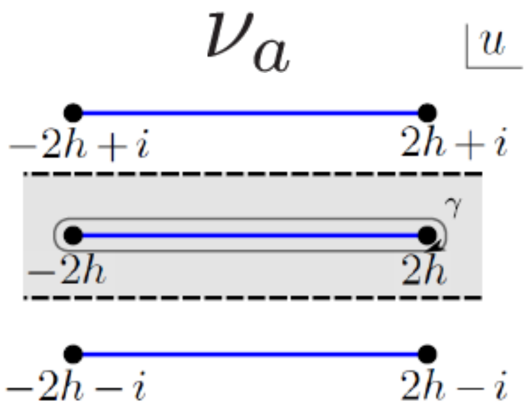
Finding Interpolation function h

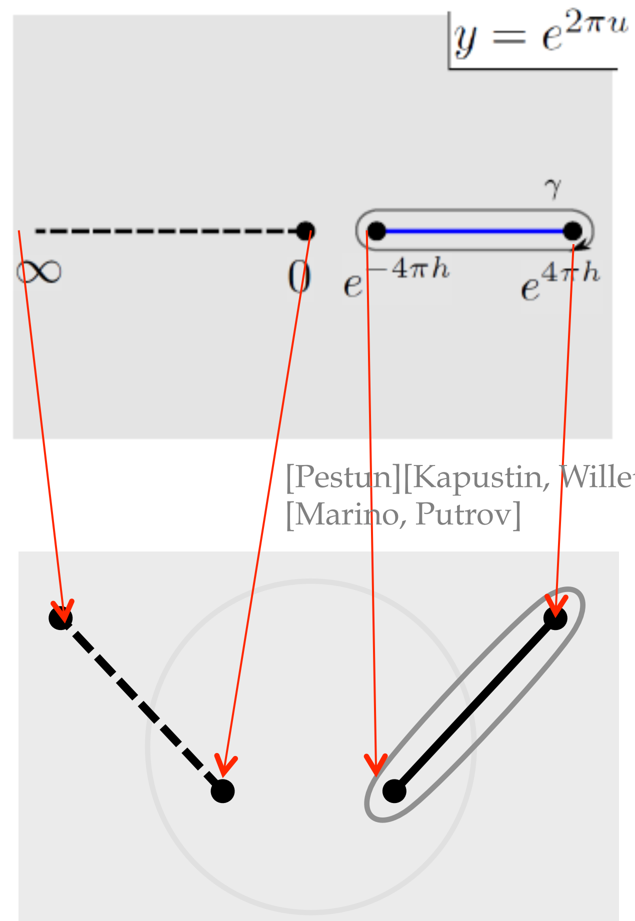
In the near BPS limit we should be able to match with localization

[N.G., Sizov]

Integrability:

Elliptic type integral





[Pestun][Kapustin, Willett, Yaakov]
[Marino, Putrov]

ABJM Matric model integral in its planar limit:

Localization:

$$\langle W^{1/6}_{m=1}\rangle = \int_{\frac{1}{A^+}}^{A^+} \frac{dZ}{2\pi^2 i\lambda}\arctan\sqrt{\frac{2+i\kappa-Z-\frac{1}{Z}}{2-i\kappa+Z+\frac{1}{Z}}}$$

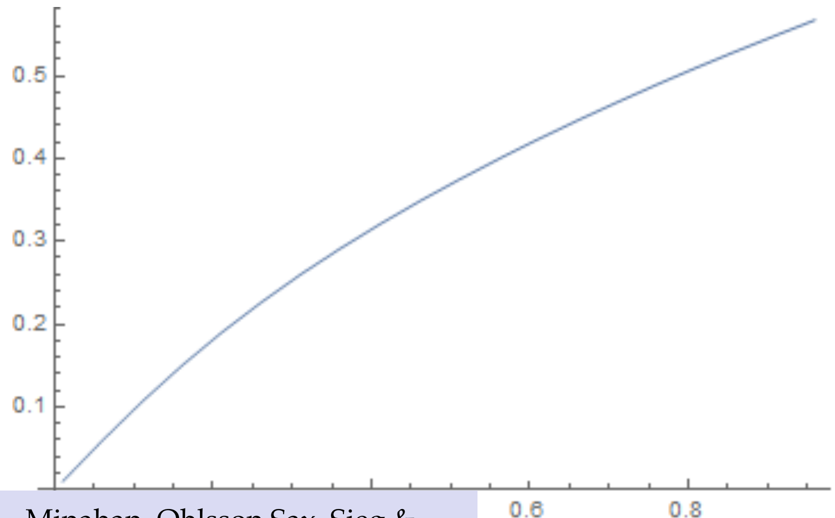
$$\lambda \;=\; \frac{\kappa}{8\pi}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};1,\frac{3}{2};-\frac{\kappa^2}{16}\right)$$

Comparing cross-ratios of the branch points:

$$\kappa = 4\sinh(2\pi h)$$

Interpolation function h

$$\lambda = \frac{\sinh(2\pi h)}{2\pi} {}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};1,\frac{3}{2};-\sinh^2(2\pi h)\right)$$



Minahan, Ohlsson Sax, Sieg & Leoni, Mauri, Minahan, Ohlsso Sax, Santambrogio, Sieg, Tartagli Mazzucchelli,

McLoughlin, Roiban Tseytlin Abbott, Aniceto, Bombardelli Lopez-Arcos, Nastase

Minahan, Zarembo

$$h(\lambda) = \lambda - \frac{\pi^2\lambda^3}{3} + \frac{5\pi^4\lambda^5}{12} - \frac{893\pi^6\lambda^7}{1260} + \mathcal{O}(\lambda^9),$$

Reproduces ~4 nontrivial coefficients!

$$h(\lambda) = \sqrt{\frac{\lambda}{2} - \frac{1}{48}} - \frac{\log 2}{2\pi} + \mathcal{O}\left(e^{-\pi\sqrt{8\lambda}}\right),$$

?

Bergman, Hirano

Conclusions

- QSC unifies all integrable structures: BFKL/ local operators, classical strings/spin chains.
- Mysterious relation between ABJM and N=4 SYM integrable structures. Sign for an unifying theory? What is QSC for AdS^3?
- Q-functions should give a way to the exact wave function in separated variables. Can we use it to compute general 3-point correlation functions to all loops?
- Established links between exact results in integrability and localization. Does there exist a unified structure which works for both non-BPS and non-planar? Discretization of Zhukovsky cut?