Current Algebra Constraints on Supersymmetric Quantum Field Theories

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Main Theme

Conserved charges $\mathcal{Q}_i$ generate continuous symmetries. Their (graded) commutators define the symmetry algebra $\mathcal{A}$.

- If the charges $\mathcal{Q}_i$ annihilate the vacuum, $\mathcal{Q}_i|0\rangle = 0$, then all states lie in representations $\mathcal{R}$ of the symmetry algebra $\mathcal{A}$.
- If $\mathcal{Q}_i|0\rangle \neq 0$ the symmetry is spontaneously broken.

In unitary theories $\mathcal{R}$ should be a unitary representation of $\mathcal{A}$.

Natural questions (many examples, long history):

- When can an algebra $\mathcal{A}$ arise as a physical symmetry algebra?
- Which representations $\mathcal{R}$ of a symmetry algebra $\mathcal{A}$ can occur?

We will examine two examples involving supersymmetric QFTs:

- $\mathcal{A} =$ superconformal algebra, $\mathcal{R} =$ local operators
- $\mathcal{A} =$ extended Poincaré SUSY algebra, $\mathcal{R} =$ particles, strings

Current Algebra

In QFT, we expect the generators $\mathcal{Q}_i$ of continuous symmetries to arise from local currents $J_i(x)$. Like all well-defined local operators, they should reside in a multiplet $\mathcal{J}$ of the symmetry algebra $\mathcal{A}$.

$$\mathcal{J} \supset \{J_i(x)\} \quad \longrightarrow \quad \mathcal{Q}_i = \int dx\, J_i(x)$$

This talk: current algebra = action of the $\mathcal{Q}_i$ on the operators in the current multiplet $\mathcal{J}$, e.g. $\mathcal{Q}_i J_j(x)$. Integrating over x, we must recover the charge algebra. This is a nontrivial constraint on $\mathcal{J}$, $\mathcal{A}$.

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Some representations of $\mathcal{A}$ may be inconsistent with the existence of local currents. Example [Weinberg,Witten]: If $\mathcal{A} =$ Poincaré algebra, then $\mathcal{J} = T_{\mu\nu}$ is the stress tensor. There are massless single-particle representations of $\mathcal{A}$ for any helicity $h \in \frac{1}{2}\mathbb{Z}$, but

$$\langle p', h | T_{\mu\nu}(q) | p, h \rangle \neq 0 \qquad \Longrightarrow \qquad |h| \leq 1 \; .$$

In the forward limit $q \to 0$ this measures the energy of the particle (via soft graviton scattering): must be IR finite and nonzero.

Maximal Supersymmetry in QFT

Massless single-particle representations of $\{Q, Q\} \sim P$ violate the Weinberg-Witten bound $|h| \leq 1$ when $d \geq 4$ and $N_Q > 16$. This leads to the standard lore that QFT requires $N_Q \leq 16$.

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- Not true in $d = 3$ (no notion of helicity for massless particles), e.g. an $\mathcal{N} = 9$ free hypermultiplet exists. It has 16 free bosons ϕ^i and 16 free Majorana fermions ψ^i_α ($\mathfrak{so}(9)_R$ spinors).
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In SCFTs $\mathcal{A} =$ superconformal algebra. Algebraically consistent $\mathcal{A}$'s are classified [Nahm], very restricted in $d \geq 3$:

$\mathfrak{osp}(\mathcal{N}\|4)$	$\mathfrak{su}(4\|\mathcal{N})$	$\mathfrak{f}(4)$	$\mathfrak{osp}(8\|\mathcal{N})$	none
$d = 3$	$d = 4$	$d = 5$	$d = 6$	$d \geq 7$

5d is exceptional (only $\mathcal{N} = 1$), 6d requires chiral $(\mathcal{N}, 0)$ SUSY. In $d = 3, 4, 6$ candidate algebras exist for every $\mathcal{N} \in \mathbb{Z}_{\geq 0}$.

Maximal Supersymmetry in QFT (cont.)

Not all superconformal algebras $\mathcal{A}$ admit a current algebra interpretation. The required current multiplet $\mathcal{T}$ contains the R-symmetry current R_μ^{ij}, the traceless SUSY current $S_{\mu\alpha}^i$ (gives Q, S-supercharges), and the traceless stress tensor $T_{\mu\nu}$ (gives P_μ, D, K_μ). The commutation relations of $\mathcal{A}$ require that

$$(\star) \quad \mathcal{T} \supset \{R_\mu^{ij}, S_{\mu\alpha}^i, T_{\mu\nu}\} , \quad QR \sim S , \; QS \sim T , \; QT \sim 0$$

Moreover, $\mathcal{T}$ must be a unitary multiplet of $\mathcal{A}$.

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We have developed a uniform procedure to tabulate the operator content of any unitary superconformal multiplet [Dolan, Osborn;...]. In particular, we analyzed all multiplets with conserved currents:

- If $\mathcal{T}$ exists, it is essentially unique, with a single lowest weight.
- No candidate $\mathcal{T}$ satisfying the constraints $(\star)$ exists if $d = 4, 6$ and $N_Q > 16$ (talk by [Vafa]).
- In 3d $\mathcal{T}$ exists for any $\mathcal{N}$. If $\mathcal{N} \geq 9$, then $\mathcal{T}$ contains higher-spin currents; the theory is free [Maldacena, Zhiboedov].

Deformations of SCFTs

Our machinery also leads to a classification of all possible SUSY deformations of SCFTs by local operators. Many applications, e.g. universal constraints on SUSY RG-flows. Example: 4d $\mathcal{N} = 2$ SCFTs [Argyres et. al.], $SU(2)_R \times U(1)_r$ symmetry ($r(Q^i_\alpha) = -1$).

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Only two kinds of Lorentz-scalar relevant or marginal deformations:

- A flavor current resides in a real multiplet $\mathcal{J}^{(ij)}$ such that

  $$Q^{(i}_\alpha \mathcal{J}^{jk)} = \overline{Q}^{(i}_{\dot\beta} \mathcal{J}^{jk)} = 0 \ , \quad \Delta_{\mathcal{J}} = 2 \ , \quad \sigma^\mu_{\alpha\dot\beta} j_\mu \sim Q^i_\alpha \overline{Q}^j_{\dot\beta} \mathcal{J}_{ij} \ .$$

  $\Delta\mathscr{L} = (Q^2)^{ij}\mathcal{J}_{ij}$ preserves SUSY, $SU(2)_R$, breaks $U(1)_r$.
- Chiral operators satisfy $\overline{Q}^i_{\dot\alpha}\mathcal{O} = 0$ and $\Delta_{\mathcal{O}} = r > 1$.
  $\Delta\mathscr{L} = Q^4\mathcal{O}$ preserves SUSY, $SU(2)_R$, typically breaks $U(1)_r$.

The upshot is that all deformed SCFTs have an $SU(2)_R$ symmetry, but generically not $U(1)_r$ (the same conclusion applies to gauging). If there is a Coulomb branch, then $SU(2)_R$ is unbroken there.

Non-Conformal 4d $\mathcal{N} = 2$ Theories

Now $\mathcal{A} =$ Poincaré SUSY algebra $\rtimes SU(2)_R$. It can be extended by p-form charges carried by p-brane excitations:

$$\{Q^i_\alpha, \overline{Q}_{j\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}\left(\delta^i{}_j P_\mu + (X_\mu)^i{}_j\right) \ ,$$

$$\{Q^i_\alpha, Q^j_\beta\} = 2\sigma^{\mu\nu}_{\alpha\beta} Y^{(ij)}_{[\mu\nu]} + 2\varepsilon_{\alpha\beta}\varepsilon^{ij} Z \ ,$$

$$[R^{(ij)}, Q^k_\alpha] = -\varepsilon^{k(i} Q^{j)}_\alpha$$

The charged states are strings for $(X_\mu)^i{}_j$, domain walls for $Y^{(ij)}_{[\mu\nu]}$, and particles for Z. Unitarity, with $\left(Q^i_\alpha\right)^\dagger = \overline{Q}_{i\dot\alpha}$, implies a BPS bound for their mass (or tension):

$$M_{\text{string}} \geq |X| \ , \qquad M_{\text{domain wall}} \geq |Y| \ , \qquad M_{\text{particle}} \geq |Z| \ .$$

When this bound is saturated, we can get BPS strings, domain walls, or particles. Which of these excitations can arise in $\mathcal{N} = 2$ QFTs, and what can we say about their quantum numbers?

$\mathcal{N} = 2$ Stress-Tensor Multiplets

The current multiplet $\mathcal{T}$ that gives rise to the SUSY algebra $\mathcal{A}$ is again the stress-tensor multiplet. All charges arise from currents:

$$(\star) \qquad \mathcal{T} \supset \left\{R_\mu^{(ij)}, S_{\mu\alpha}^i, T_{\mu\nu}, \left(x_{[\mu\nu]}\right)^i{}_j, \left(y_{[\mu\nu\rho]}\right)^{(ij)}, z_\mu\right\}$$

Now $T_{\mu\nu}$, $S_{\mu\alpha}$ are not traceless. The charge algebra $\mathcal{A}$ fixes

$$(\dagger) \quad \overline{Q}S \sim T + x \ , \quad QS \sim y + z \ , \ Q\left(T, x, y, z\right) \sim 0 \ , \quad QR \sim S$$

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Qualitative differences with the stress-tensor multiplet in SCFTs:

- $\mathcal{A}$ may admit distinct representations satisfying $(\star)$, $(\dagger)$.
- A given theory may have two (or more) multiplets $\mathcal{T}$,$\mathcal{T}'$. Then $T_{\mu\nu}$, $T'_{\mu\nu}$ and $S^i_{\mu\alpha}$, $S'^i_{\mu\alpha}$ differ by improvement terms.
- The other currents in $\mathcal{T}$, $\mathcal{T}'$ need not differ by improvements. Example: $R_\mu^{(ij)}$ can mix with an $SU(2)$ flavor current.

A complete list of possible $\mathcal{N} = 2$ stress-tensor multiplets is not available, but we know several examples. Is there a preferred one?

The Sohnius Stress-Tensor Multiplet

Nearly all non-conformal $\mathcal{N}=2$ theories with $SU(2)_R$ symmetry seem to admit a stress-tensor multiplet $\mathcal{T}$ introduced by [Sohnius]:

$$(\mathcal{T})^\dagger = \mathcal{T} \ , \quad \varepsilon^{\alpha\beta} Q_\alpha^{(i} Q_\beta^{j)} \mathcal{T} = \mathcal{Z}^{(ij)} \ , \quad Q_\alpha^{(i} \mathcal{Z}^{jk)} = \overline{Q}_{\dot\alpha}^{(i} \mathcal{Z}^{jk)} = 0 \ .$$

- $\mathcal{Z}^{(ij)}$ is a complex flavor current multiplet that contains z_μ. When it vanishes, we recover the superconformal multiplet.
- An $\mathcal{N}=2$ version of the $\mathcal{N}=1$ multiplet [Ferrara, Zumino].

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$$\mathcal{T} \ \rightarrow\ \psi_\alpha^i \ \rightarrow\ \mathcal{Z}^{(ij)}, W_{[\mu\nu]}, R_\mu^{(ij)}, r_\mu \ \rightarrow\ S_{\mu\alpha}^i, \chi_\alpha^i \ \rightarrow\ T_{\mu\nu}, z_\mu, C$$

- $W_{[\mu\nu]}$, r_μ are not conserved. SCFT: $r_\mu = U(1)_r$ current.
- There are no genuine currents $\left(x_{[\mu\nu]}\right)^i{}_j$ or $y_{[\mu\nu\rho]}^{(ij)}$. Hence there are no BPS strings or domain walls.
- Consistent with $\mathcal{N}=1$ [TD, Seiberg]: no BPS strings with an [FZ]-multiplet, no BPS domain walls with an R-symmetry.

BPS Particles in 4d $\mathcal{N} = 2$ Theories

Typically studied on Coulomb branch, where $SU(2)_R$ is unbroken.

$$\{Q^i_\alpha, S^j_{\mu\beta}\} = 2\varepsilon^{ij}\varepsilon_{\alpha\beta}\left(z_\mu + \partial^\nu W^+_{[\mu\nu]}\right) \;, \; W^+_{[\mu\nu]} \sim F^+_{[\mu\nu]}$$ [Witten, Olive]

Pick a vacuum and charge sector. Then $Z \in \mathbb{C}$ is fixed and can be aligned with $\mathbb{R}$: particles have $Z > 0$, antiparticles have $Z < 0$. In the rest frame $P^\mu = (M, \mathbf{0})$, little group is $SU(2)_J \times SU(2)_R$.

$$A^{(\pm)i}_\alpha = Q^i_\alpha \pm \sigma^0_{\alpha\dot\beta}\overline{Q}^{i\dot\beta} \;, \qquad \left(A^{(\pm)i}_\alpha\right)^\dagger = \pm A^{(\pm)\alpha}_i$$

$$\left\{A^{(\pm)i}_\alpha, A^{(\mp)j}_\beta\right\} = 0 \;, \qquad \left\{A^{(\pm)i}_\alpha, A^{(\pm)j}_\beta\right\} = 4\varepsilon^{ij}\varepsilon_{\alpha\beta}\left(Z \pm M\right)$$

- BPS particles satisfy $M = Z > 0$, and hence $A^{(-)} = 0$. Four states in a half hypermultiplet: $|\uparrow\rangle \xleftrightarrow{A^{(+)}} |i = 1, 2\rangle \xleftrightarrow{A^{(+)}} |\downarrow\rangle$.
- Anti-BPS particles: $M = -Z > 0$, roles of $A^{(\pm)}$ are reversed.
- Long multiplets: $M > |Z|$, $A^{(\pm)} \neq 0$. This leads to 16 states.

The NEC and its Consequences

More generally, we can tensor the half hypermultiplet with any representation $(j;r)$ of the $SU(2)_J \times SU(2)_R$ little group:

$$|\uparrow; m=-j,\ldots,j\,,\ s=-r,\ldots,r\rangle \xleftrightarrow{A^{(+)}} |i=1,2\,;\,m,s\rangle \xleftrightarrow{A^{(+)}} |\downarrow;\,m,s\rangle$$

- Multiplets with $j \neq 0$ occur, e.g. $(j=\frac{1}{2}; r=0)$ is a W-boson.
- Empirically, multiplets with $r \neq 0$ do not seem to occur in QFT. This was formalized in the no-exotics conjecture (NEC) of [Gaiotto, Moore, Neitzke]. Putative multiplets with $r \neq 0$ are called exotic. Further work by [Diaconescu et.al.; del Zotto, Sen].

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The conjecture has implications for physics and mathematics:

- Long multiplets cannot hit the BPS bound and decay into short ones, because some fragments would have to be exotic.
- Protected indices, which count BPS states with signs, actually coincide with the physical degeneracies (cf. BH microstates).
- Implies constraints on the cohomology of moduli spaces that arise in counting BPS states [Moore, Royston, van Den Bleeken].

Flavor Symmetries and Mixing

In the presence of an $SU(2)_{\text{flavor}}$ symmetry, the $SU(2)_R$ symmetry is not unique: $\widetilde{SU}(2)_R = SU(2)_R \times SU(2)_{\text{flavor}}\big|_{\text{diag}}$ is just as good.

Example: massless hypermultiplet $q^{i,a}, \psi^a_\alpha$. Here i an R-symmetry doublet index, and a is a flavor doublet index.

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Mixing: $i \to \widetilde{i}, a \to \widetilde{j}$, where $\widetilde{i}, \widetilde{j}$ are $\widetilde{SU}(2)_R$ doublet indices.

Therefore the hypermultiplet is exotic with respect to $\widetilde{SU}(2)_R$.

- The presence of $SU(2)_{\text{flavor}}$ renders the NEC ambiguous.
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While the two R-symmetries are indistinguishable at the level of charges, they arise from different current algebras. The current $R^{(ij)}_\mu$ resides in the Sohnius stress-tensor multiplet, while $\widetilde{R}^{(ij)}_\mu$ resides in a structurally different, less familiar multiplet.

Current-Algebra Proof of the NEC

Goal: prove the NEC with respect to the $SU(2)_R$ current $R_\mu^{(ij)}$ in the Sohnius multiplet. We will examine its forward matrix elements between BPS states, where it measures the charges $R^{(ij)}$. For now, we assume that all forward limits exist, postponing a small subtlety.

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Argue by contradiction: consider $\langle \uparrow; s \,|\, R_0^{22} \,|\, \uparrow; s' \rangle$. If $r \neq 0$ (exotic), choose $s = r - 1$, $s' = r$ to get a nonzero matrix element for the lowering operator R^{22}. Claim: in fact, it actually vanishes.

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Sohnius multiplet: $R_0^{22} \sim Q\overline{Q}\mathcal{T}$. BPS state: $A^{(-)} \sim Q - \overline{Q} = 0$. We can move the Q's around to derive a Ward identity:

$$\langle \uparrow; r-1 |\, R_0^{22} \,| \uparrow; r \rangle = -2M \, \langle 1; r-1 |\, \mathcal{T} \,| 2; r \rangle$$

There are many other such Ward identities (interesting), but they are not sufficient to show that the matrix element vanishes.

Current-Algebra Proof of the NEC (cont.)

Extra tool: the $\Theta = \mathrm{CPT}$ symmetry of relativistic QFT.
Since $\Theta^2 = (-1)^F$, the SUSY algebra determines (up to a sign)

$$\Theta Q^i_\alpha \Theta^{-1} = i\overline{Q}_{i\dot{\alpha}} \ , \qquad \theta Z \theta^{-1} = -\overline{Z} \ .$$

This fixes the Θ-transformations of all Sohnius multiplet operators.

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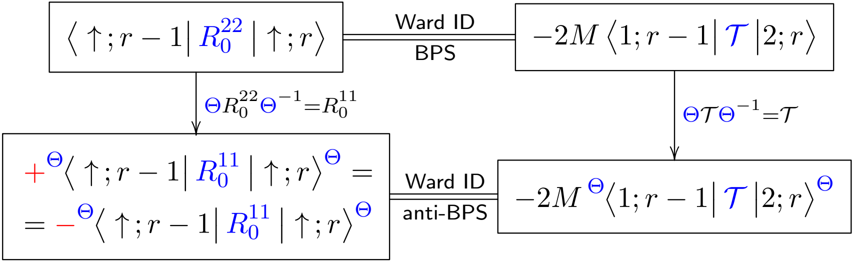

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$$\left\langle \uparrow; r-1 \right| R_0^{22} \left| \uparrow; r \right\rangle \quad \overset{\text{Ward ID}}{\underset{\text{BPS}}{=\!=\!=}} \quad -2M \left\langle 1; r-1 \right| \mathcal{T} \left| 2; r \right\rangle$$

$\downarrow\ \Theta R_0^{22} \Theta^{-1} = R_0^{11}$ $\qquad\qquad$ $\downarrow\ \Theta \mathcal{T} \Theta^{-1} = \mathcal{T}$

$$+\,{}^{\Theta}\!\left\langle \uparrow; r-1 \right| R_0^{11} \left| \uparrow; r \right\rangle^{\Theta} = -\,{}^{\Theta}\!\left\langle \uparrow; r-1 \right| R_0^{11} \left| \uparrow; r \right\rangle^{\Theta} \quad \overset{\text{Ward ID}}{\underset{\text{anti-BPS}}{=\!=\!=}} \quad -2M\, {}^{\Theta}\!\left\langle 1; r-1 \right| \mathcal{T} \left| 2; r \right\rangle^{\Theta}$$

Subtlety: forward matrix elements of $\mathcal{T}$ are divergent, due to soft single-photon exchange. This IR effect can be computed exactly and subtracted: $\mathcal{Z}^{(ij)} \to \mathcal{Z}^{(ij)}_{\text{eff}}$ (E&M boundary terms in Z) .

Conclusions and Extensions

- General lesson (not new): in QFT, current algebra can exclude phenomena that are allowed at the level of the charge algebra.
- Two examples:
  - SCFTs with $N_Q > 16$ in $d \geq 3$ (in $d = 3$, interacting SCFTs).
  - Exotic BPS states in 4d $\mathcal{N} = 2$ theories [GMN]
- The argument against exotics did not require a UV-complete theory. Consider a 5d $\mathcal{N} = 1$ QFT with a Sohnius multiplet, compactified on S^1. Some $4d$ BPS states come from BPS strings wrapping S^1, so the strings cannot carry R-charge.
- The discussion can be repeated for BPS particles in 5d $\mathcal{N} = 1$ theories and BPS strings in 6d $(1, 0)$ theories. There is also a 3d version (richer due to $SU(2)_R \times SU(2)'_R$ symmetry).
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Thank You for Your Attention!