

Holographic Lattices, Metals and Insulators

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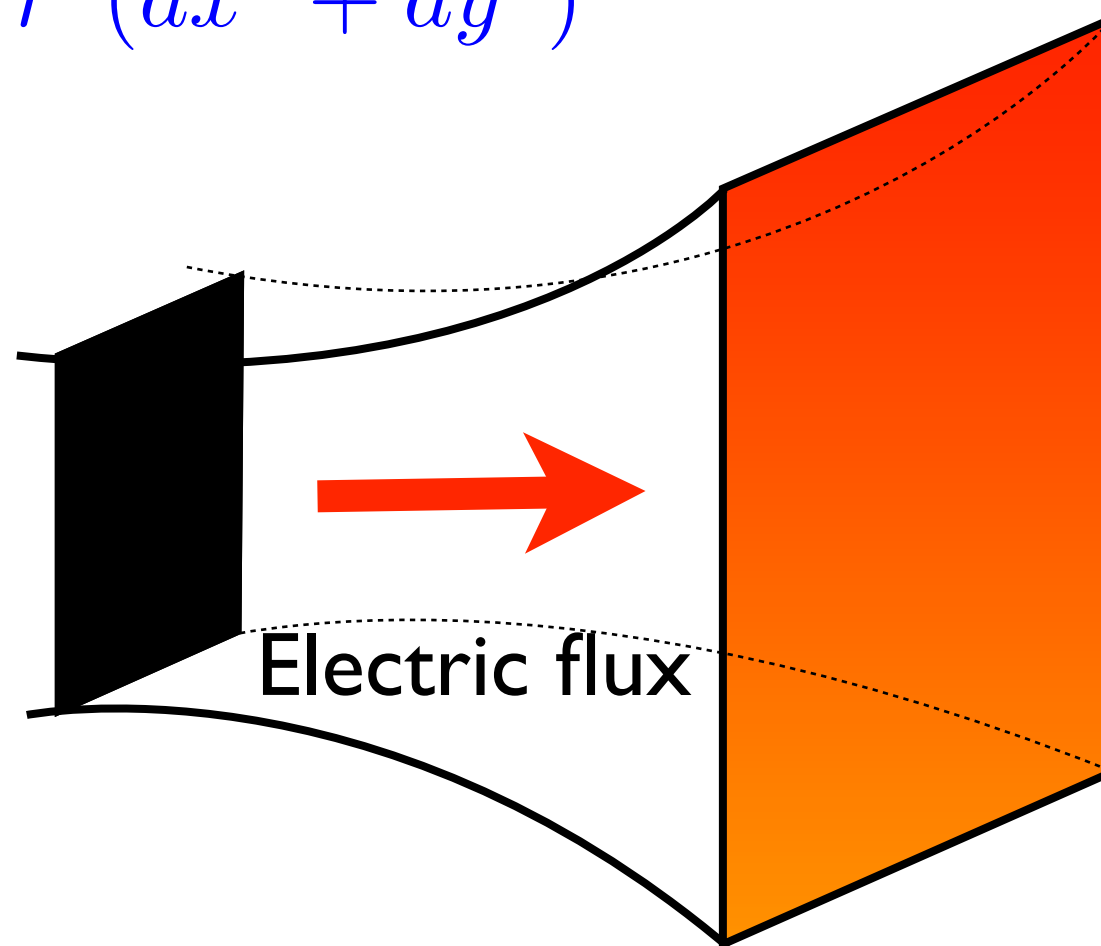
Aristomenis Donos

Electrically charged AdS-RN black hole (brane)

Describes holographic matter at finite charge density that is translationally invariant

$$ds^2 = -U dt^2 + \frac{dr^2}{U} + r^2(dx^2 + dy^2)$$

$$A_t = \mu \left(1 - \frac{r_+}{r}\right)$$



d=3 CFT

μ T

T=0 limit:

$AdS_2 \times \mathbb{R}^2$

IR



AdS_4

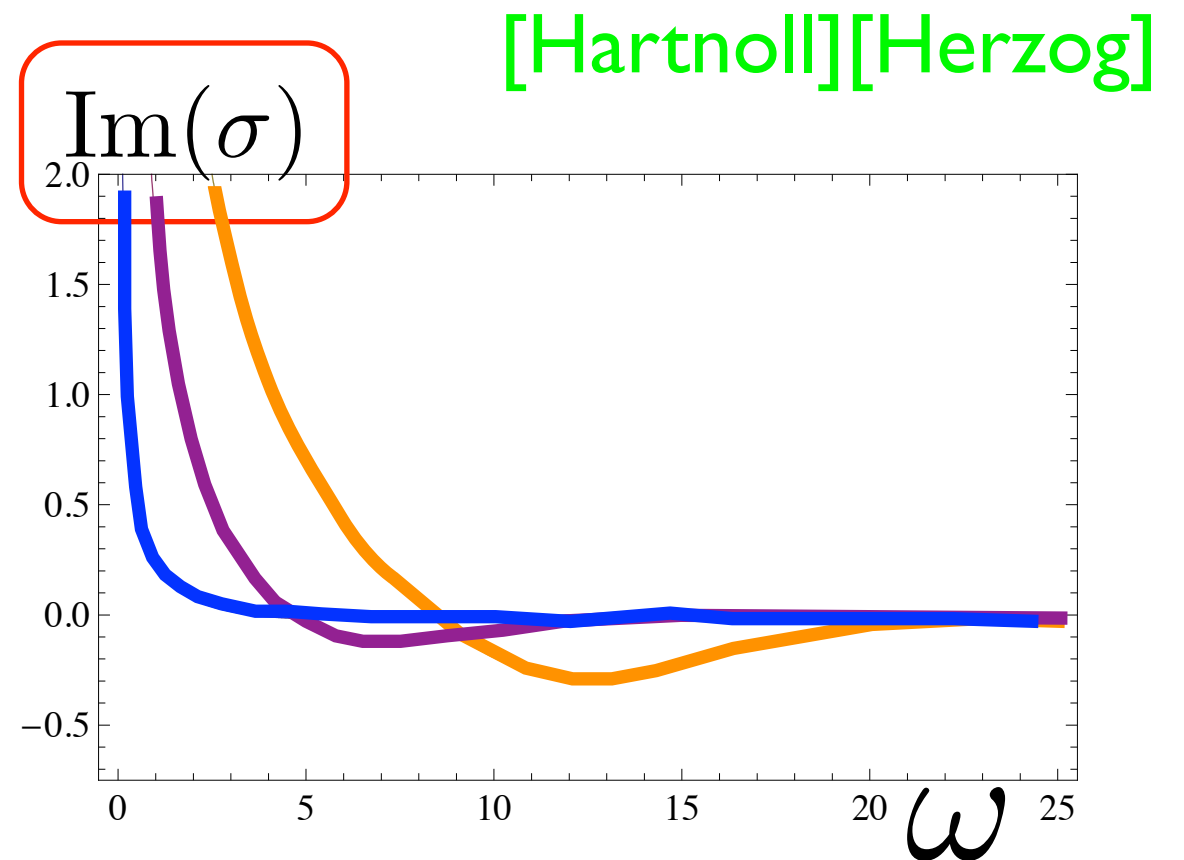
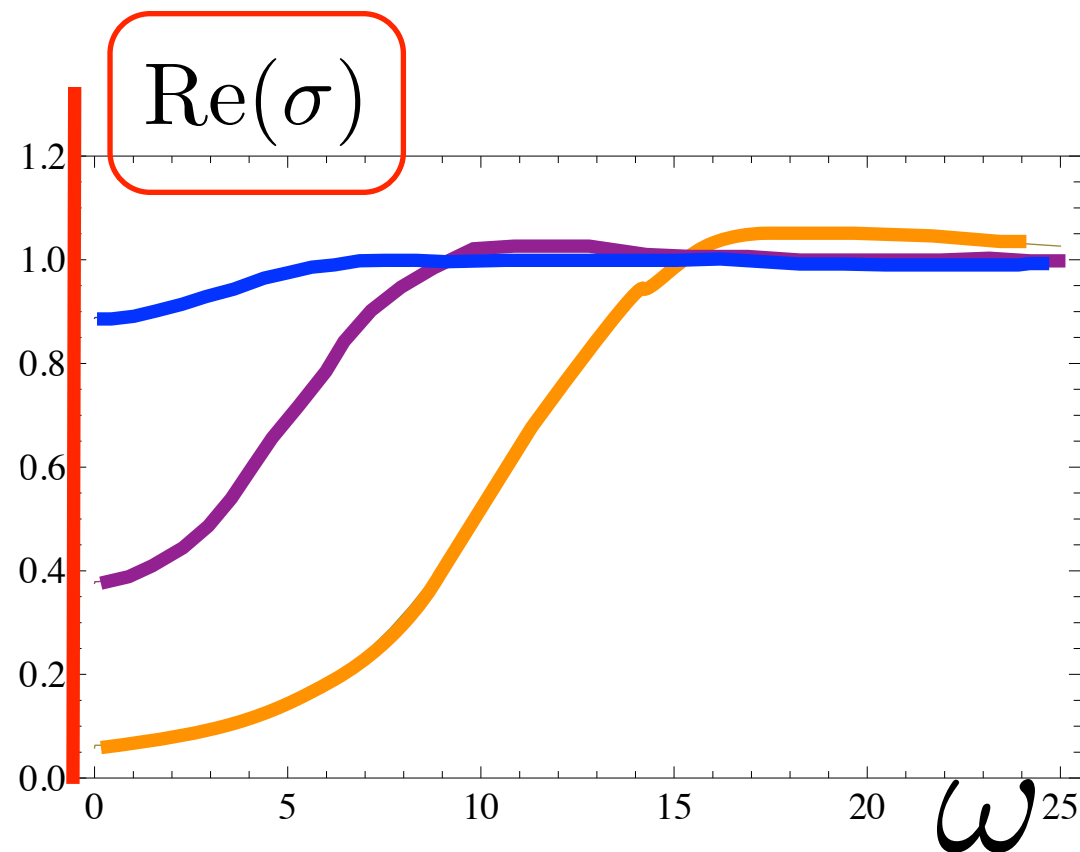
UV

Conductivity calculation

$$\sigma(\omega) = -i \frac{G_{J_x J_x}(\omega)}{\omega}$$

$$\delta A_x = e^{-i\omega t} a_x(r)$$

$$\delta g_{tx} = e^{-i\omega t} h_{tx}(r)$$



More precisely $\sigma(\omega) \sim \delta(\omega) + \frac{i}{\omega}$ near $\omega \sim 0$

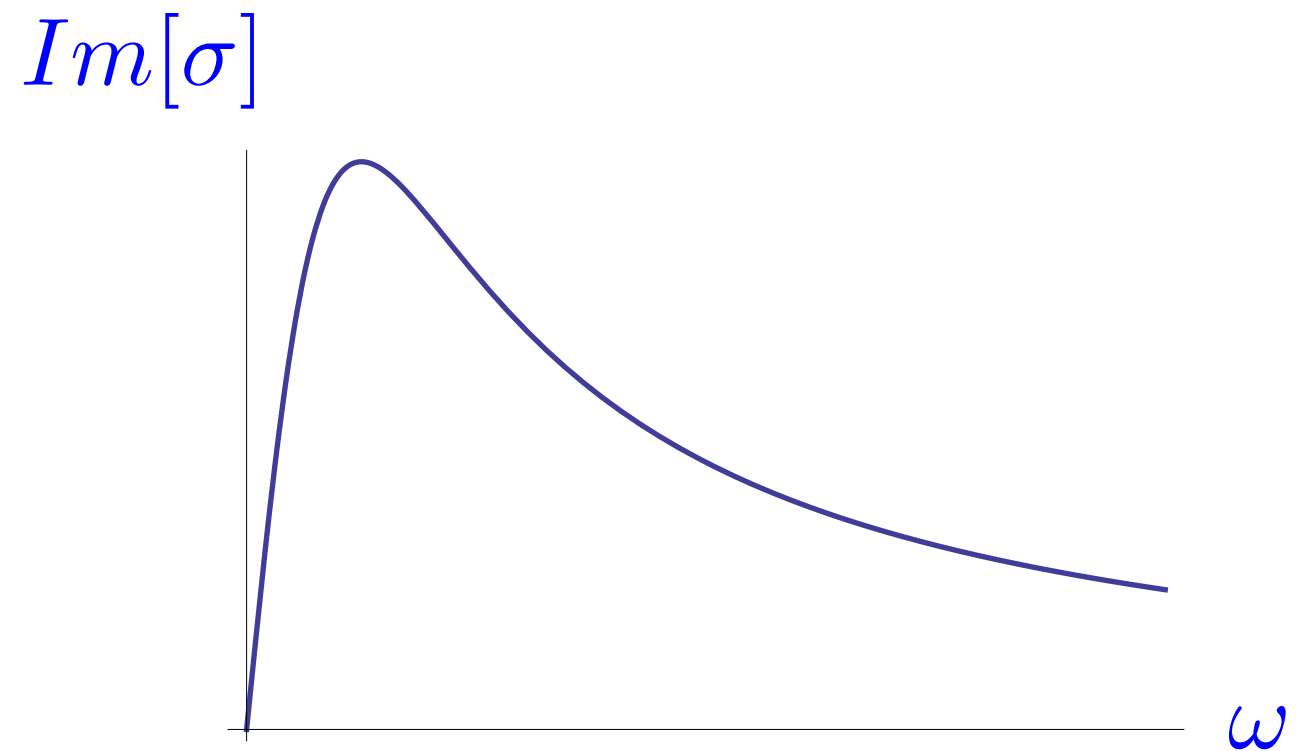
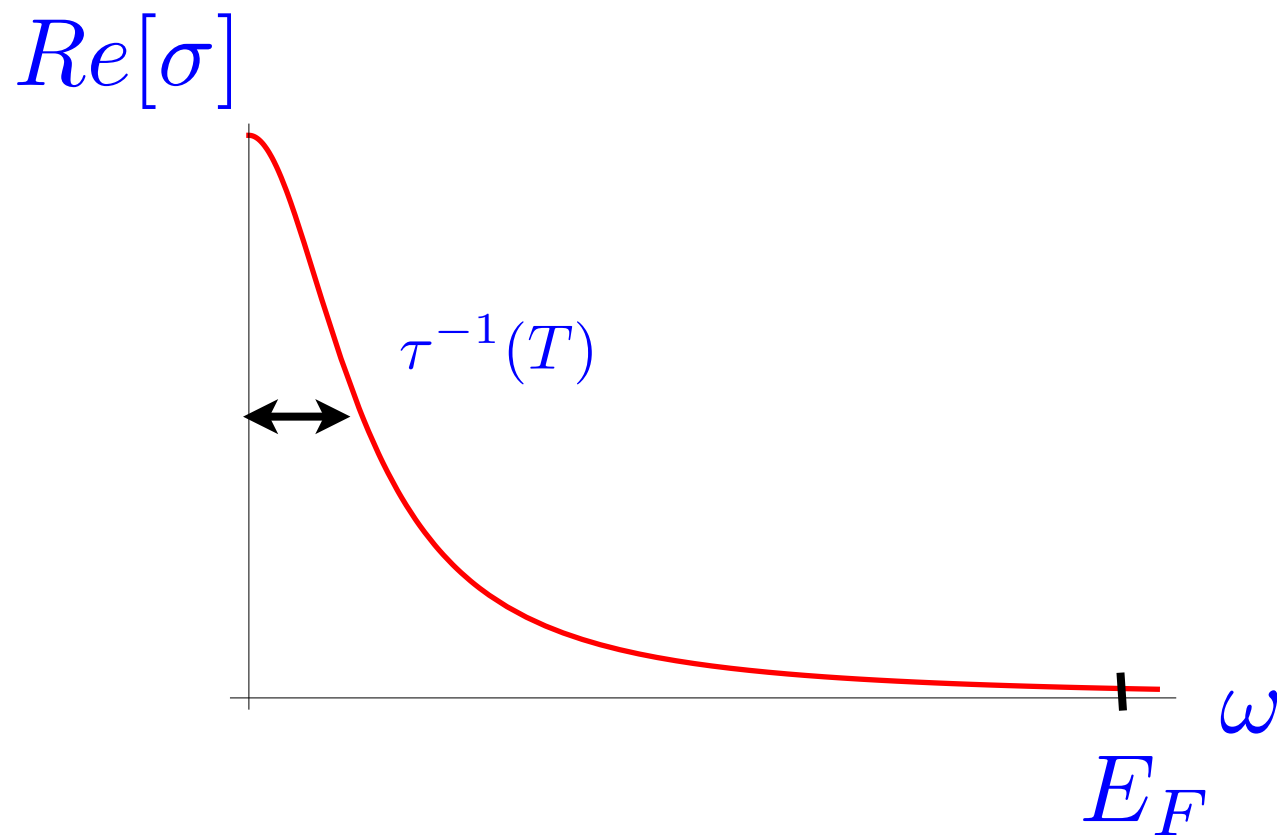
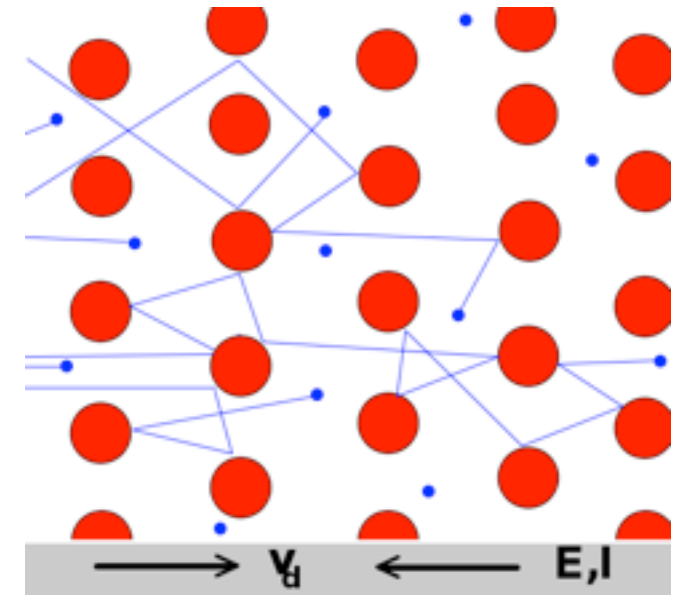
Infinite DC conductivity arises because translation invariance implies there is no momentum dissipation

Drude Model of transport in a metal
 e.g. quasi-particles and no interactions

$$J(\omega) = \sigma(\omega)E(\omega)$$

$$\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau}$$

$$\sigma_{DC} = \frac{nq^2\tau}{m}$$



“Coherent” or “good” metal

$$\tau \rightarrow \infty \quad \sigma(\omega) \sim \delta(\omega) + \frac{i}{\omega}$$

- Drude physics doesn't require quasi-particles

Arises when momentum is nearly conserved

Can be studied using “memory matrix” formalism

[Hartnoll, Hofman]

These coherent non Fermi-liquids can be modelled in holography

- There are also “incoherent” metals without Drude peaks
- Insulators with $\sigma_{DC} = 0$
- Metal-insulator transitions involve dramatic reorganisation of degrees of freedom

Want to study these within holography

Holographic Lattices and metals

To realise more realistic metals and/or insulators we want to consider charged black holes that explicitly break translations using a deformation of the CFT

A few examples of periodic monochromatic lattices have been studied [Horowitz, Santos, Tong]

E.g. add a real scalar field to Einstein-Maxwell and consider

$$\phi(r, x) \sim \frac{\lambda \cos(kx)}{r^{3-\Delta}} + \dots$$

Need to solve PDEs

Can we simplify? Find some agreement and some differences

Plan

- Holographic Q-lattices - solve ODEs

(Aside: D=5 helical lattices [Donos,Hartnoll][Donos,Gouteraux,Kiritsis])

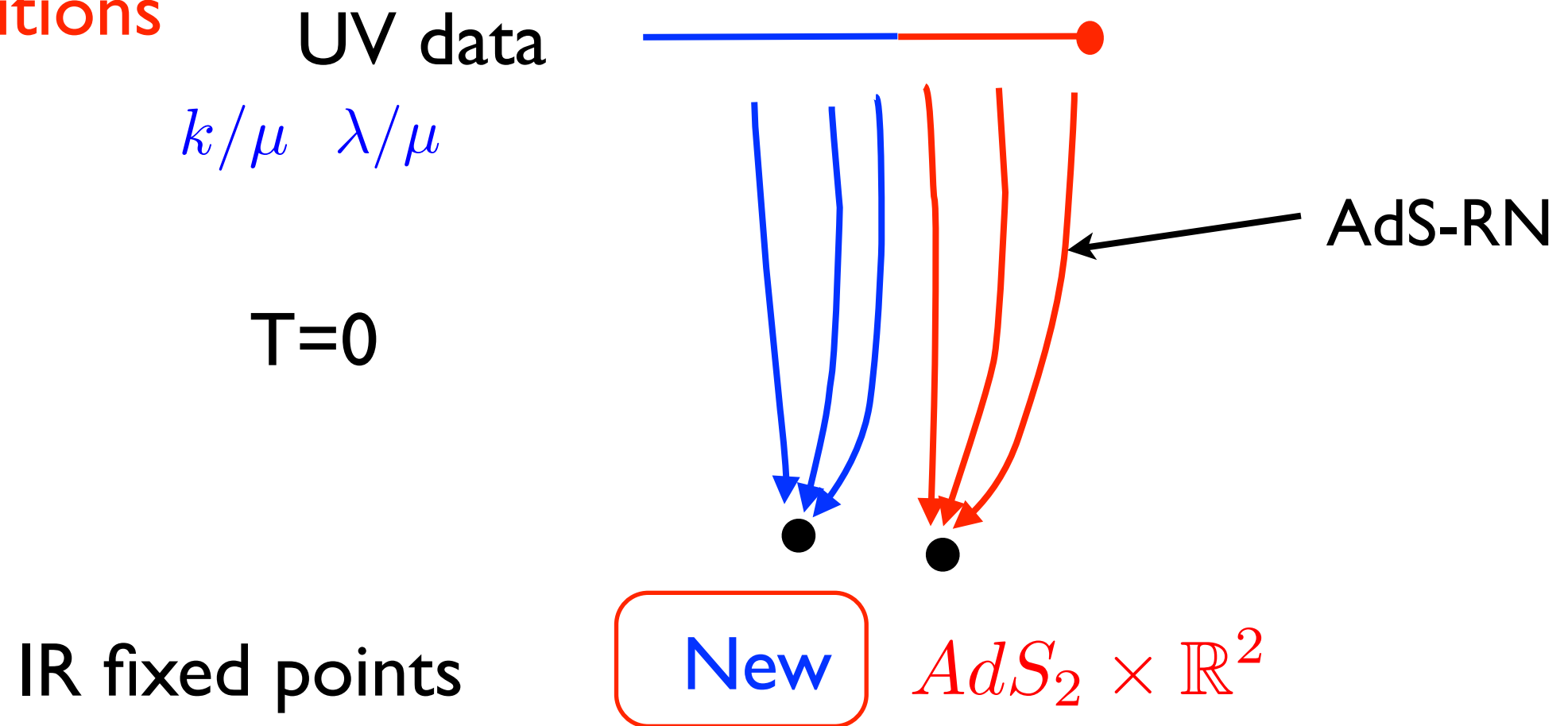
- Calculation of thermoelectric DC conductivity σ_{DC} , α_{DC} , $\bar{\kappa}_{DC}$ in terms of black hole horizon data

Analogous to $\eta = \frac{s}{4\pi}$ [Policastro,Kovtun,Son,Starinets]

For σ_{DC} c.f. [Iqbal,Liu][Davison][Blake,Tong,Vegh][Andrade,Withers]

- Q-lattices can give coherent metals, incoherent metals and insulators and transitions between them.

Holographic transitions



Coherent metals with Drude peaks
[Hartnoll,Hofman][Horowitz,Santos,Tong]

Strategy: vary UV data aiming to RG destabilise $AdS_2 \times \mathbb{R}^2$

First examples **M-I** transitions in D=5 helical lattices [Donos,Hartnoll]

We will find new **M-I** transitions as well as **M-M** transitions

Comment: not necessary that $AdS_2 \times \mathbb{R}^2$ is RG destabilised

Holographic Q-lattices

- Illustrative D=4 model

$$\mathcal{L} = R - \frac{1}{2} [(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2] + V(\phi) - \frac{Z(\phi)}{4} F^2$$

- Choose Φ, V, Z so that we have an AdS_4 vacuum and that AdS-RN is a solution at $\phi = 0$

- Particularly interested in cases where χ is periodic.

eg if it is the phase of a complex scalar field $\varphi = \phi e^{i\chi}$

with $\Phi = \phi^2$

Analysis covers cases when χ is not periodic e.g.

[Azeneyagi, Takayanagi, Li][Mateos, Trancanelli][Andrade, Withers]

- The model has a gauge $U(1)$ and a global $U(1)$ symmetry
Exploit the **global** bulk symmetry to break translations

Ansatz for fields

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx_1^2 + e^{2V_2} dx_2^2$$

$$A = a(r) dt$$

$$\chi = kx_1, \quad \phi = \phi(r)$$

UV expansion:

$$U = r^2 + \dots, \quad e^{2V_1} = r^2 + \dots, \quad e^{2V_2} = r^2 + \dots$$

$$a = \mu + \frac{q}{r} \dots, \quad \phi = \frac{\lambda}{r^{3-\Delta}} + \dots$$

IR expansion: regular black hole horizon

Homogeneous and anisotropic and periodic holographic lattices

$$\text{UV data: } T/\mu \quad \lambda/\mu^{3-\Delta} \quad k/\mu$$

Analytic result for DC in terms of horizon data

Apply electric fields and thermal gradients and find linear response

Generalised Ohm/Fourier Law:

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

J^a

Electric current

$$Q^a = T^{ta} - \mu J^a$$

Heat current

For Q-lattice black holes the DC matrices $\sigma, \alpha, \bar{\alpha}, \bar{\kappa}$ diagonal

- Calculating σ and $\bar{\alpha}$

Switch on constant electric field perturbation

$$A_x = -Et + \delta a_x(r)$$

supplemented with $\delta g_{tx}(r)$ $\delta g_{rx}(r)$ $\delta \chi(r)$

Gauge equation of motion:

$$\nabla_\mu (Z(\phi) F^{\mu\nu}) = 0 \quad \Rightarrow \quad \partial_r (\sqrt{-g} Z(\phi) F^{rx}) = 0$$

$$J = -e^{V_2 - V_1} Z(\phi) U \delta a'_{x_1} + q e^{-2V_1} \delta g_{tx_1}$$

Use Einstein equations and regularity at the black hole horizon to relate J and E to get σ

Perturbed metric has a timelike Killing vector k^μ

$$G^{\mu\nu} = \nabla^\mu k^\nu + \dots \quad \Rightarrow \quad \nabla_\mu G^{\mu\nu} = -\frac{V}{2} k^\mu$$

Similar steps then relate Q and E to get $\bar{\alpha}$

- Calculating α and $\bar{\kappa}$

Consider a source for electric and heat currents

$$g_{tx} = t\delta f_2(r) + \delta g_{tx_1}(r)$$

$$A_x = t\delta f_1(r) + \delta a_x(r)$$

Similar steps, with a subtlety that there is both a static and time-dependent heat current

$$\mathcal{L} = R - \frac{1}{2} [(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2] + V(\phi) - \frac{Z(\phi)}{4} F^2$$

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx_1^2 + e^{2V_2} dx_2^2$$

$$\chi = kx_1 \quad A = a dt$$

$$\alpha_{DC} = \bar{\alpha}_{DC} = - \left[\frac{4\pi q}{k^2 \Phi(\phi)} \right]_{r=r_+} \quad \bar{\kappa}_{DC} = \left[\frac{4\pi s T}{k^2 \Phi(\phi)} \right]_{r=r_+}$$

$$\sigma_{DC} = \left[e^{-V_1+V_2} Z(\phi) + \frac{q^2 e^{-V_1-V_2}}{k^2 \Phi(\phi)} \right]_{r=r_+}$$

First term in σ is finite for AdS-Schwarzschild [Iqbal,Liu]

“Pair creation” term. In general it is $(J/E)_{Q=0} \equiv \sigma - \alpha^2 \bar{\kappa}^{-1} T$

Second term “Dissipation” term

Different ground states can be dominated by first or second term

- Some general results

Define thermal conductivity at zero current

$$\kappa = \bar{\kappa} - \alpha \bar{\alpha} T / \sigma$$

For dissipation dominated $T=0$ ground states κ and $\bar{\kappa}$ can have different low temperature scaling (n.b. $\kappa = \bar{\kappa}$ for FL)

- $$\bar{L} \equiv \frac{\bar{\kappa}}{\sigma T} \leq \frac{s^2}{q^2}$$

Bound is saturated for dissipation dominated systems

c.f. Wiedemann-Franz Law.

Complementary result using memory matrix [Mahajan, Barkeshli, Hartnoll]

- $$\frac{\bar{\kappa}}{\alpha} = -\frac{T s}{q}$$

Coherent metal phases

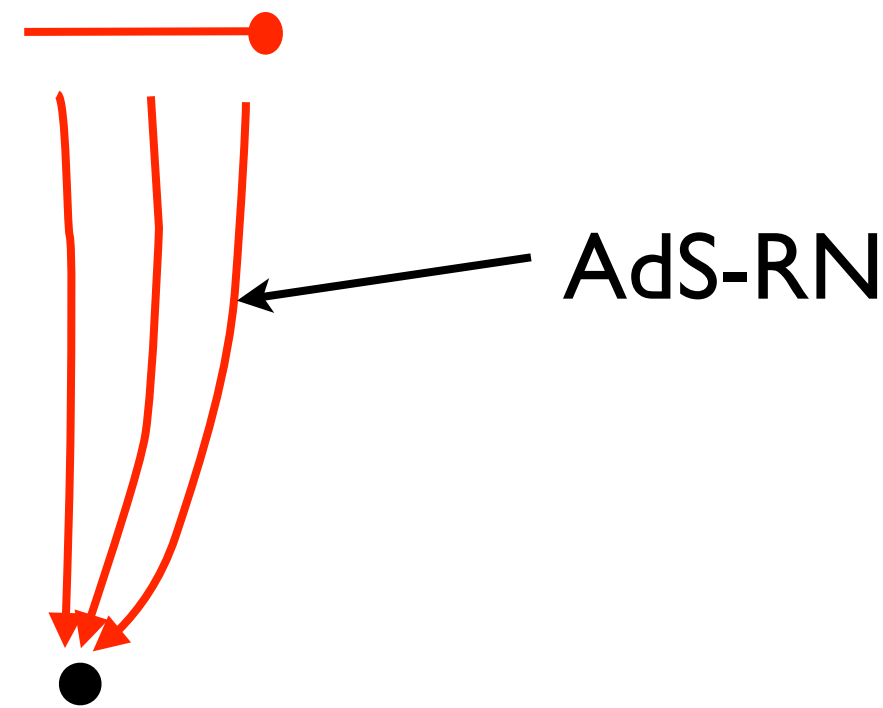
Dissipation dominated

UV data

T=0

IR fixed point

$AdS_2 \times \mathbb{R}^2$



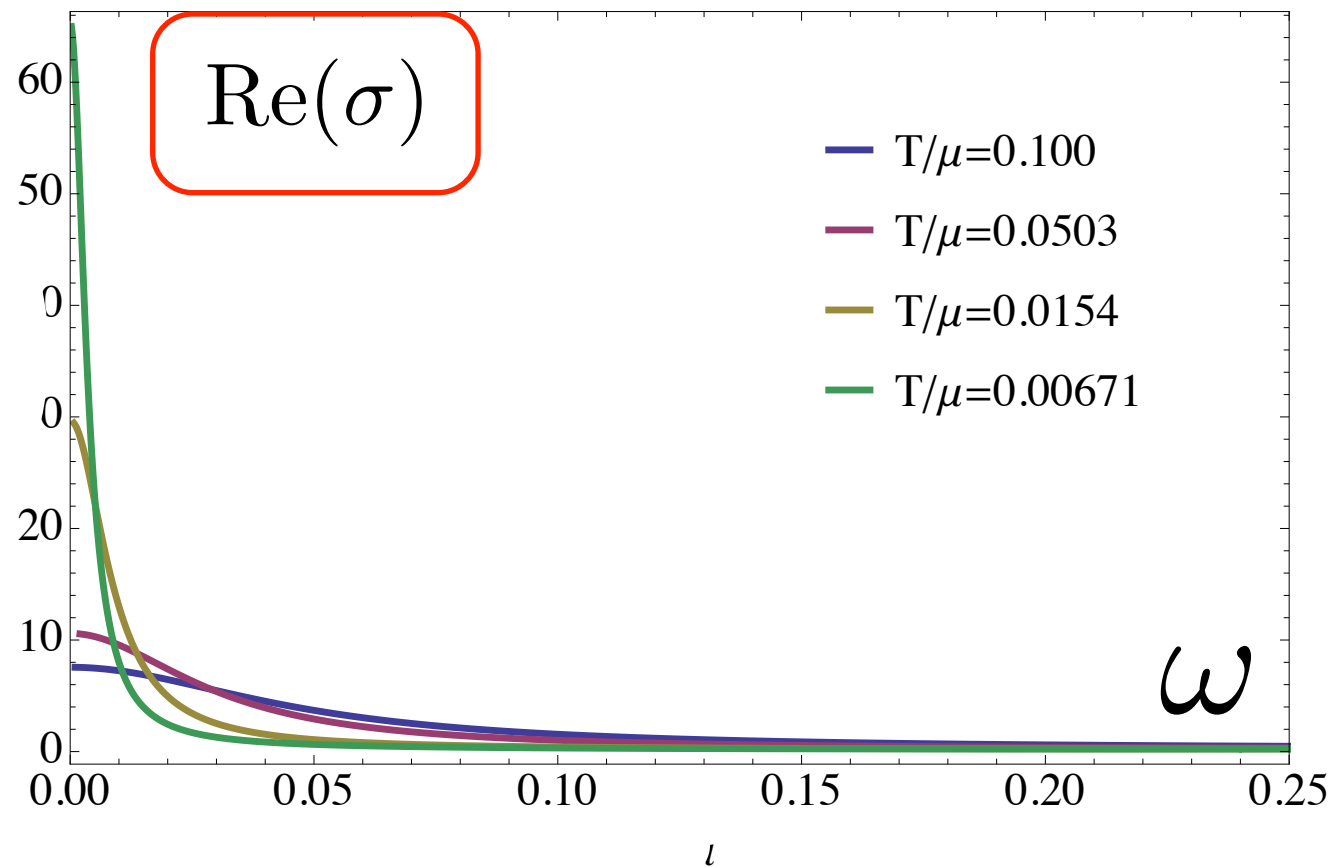
At T=0 the black holes approach $AdS_2 \times \mathbb{R}^2$ in the IR
perturbed by irrelevant operator [Hartnoll, Hoffman]

$$\sigma \sim T^{2-2\Delta(k_{IR})}$$

note: k_{IR} depends on RG flow

Always have $\kappa \sim T$ but $\bar{\kappa} \sim T^{3-2\Delta(k_{IR})}$ and $\bar{\kappa} \rightarrow 0, \infty$

Drude peaks at finite T



Similar to what was seen by [Horowitz,Santos,Tong]

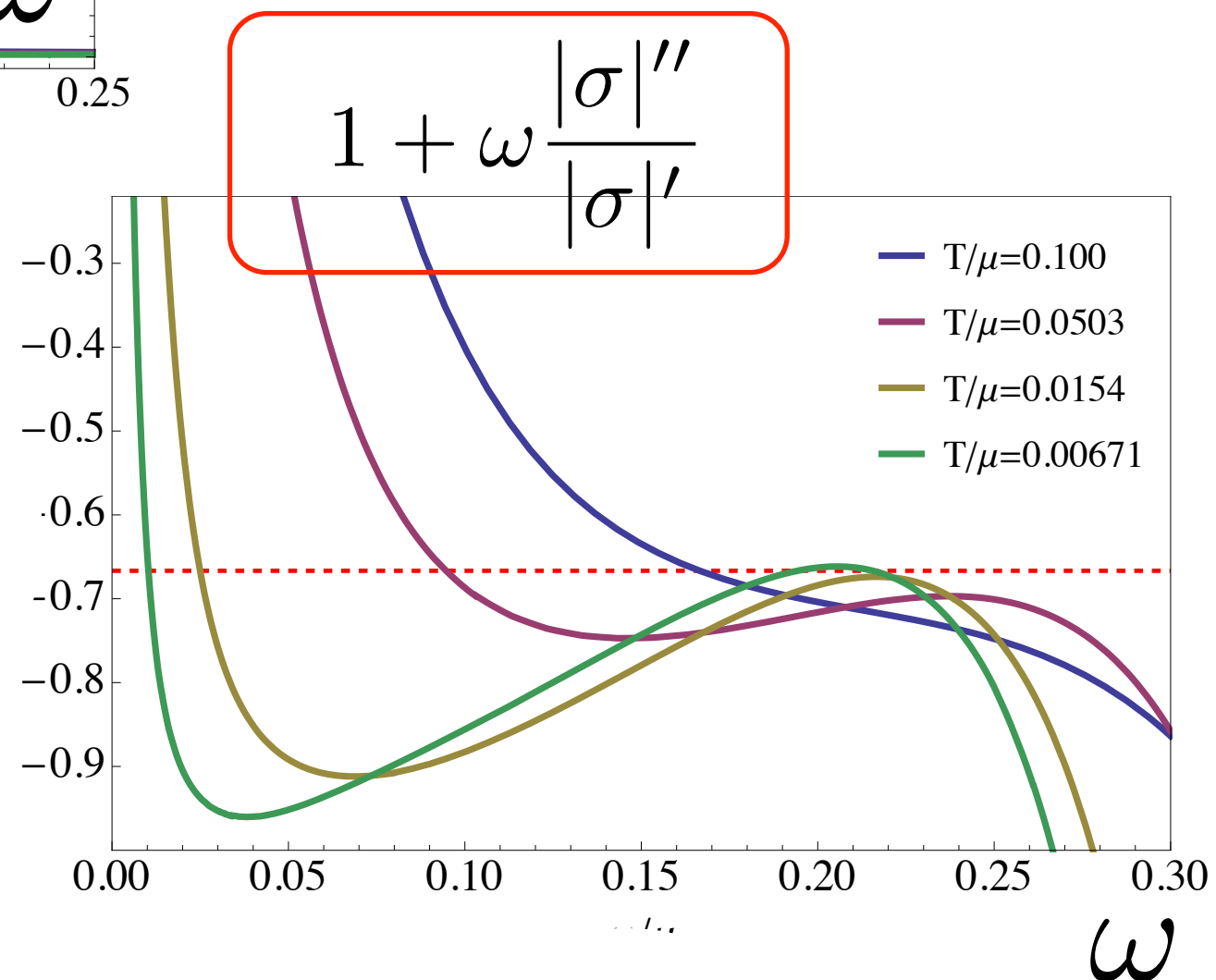
Intermediate scaling?

[Horowitz,Santos,Tong]

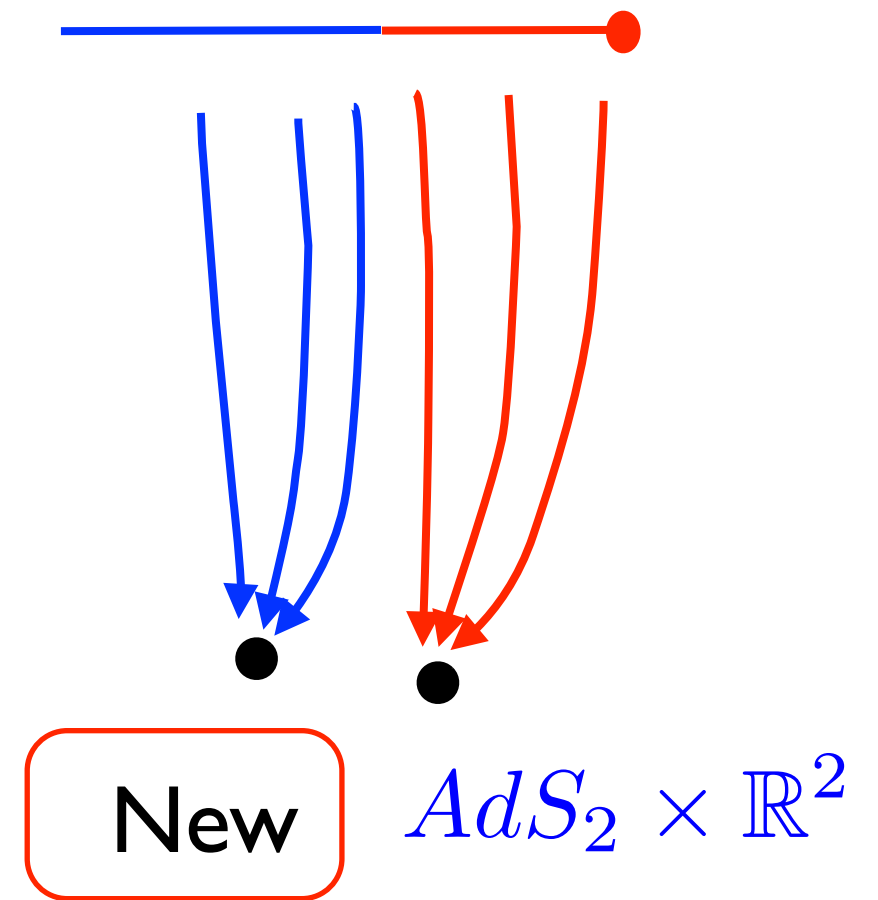
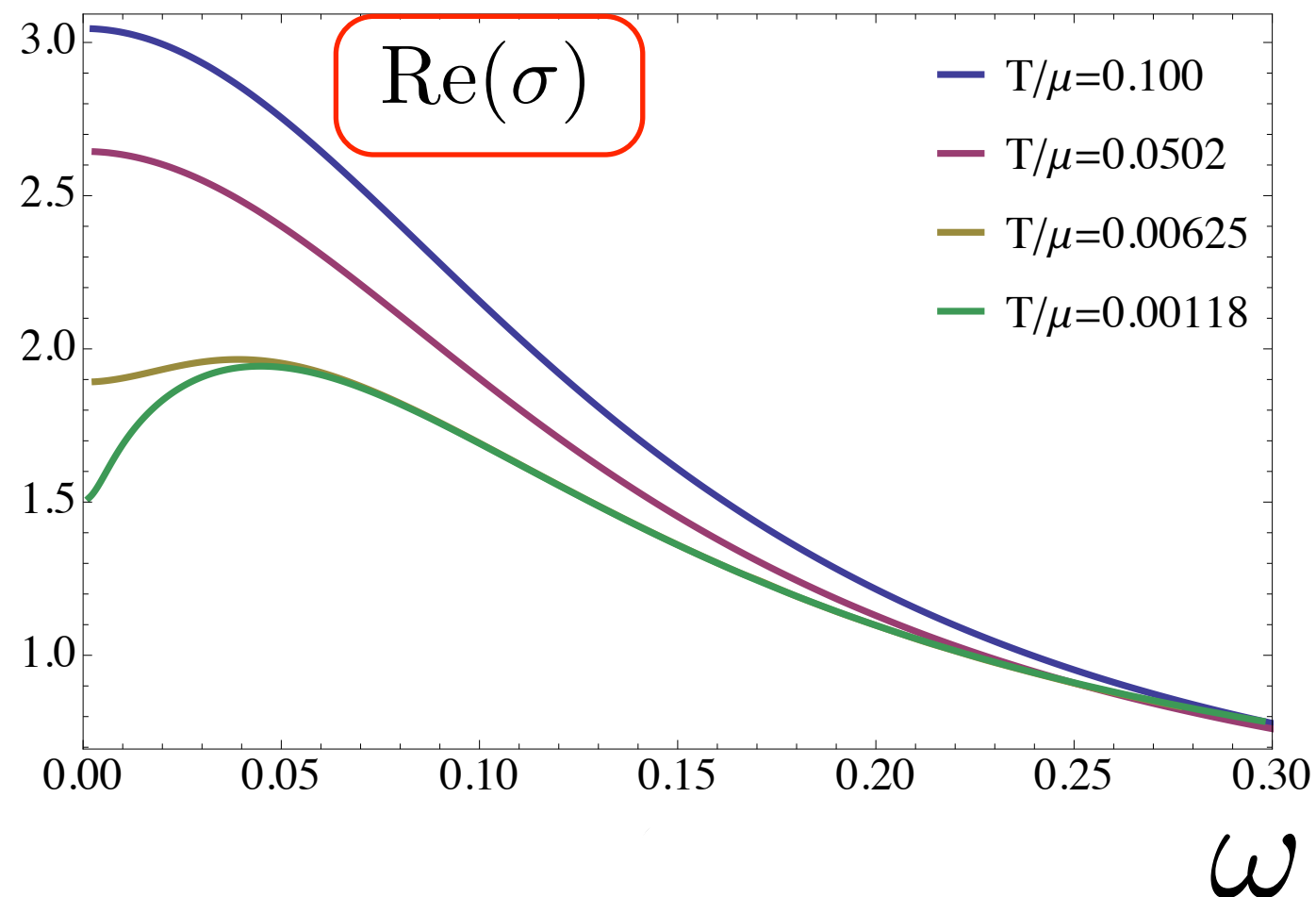
$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

reminiscent of cuprates

Scaling is not universal



Insulating phases



Appearance of a mid-frequency hump.

Spectral weight is being transferred, consistent with sum rule

What are the $T=0$ insulating ground states??

Focus on specific models (see also [\[Gouteraux\]](#))

New Insulating and Metallic ground states - Anisotropic

$$\mathcal{L} = R - \frac{1}{2} [(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2] + V(\phi) - \frac{Z(\phi)}{4} F^2$$

Focus on models and $T=0$ ground states which are solutions with $\phi \rightarrow \infty$ as $r \rightarrow 0$

and
$$\mathcal{L} \rightarrow R - \frac{3}{2} [(\partial\phi)^2 + e^{2\phi}(\partial\chi)^2] + e^\phi - \frac{e^{\gamma\phi}}{4} F^2$$

IR “fixed point” solutions

$$ds^2 \sim -r^u dt^2 + r^{-u} dr^2 + r^{v_1} dx_1^2 + r^{v_2} dx_2^2$$

$$e^\phi \sim r^{-\phi_0} \quad A \sim r^a dt \quad \chi = kx_1$$

with exponents fixed by γ

- Calculate AC conductivity

Obtained using a matching argument [Faulkner,Liu,McGreevy,Vegh] with ground state correlators at $T=0$. Valid when $T \ll \omega \ll \mu$

$$\sigma_{AC} \sim \omega^{c(\gamma)}$$

- Calculate DC conductivity using analytic formula

For $T \ll \mu$ the scaling is obtained from the IR fixed point solutions

$$\sigma_{DC} \sim T^{b(\gamma)}$$

In these models we have $b = c$
(as we have for the $AdS_2 \times \mathbb{R}^2$ coherent metals)

$$\sigma_{DC} \sim T^{b(\gamma)} \quad \sigma_{AC} \sim \omega^{c(\gamma)}$$

$$b = c > 0$$

Have new type of insulating ground states

$$b = c < 0$$

Have new type of incoherent metallic ground states not associated with Drude physics

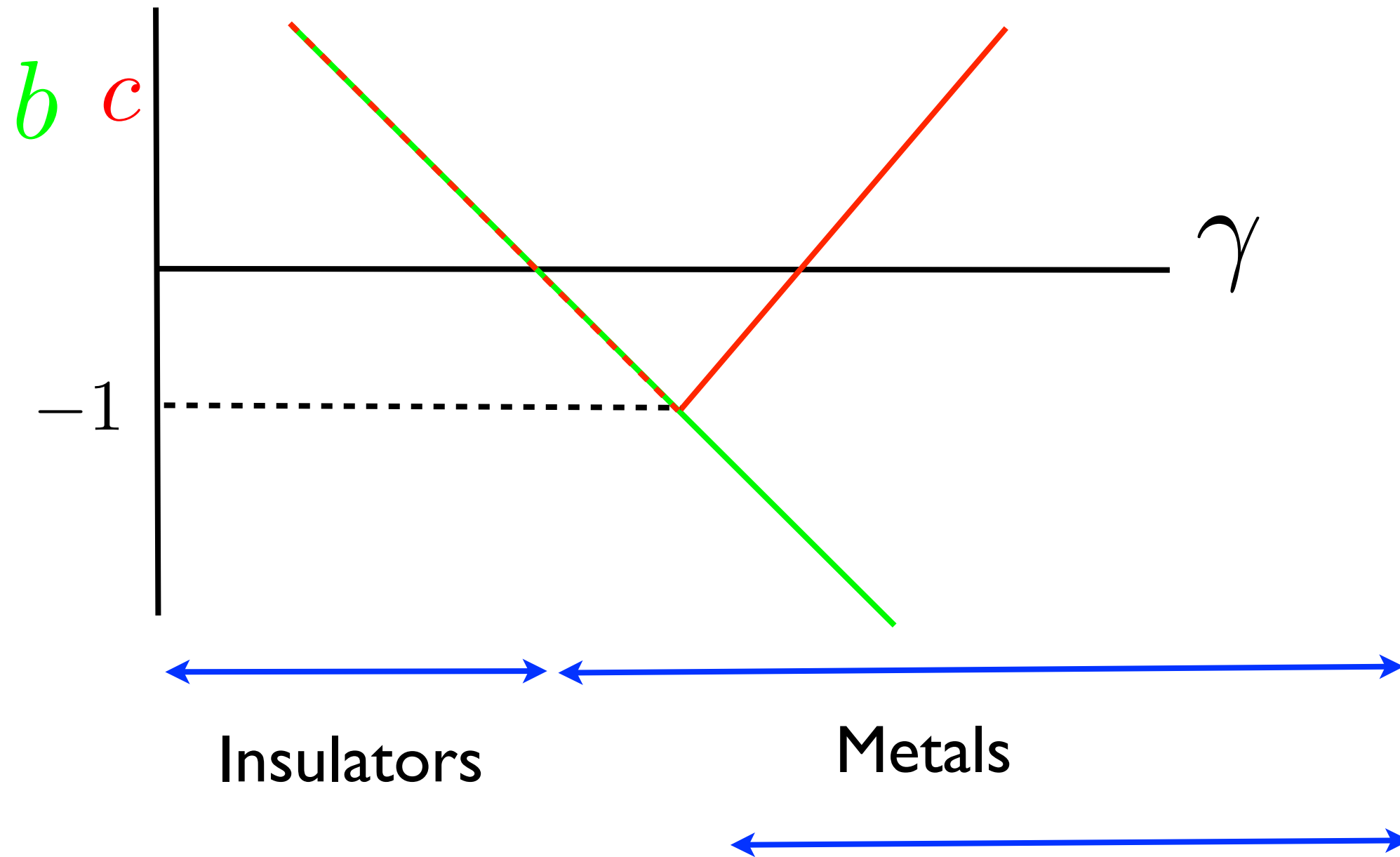
$$b = c = 0$$

Novel metallic ground states with finite conductivity at $T=0$

Metallic ground states are all thermal insulators: $\bar{\kappa} \rightarrow 0$
electric conductivity is due to “pair creation”

New Insulating and Metallic ground states - Isotropic (χ_1 and χ_2)

$$\sigma_{DC} \sim T^b \quad \sigma_{AC} \sim \omega^c$$



Reappearance of sharp peaks not related to the charge density and Drude physics

Summary

- Holographic Q-lattices are simple and illuminating
- Analytic result for DC conductivity in terms of horizon data.
Can be generalised to inhomogeneous lattices [to appear]
- No intermediate $2/3$ scaling in AC conductivity
Absent in another recent example [Taylor, Woodhead]
We find it to be absent in inhomogeneous lattices [to appear]
- Coherent metallic phases with Drude peaks
- Also find novel metallic phases and insulating phases
Metal-Insulator and Metal-Metal transitions