Topological properties of N=1 SYM

Strings 2014

SU(N) Super Yang Mills

- Gauge fields + adjoint Majorana fermion (gaugino)
- Four supercharges
- Confinement
- Spontaneously chiral symmetry breaking

Symmetry breaking

- R-symmetry: naive U(1) rotating fermions
- Anomaly: $U(1) \to Z_{2N}$
- Gaugino bilinear vev: $\langle \lambda^\alpha \lambda_\alpha \rangle = \Lambda^3 e^{2\pi i \frac{k}{N}}$
- N vacua: $Z_{2N} \to Z_2$

BPS domain walls

- Acharya-Vafa:
  - (n+k,n) domain wall supports Chern Simons
    - Supersymmetric $U(k)_N$ CS theory
- Topological order!
  - SUSY alone cannot enforce that

SYM as topological phase

- Bulk vacua have no long range entanglement
- It could be a SPT phase:
  - Global symmetry
  - Bulk theory with ’t Hooft anomaly on boundaries
  - Boundary dof required to cancel it.

Exotic SPT phase

- Kapustin,Thorngreen; Aharony, Seiberg, Tachikawa; Gukov, Kapustin, …
- SU(N) SYM has a one-form flavor symmetry.
- Vacua can be SPT phases for that.
- $U(k)_N$ has appropriate t'Hooft anomaly!
  - Dierigl, Pritzel

Boundary conditions

- N=1 SYM has BPS boundary conditions
  - Dirichlet, Neumann, Neumann+matter, etc.
- Some (Neumann) preserve one-form symmetry
  - Low energy topological order on boundaries?

Borrow from 2d

- SU(N) SYM on two-torus = (2,2) CP^{N-1} σ-model
- Both have N vacua
- Domain walls = BPS solitons
- Boundaries = Branes

Domain walls 4d/2d

- 2d theory has $\binom{N}{k}$ BPS solitons
  - k-th antisymmetric of SU(N) flavor
- Same as vacua of $U(k)_N$ on two-torus!

  Acharya-Vafa
- Branes in 2d have computable ground states
  - Predicts nr. vacua of 4d boundary condition

Boundary conditions

- 4d Neumann = 2d Neumann
  - Chern Simons level n= magnetic flux on brane
- ground states:
  - “left” vacua: (n+k)-th symmetric of SU(N) flavor
  - “right” vacua: L-shaped of SU(N) flavor

Tentative matching

- left vacua: $SU(N)_{n+k}$
- right vacua: unknown 3d TFTs?
  - "bound state" of $SU(N)_n$ and $U(k)_N$

How to identify 3d TFTs?

- Domain walls have junctions: $\frac{U(N)_N}{\prod_a U(n_a)_N}$
    - Elliptic genus matches 2d junction calculation
- In 2d we compute boundary-domain wall junctions
    - Information about 3d TFT? Modular matrix?

Conclusions

- Massive phases of SUSY gauge theories may have intricate topological properties

- SUSY theories may give concrete examples of novel topological phases.