A Quantum Model Of Black Hole

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Building and solving quantum models of BHs is an outstanding open problem. It is a crucial endeavor and probably the only way to address consistently the fundamental questions, in a field that desperately lacks equations, as Joe Polchinski put it on tuesday.

My goal will be to very briefly review a work in progress that seems extremely promising and allows to tackle, in an extremely and surprisingly simple way (in view of the complexity of the problems), some of the deep questions of BH physics:

- How does the quasi-normal behavior emerges?
- How is unitarity restored?
- How can we describe a local probe falling into the horizon?

Hurdles that we need to pass to answer these questions are as follows:
- What kind of QM models can describe a BH?
- How can we find a way to reliably compute in these models the observables that are relevant to describe the fundamental properties of BHs (like the quasi-normal behavior, unitarity restoration, etc)? (very hard in principle)
- How can we study local probes and find the answers we seek from a calculation (and not simply an educated guess)?

The models

They are suggested by holography. A basic model would be the N=4 SYM theory on a spatial 3-sphere of radius R. Doing the dimensional reduction on the three-sphere, we get a gauged QM, with gauge group U(N) and an infinite number of adjoint variables.
BHs can be obtained with only a finite number of adjoint variables (that's what happens e.g. for the D0 brane QM).
So a typical model we consider is a gauged QM with adjoint variables and an action of the form

$$\int \mathrm{d}t \ \mathrm{tr}\Big[\big(\mathcal{D}X\big)^2 + \omega^2 X^2 + \text{interactions} + \text{fermions}\Big]$$

where X is a set of bosonic matrix variables, interactions terms can be the usual commutator square term $[X,X]^2$, X^4 terms, etc., and fermion terms can be added as for D0 branes.

Computing: “easy”

a) Partition function: OK at weak coupling, by reducing the problem to a unitary matrix model corresponding to the holonomy along the thermal circle (Sundborg 1999, Aharony et al. 2005). Find Hagedorn phase transition, plausibly related to the Hawking-Page transition at strong coupling (at least in some cases). But the partition is far from being enough to derive the essential features of BH physics.

Computing: hard

b) A fundamental set of observables to consider are real time thermal correlators. One needs a reliable way to compute these at large N and at very long times, in order to see the emergence of an arrow of time (the quasi-normal behavior of BHs), which is nothing but thermalization in the strongly coupled gauged QM. One also needs a way to understand these correlators at finite N, in order to see how unitarity is restored.

Such calculations are notoriously difficult and no example is known in a full-fledged gauged matrix QM model (see however Iizuka, Okuda, Polchinski 2008, 2009 for interesting toy models).
Difficulty: we need a non-perturbative resummation (the quasi-normal behavior is not seen at any finite order of perturbation theory, even at weak coupling!, see Festuccia and Liu 2006); and we need finite N to study unitarity restoration!

c) Local probe in the bulk (modeling a “local observer going through the horizon”): even the definition of such an object is non-trivial.

Hint at a possible breakthrough:

Emparan et al. (2013, 2014) have shown very recently that there exists a well-defined large D limit of classical GR (D is the number of space dimensions), which captures, in particular, the essential features of BH physics, including the quasi-normal behavior.

This is ringing a bell! From the point of view of the matrix QM, the large D limit is the limit of a large number of matrices. Might seem complicated at first sight, but actually this limit is well-behaved and easy to study.

Typically, we can arrange the variables into SO(D) vectors $\vec{X}$ of N times N matrices and the large D limit can be treated along the lines of the large n limit of O(n) vector models. There are a couple of *fundamental* differences due to the fact that we deal with a matrix model as well, that we briefly mention in the next transparency.
This yieds an extremely reliable approximation scheme (for example, can be checked vs Monte Carlo in zero dimension), that is able to capture all the non-trivial features of BH physics (including the arrow of time), and which is tractable at finite N!

The large D limit in models of vector of matrices

1) The usual auxiliary fields used to solve the large n O(n) vector models are now replaced by SO(D) invariant bilinears in the matrix variables

$$\sigma^{ab}_{cd} = (X_\mu)^a{}_c (X_\mu)^b{}_d$$

The model is then reformulated in terms of a U(N) tensor model, with an effective action $S_{\text{eff}}(\sigma)$ which is proportional to D. This is quite unusual but is tractable, both at finite N and at large N (generalizing the usual matrix models).

2) The large D saddle point for the tensor breaks U(N) because of the non-trivial holonomy $e^{-\beta A}$ of the gauge field A over the thermal circle. The equations yield the spectrum of σ in terms of the spectrum of A.

3) This then yields the spectrum of Bohr frequencies, with gaps of order $1/N^2$. The spectrum then becomes continuous at large N and implies the quasi-normal behavior, but it is discrete at finite N, which implies unitarity.

Local probe of the geometry

We can use D-brane probes to study the resulting BH geometry, combining the first principle construction given in Ferrari 2013 with the large D computational technique described above.

An important feature of the construction is that the local probe action depends on a choice of equivariant gauge-fixing, even though the physical quantities, like the on-shell probe action, are of course independent of any gauge choice. This fundamental equivariant gauge-dependence of the local description is related in this formalism to the fact that there is no local observable in QG and provides an alternative to the state-dependent "operators" to reconstruct the bulk geometry.