

Through a Glass Darkly

St Paul

Michael B. Green
University of Cambridge

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QUESTION:

WHAT IS A VISION TALK ?

This week we have seen a wider range of topics covered by “String Theory” than ever - and better talks than ever!

As time goes by and String Theory evolves, it is more and more apparent that it is not just a

Theory of String-like Elementary Particles

but is a

Magnificent theoretical framework that interrelates a very wide range of topics in physics and mathematics.

The unpredictable trajectory of String Theory since its inception is part of what makes our subject so exciting and so challenging –

BUT makes it impossible to foresee future developments – so there is limited “vision” in the following.

This talk is based partly on pestering colleagues for their views on questions and issues that might be advanced in a 5-year time frame.

The following GRANDEST QUESTIONS are easy to pose but have the most speculative answers - so I will leave them for others:

EARLY UNIVERSE – **The Multiverse; Inflation; Holographic Cosmology.**

The fate of BLACK HOLE INFORMATION - **Firewalls, EP=EPR,**

The STANDARD MODEL within String Theory,

In this short talk I will also have to ignore many fascinating topics:

....., localisation; dS holography; symmetries of Minkowski space gravity; Dualities of ABJM; Horizonless black holes; F-theory

HERE I WILL PICK SPECIFIC ISSUES IN A FEW THEMES

NOVEL USES OF NUMERICAL TECHNIQUES

CFT BOOTSTRAP.

Spectrum of finite N theories. $D=3$ Ising and $O(N)$ models; $D=4$ maximal SUSY Yang-Mills; A tool for probing the $D=6$ $(0,2)$ theory.

SUPERSYMMETRIC $SU(N)$ YANG-MILLS ON LATTICE.

e.g. Catterall, Damgaard, DeGrand, Giedt, Schaich

Lattice formulation of twisted maximal SYM $SU(N)$ gauge group.

Potential window into quantum gravity via holography – finite N .

HOLOGRAPHY AND CONDENSED MATTER

New arenas for Numerical Relativity - weird and wonderful boundary conditions breaking a variety of symmetries. Interface with novel techniques for cosmological computations.

CIRCLE OF IDEAS THAT AVOID SPACE-TIME: - a host of questions

ENTANGLEMENT, EMERGENCE, HIGHER SPIN, ON-SHELL AMPLITUDES.

This conference will be remembered for the extraordinary
CONFLUENCE OF IDEAS CONCERNING ENTANGLEMENT.

Novel tools from quantum criticality:

Entanglement Renormalization Group (MERA) and Holography.

Tensor networks. - Vidal, Swingle

Emergence of nonlinear Einstein theory?

Geometry from entanglement “EPR = ER”?

Generalisation of Ryu-Takayanagi entropy to higher spin theories?

Many other questions concerning HIGHER SPIN: In particular

Is String Theory a broken symmetry phase of an underlying theory?

SCATTERING AMPLITUDES

From Higgs production to number theory.

AMPLITUDES IN QUANTUM FIELD THEORY

Heroic calculations to high orders using various ON-SHELL TECHNIQUES that package together large classes of Feynman diagrams.

c.f. *Amplitudes 2014 (Paris, June 9 – 13)*

ADVANCES IN EXPLOITING INTEGRABILITY for $\mathcal{N} = 4$, $d = 4$ SYM: e.g.

NULL WILSON LOOPS: Towards exact planar scattering amplitudes.

esp. Basso, Sever, Vieira, Alday, Gaiotto, Maldacena

AMPLITUHEDRON. Expresses planar integrand in terms of mathematically intriguing simplex. - esp. Arkani-Hamed, Trnka, ...

But (How) are these related to each other?

Non-planar extensions? Gravity?

Relation to scattering equations? esp. Cachazo et al.
twistors, ambitwistors.

SYMBIOSIS WITH INTERESTING MATHEMATICS

Many ever-deepening connections between structure of amplitudes and recent mathematics:

periods; (motivic) Multi-Zeta Values, Goncharov polylogs, Chen iterated integrals, the Drinfeld associator, the Deligne associator, Hopf algebras,

Even more wonderful connections in String Theory amplitudes.

PERTURBATIVE STRING AMPLITUDES

Subtle interplay: Quantum field theory and string theory;
Low energy expansion and perturbation theory;
Ramond-Neveu-Schwarz and pure spinor.

Particular features: A single diagram at any order; No UV divergences.

LOW ENERGY EXPANSION.

Tree-level: Beautiful connections with mathematics (multiple polylogs,). esp. Mafra, Schlotterer, Stieberger Broedel,
Bjerrum-Bohr, Damgaard, Sondegaard, Vanhove,

Relation between open and closed strings has its math counterpart in
Single-valued multiple polylogarithms - Brown 2012

Systematics of expansion of HIGHER-GENUS string amplitudes?
Elliptic and higher genus multiple polylogs??

PURE SPINOR FORMALISM (Berkovits)

- Manifest SUSY; No world-sheet spinors – no spin structures
- Efficient low genus calculations – up to three loops.
- N-point functions of open and closed strings at genus zero and one.
- Ramond - Ramond backgrounds (in principle).

BUT it lacks a geometrical origin - created by magic!

Complicated expressions – specially composite b ghost.

Is the formalism well defined for all loops?

Formulate in an interesting RR background - $AdS_5 \times S^5$?

How is pure spinor formalism related to the RNS formalism??

Recent progress. Find a larger formalism that can be “gauge fixed” to either formalism.

More insight into connection between world-sheet supersymmetry and space-time SUSY.

FIELD THEORY LIMIT OF STRING PERTURBATION THEORY.

String perturbation theory is UV finite at all loops and contains all field theory diagrams from boundary of moduli space.

A fantasy:

Can one obtain multi-loop field theory amplitudes from string perturbation theory?

Use general features concerning degeneration limits of higher-genus super Riemann surfaces – Donagi, Witten,

Onset of UV divergences in SUGRA for a given number of loops ?

NON-PERTURBATIVELY, SUGRA is known not to be a limit of string theory.

STRING AMPLITUDES BEYOND PERTURBATION THEORY

NON-PERTURBATIVE DUALITIES constrain perturbative and non-perturbative features -

e.g. Non-renormalisation theorems for BPS operators in type II.

Coefficients of BPS interactions at low orders in type II in $(10 - n)$ dimensions are automorphic forms for type II duality groups

$SL(2, \mathbb{Z})$, $SL(2, \mathbb{Z})$, $SL(2, \mathbb{Z}) \times SL(3, \mathbb{Z})$, $SL(5, \mathbb{Z})$, $SO(5, 5, \mathbb{Z})$, $E_{6(6)}(\mathbb{Z})$, $E_{7(7)}(\mathbb{Z})$, $E_{8(8)}(\mathbb{Z})$

This involves specific **LANGLANDS EISENSTEIN SERIES** at rather magical values of their parameters. esp. Miller, Russo, Vanhove, Pioline, Obers, Kiritsis, MBG

Structure of higher order terms in low energy expansion, beyond BPS terms?

Further connections with number theory?

Recall different and wonderful connections with number theory:

In particular, the roles of **Ramanujan's MOCK MODULAR FORMS**

Mathieu moonshine for Mathieu group M24 **Eguchi, Ooguri, Tachikawa**
- generalisation of Monstrous Moonshine - a highlight of 1980's.

Umbral moonshine connects mock modular forms to all 23
Niemeier lattices. **Cheng, Duncan, Harvey, Gaberdiel, Volpato,**

Intriguing connections with BPS state counting.

Is this related to the occurrence of Mock modular forms in
counting of states of 1/4-BPS black holes in type II on $K3 \times T^2$?
Dabholkar, Murthy, Zagier.

Thanks for so many wonderful talks

THANKS FOR A WONDERFUL CONFERENCE!