Bulk Reconstruction in the Entanglement Wedge

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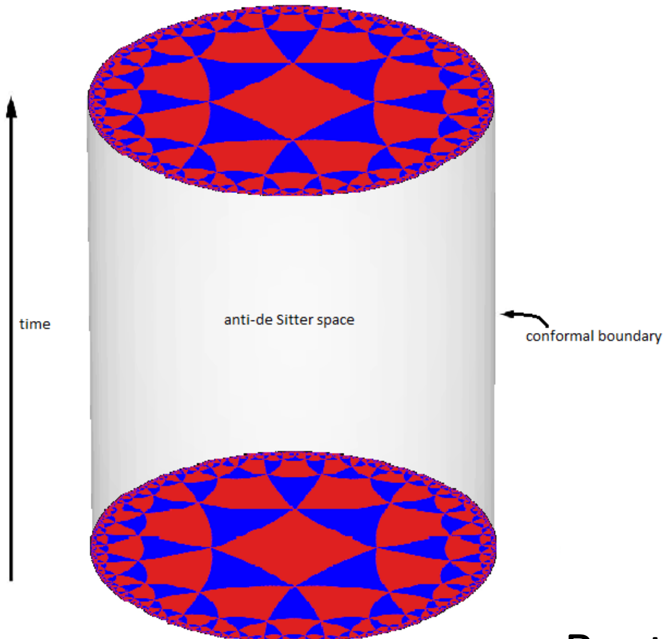
[XD, Harlow, Wall, Phys. Rev. Lett. 117, 021601 (2016)]
[Almheiri, XD, Harlow, JHEP 1504, 163 (2015)]

Strings 2016, Tsinghua University, Beijing

Anti-de Sitter/Conformal Field Theory Correspondence

[Maldacena '97]

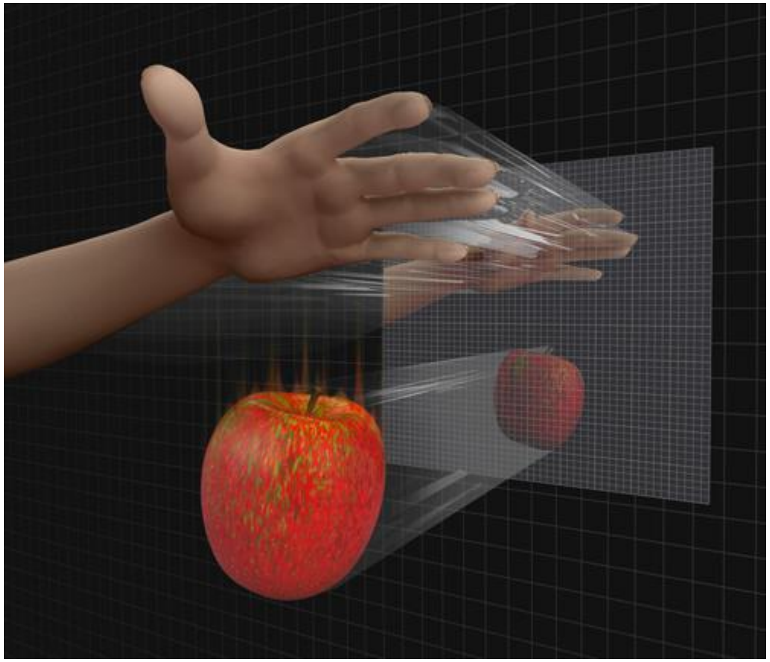


Quantum gravity in AdS_{d+1} (bulk)	**Holographic CFTs on ∂AdS_{d+1} (boundary)**
Isometry group $O(d,2)$	Conformal group $O(d,2)$
Black hole states	Thermal states
Gauge symmetry	Global symmetry
States and operators	States and operators

- Best-understood model of quantum gravity
- Concrete example of emergent spacetime/gravity
- Easy to extract CFT quantities from the bulk
- Difficult to extract bulk quantities from the CFT
- Understand black hole interior?

AdS/CFT: best-understood model of quantum gravity

[Maldacena '97]



Quantum gravity in AdS_{d+1}	Holographic CFTs on ∂AdS_{d+1}
Isometry group $O(d,2)$	Conformal group $O(d,2)$
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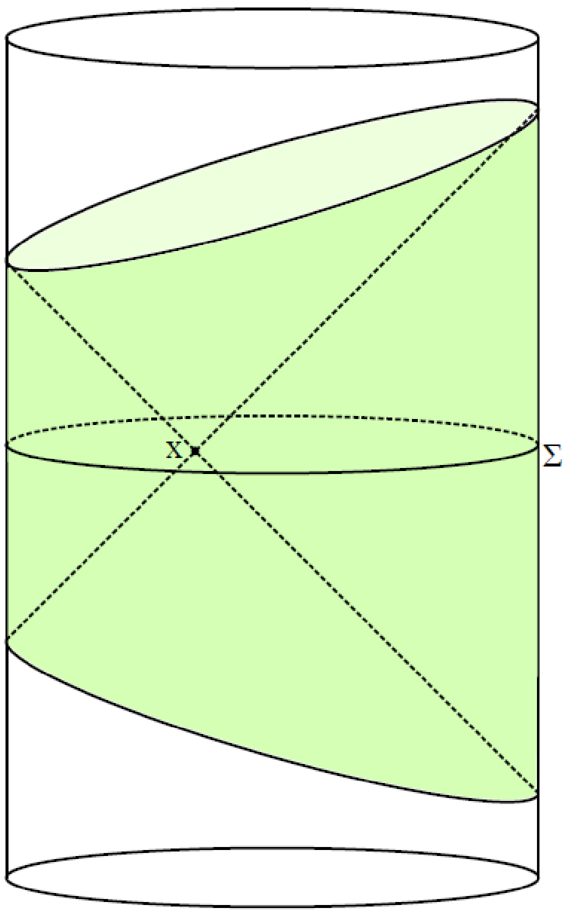
$$\lim_{r\to\infty} r^{\Delta}\phi(r,x) = O(x)$$
$$\phi(r,x) = ?$$

- What operator in CFT represents a local bulk operator?
- Once we know this, we have full access to bulk information.
- Answering this question helps us reconstruct the bulk.

Global AdS reconstruction

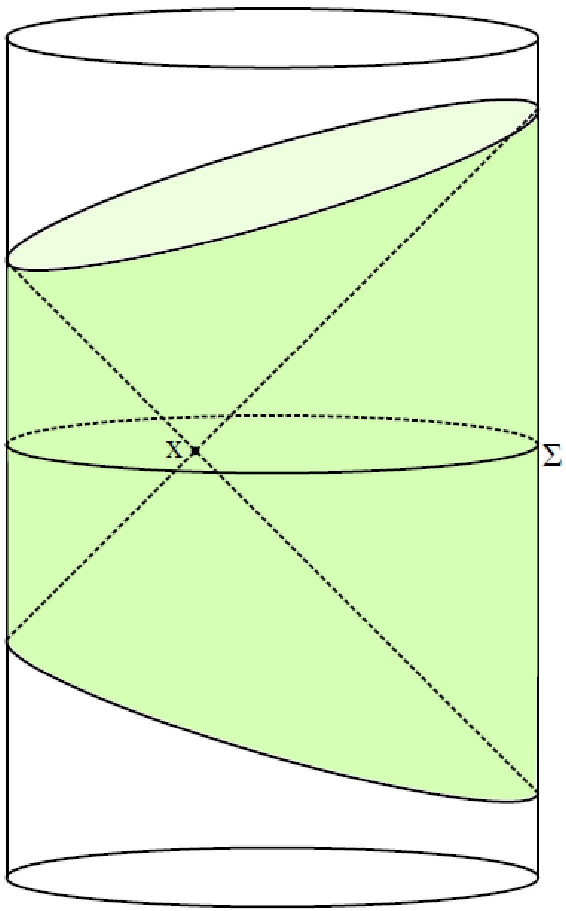


$$\phi(x) = \int_{\mathbb{S}^{d-1}\times\mathbb{R}} dY\, K(x;Y)\mathcal{O}(Y)$$

[Hamilton, Kabat, Lifschytz & Lowe ’06]

- $O(1/N)$ corrections

Global AdS reconstruction



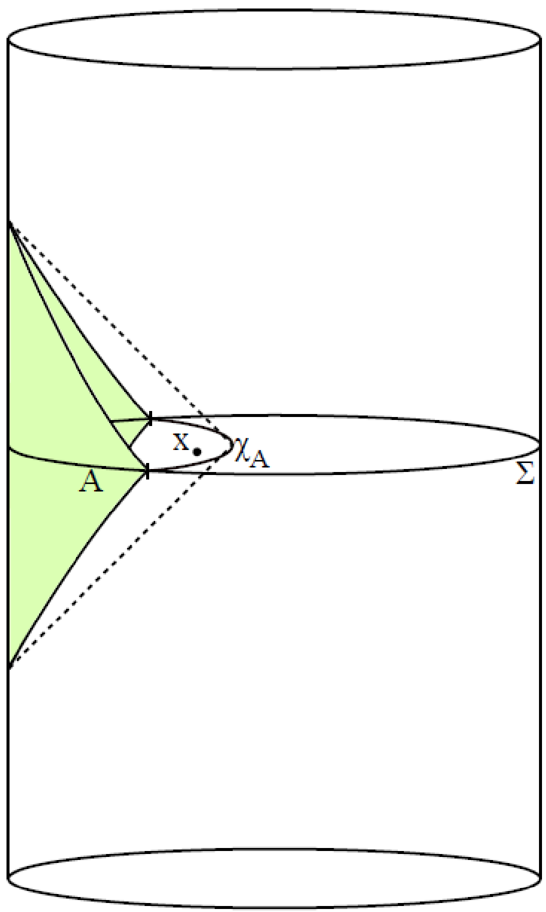
$$\phi(x) = \int_{\mathbb{S}^{d-1}\times\mathbb{R}} dY\, K(x;Y)\mathcal{O}(Y)$$

[Hamilton, Kabat, Lifschytz & Lowe '06]

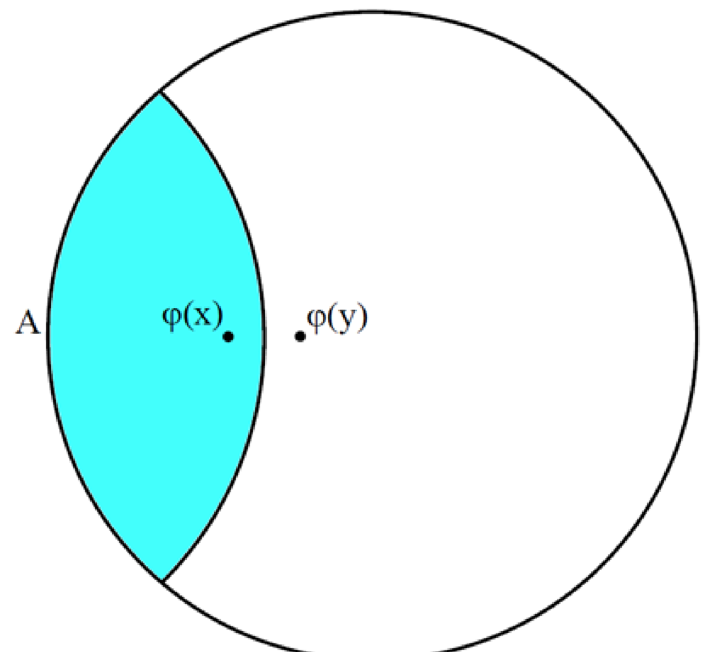
- $O(1/N)$ corrections
- Reconstruct bulk operators from a limited set of CFT data?

"Subregion duality"

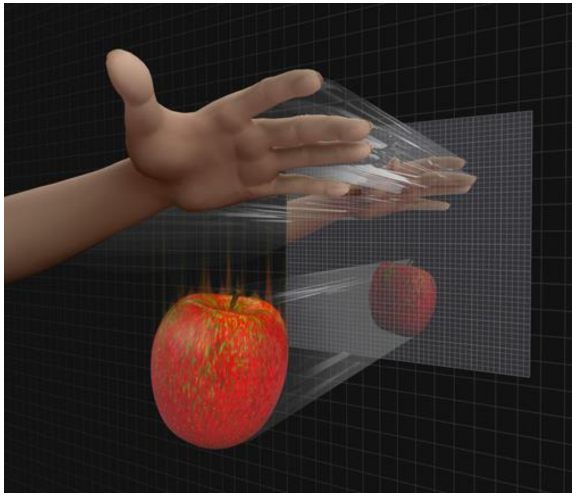
AdS-Rindler reconstruction for disk A



$$\phi(x) \sim \int_{D[A]} dY\, K_A(x;Y)\mathcal{O}(Y)$$



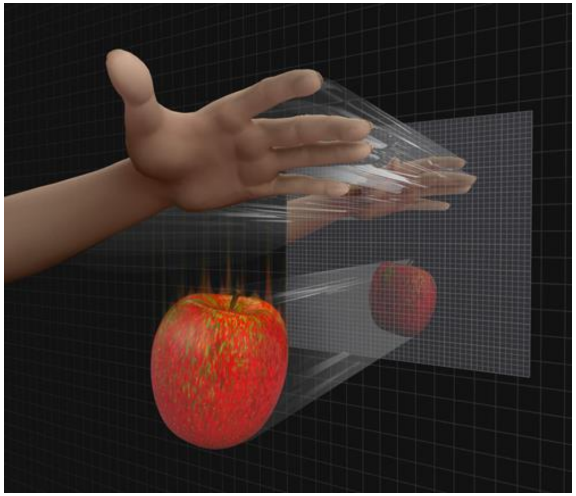
What region of the dual spacetime is described by a general subregion in a holographic CFT?

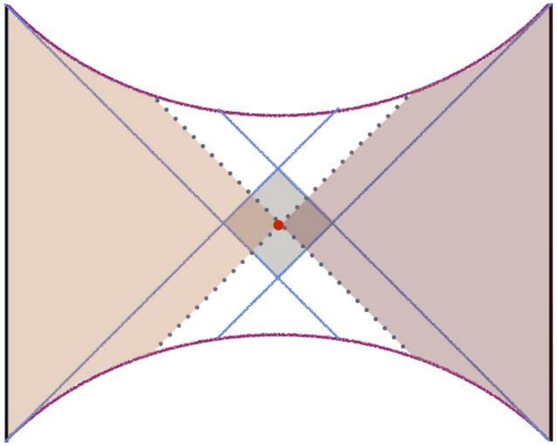


- HKLL works only in (smaller) causal wedge.

What region of the dual spacetime is described by a general subregion in a holographic CFT?

- HKLL works only in (smaller) causal wedge.
- Conjecture: bulk reconstruction works in (larger) entanglement wedge.
- Goes beyond black hole horizon and reconstructs interior.





Holographic Entanglement Entropy

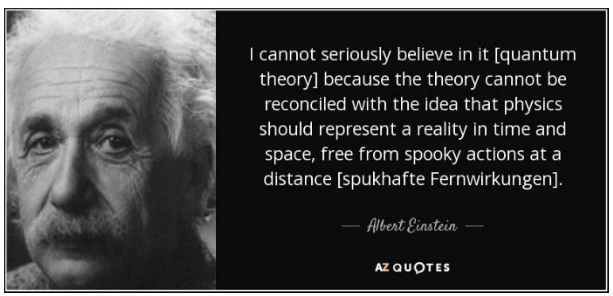
A simple and powerful prescription for entanglement entropy:

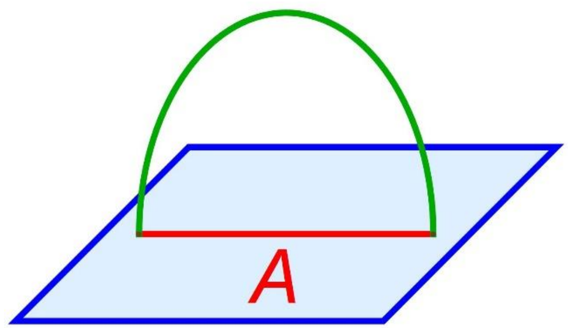
[Ryu & Takayanagi '06]

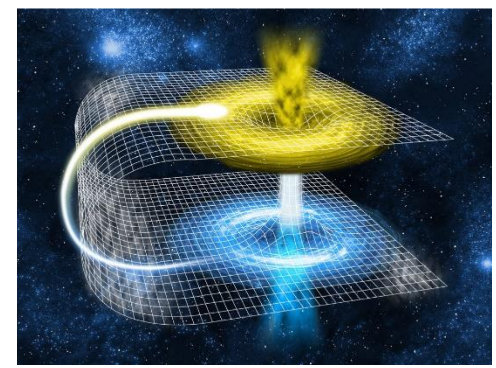
$$S = \frac{\text{Area}(\text{Minimal Surface})}{4G_N}$$

"Spooky action at a distance"

Spacetime geometry







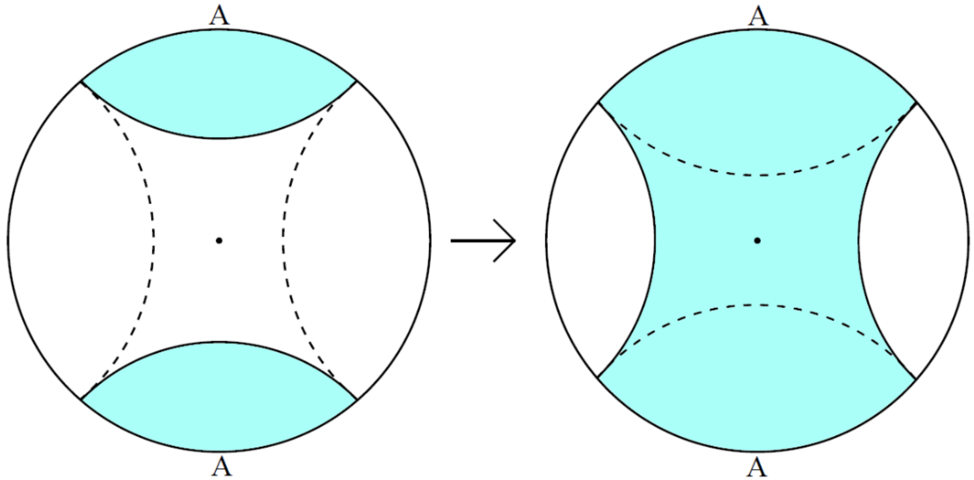
Recall the definition: $S \stackrel{\text{def}}{=} -\text{Tr}(\rho_A \ln \rho_A)$

Covariant generalization:

[Hubeny, Rangamani & Takayanagi '07] [XD, Lewkowycz & Rangamani 1607.07506]

Reconstruction conjecture for entanglement wedge

- Entanglement wedge is a bulk region bounded by the Ryu-Takayanagi minimal surface.
- It may change discontinuously.
- Conjecture: Any bulk operator in entanglement wedge of A may be represented as a CFT operator on A.



[Czech, Karczmarek, Nogueira & Van Raamsdonk ’12] [Wall ’12] [Headrick, Hubeny, Lawrence & Rangamani ’14]

Conjecture:
Any bulk operator in entanglement wedge of A may be represented as a CFT operator on A.

Proving this conjecture

Quantum error correction	RT with bulk quantum corrections

↓

CFT relative entropy = bulk relative entropy

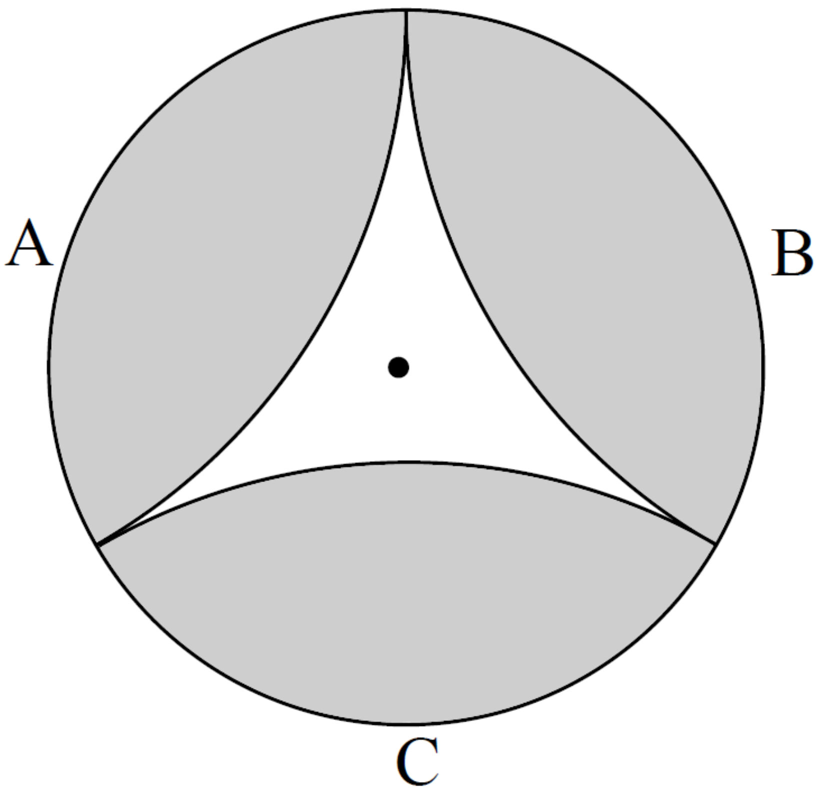
↓

Bulk operator in EW commutes with any $X_{\bar{A}}$

↓

Reconstruction in entanglement wedge

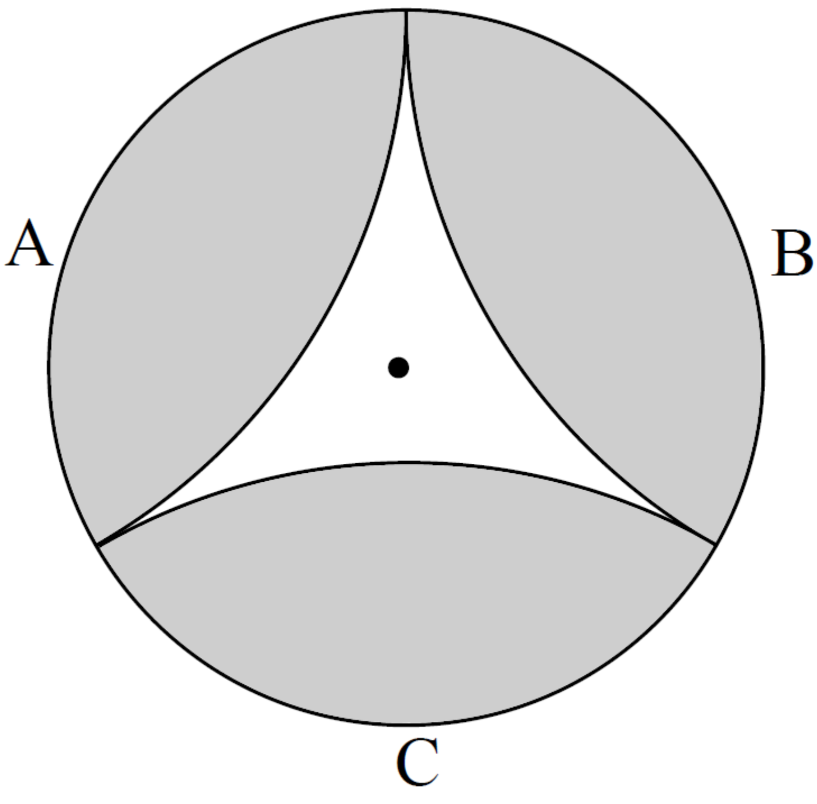
Why quantum error correction?



- $\phi(x)$ can be represented on $A \cup B$, $B \cup C$, or $A \cup C$.
- Obviously they cannot be the same CFT operator.

[Almheiri, XD, Harlow '14]

Why quantum error correction?



- $\phi(x)$ can be represented on $A \cup B$, $B \cup C$, or $A \cup C$.
- Obviously they cannot be the same CFT operator.
- Defining feature for quantum error correction.
- Holography is a quantum error correcting code.
- Reconstruction works in a code subspace of states.

[Almheiri, XD, Harlow '14]

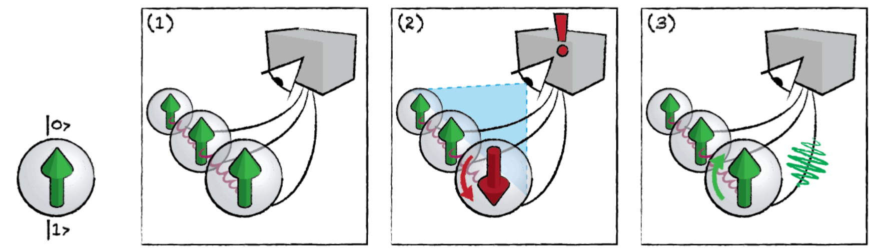
Three-qubit model

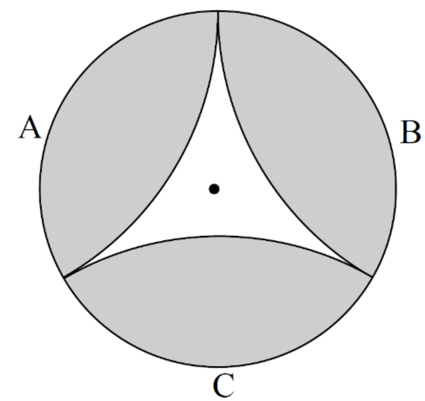
- Alice wants to send a qutrit by mail.
- She encodes it into the Hilbert space of 3 qutrits.

$$|\widetilde{0}\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)$$

$$|\widetilde{1}\rangle = \frac{1}{\sqrt{3}}(|012\rangle + |120\rangle + |201\rangle)$$

$$|\widetilde{2}\rangle = \frac{1}{\sqrt{3}}(|021\rangle + |102\rangle + |210\rangle)$$





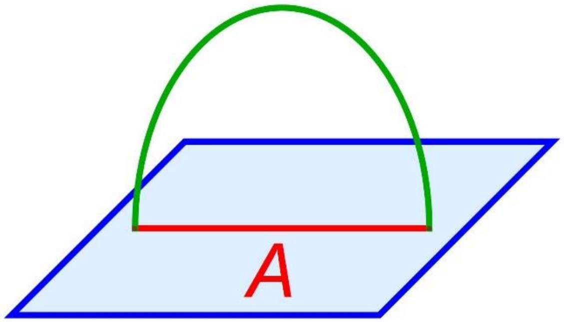
- These states span the code subspace.
- In holography, code subspace contains bulk states.

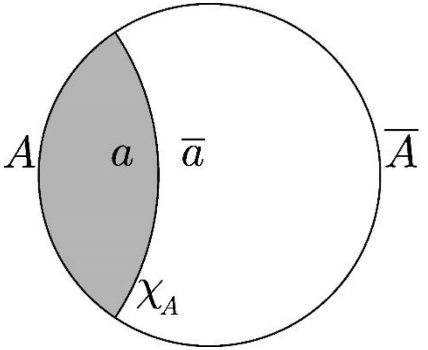
[Almheiri, XD, Harlow '14]

Ryu-Takayanagi with bulk quantum corrections

[Faulkner, Lewkowycz & Maldacena '13]

$$S = \frac{\text{Area}(\text{Minimal Surface})}{4G_N} + S_{\text{bulk}}$$



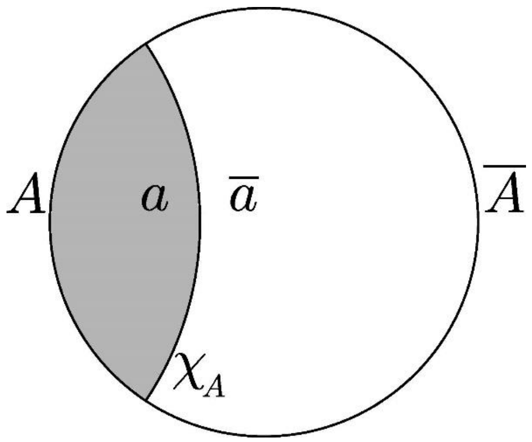


- S_{bulk}: von Neumann entropy of entanglement wedge
- Also has higher derivative corrections. [XD '13] [...]
- Derived by FLM up to $O(G_N)$ corrections; conjectured true to all orders. [Engelhardt & Wall '14] [XD & Lewkowycz, to appear]
- Intuitively, ρ_A has information in entanglement wedge of A!

CFT relative entropy is bulk relative entropy

- Define a subspace H_c of states where bulk EFT is valid.
- E.g. $H_c = \{$All states with $E < M_{Pl}$ (& conformal images)$\}$.
- Will call it a code subspace.
- Rewrite RT with quantum corrections:

$$S(\rho_A) = \mathrm{Tr}(\rho_a A_{loc}) + S(\rho_a)$$



- Small change of state $\rho \to \rho + \delta\sigma$ in H_c:

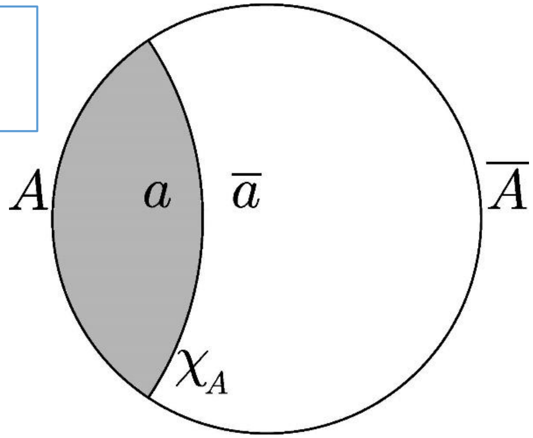
$$\mathrm{Tr}\left(\delta\sigma_A K_{\rho_A}\right) = \mathrm{Tr}\left[\delta\sigma_a\left(A_{loc} + K_{\rho_a}\right)\right]$$

- Modular Hamiltonian: $K_\rho \stackrel{\text{def}}{=} -\log\rho$.
- Integrate $\delta\sigma$:

$$\mathrm{Tr}\left(\sigma_A K_{\rho_A}\right) = \mathrm{Tr}\left[\sigma_a\left(A_{loc} + K_{\rho_a}\right)\right]$$

CFT relative entropy is bulk relative entropy

- From $$\mathrm{Tr}(\sigma_A K_{\rho_A}) = \mathrm{Tr}[\sigma_a(A_{loc} + K_{\rho_a})]$$



$$\Pi_c K_{\rho_A} \Pi_c = A_{loc} + K_{\rho_a}$$

- Relative entropy: $S(\rho|\sigma) \stackrel{\text{def}}{=} \mathrm{Tr}(\rho \log \rho) - \mathrm{Tr}(\rho \log \sigma)$

$$S(\rho_A|\sigma_A) = S(\rho_a|\sigma_a)$$

[Jafferis, Lewkowycz, Maldacena & Suh 1512.06431]

- States are as distinguishable in the bulk as in the CFT.
- Intuitively, this means we must be able to reconstruct in entanglement wedge.

Conjecture:
Any bulk operator in entanglement wedge of A may be represented as a CFT operator on A.

Proving this conjecture

Quantum error correction	RT with bulk quantum corrections ✓

↓

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Bulk operator in EW commutes with any $X_{\bar{A}}$

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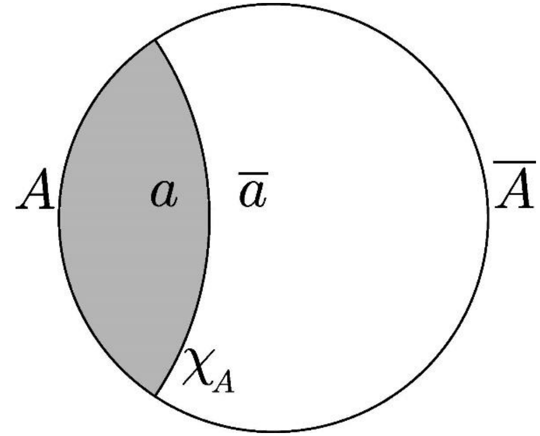
Reconstruction in entanglement wedge

A reconstruction theorem



- Goal is to prove: $\langle\phi|[O_a, X_{\bar{A}}]|\phi\rangle = 0$
- This is necessary and sufficient for

$$\exists\, O_A,\ s.t.\ O_A|\phi\rangle = O_a|\phi\rangle \text{ and } O_A^\dagger|\phi\rangle = O_a^\dagger|\phi\rangle$$

- WLG assume O_a is Hermitian.

[Almheiri, XD, Harlow '14]

- Consider two states $|\phi\rangle$, $e^{i\lambda O_a}|\phi\rangle$ in H_c:

$$S(\rho_{\bar{A}}|\sigma_{\bar{A}}) = S(\rho_{\bar{a}}|\sigma_{\bar{a}}) = 0$$

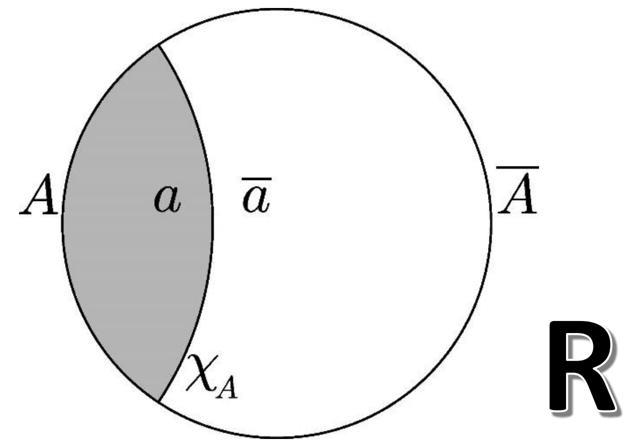
$$\langle\phi|e^{-i\lambda O_a} X_{\bar{A}} e^{i\lambda O_a}|\phi\rangle = 0 \quad \Rightarrow \quad \langle\phi|[O_a, X_{\bar{A}}]|\phi\rangle = 0$$

[XD, Harlow & Wall 1601.05416]

Explicit reconstruction (in principle)

- Add a reference system R to CFT

$$|\Psi\rangle = \sum_i |i\rangle_R \otimes |i\rangle_{A\bar{A}}$$



- Can mirror O_a to an operator O_R.
- View O_R as $O_R \otimes I_{\bar{A}}$ and mirror it back onto A as O_A.
- Use Schmidt decomposition: $|\Psi\rangle = \sum_\alpha c_\alpha |\alpha\rangle_A \otimes |\alpha\rangle_{R\bar{A}}$.
- Obstruction: $O_R \otimes I_{\bar{A}}$ may mix zero c_α with nonzero ones.
- This cannot happen if

$$[O_R \otimes I_{\bar{A}}, \rho_{R\bar{A}}] = 0 \quad \Leftrightarrow \quad \langle\phi|[O_a, X_{\bar{A}}]|\phi\rangle = 0$$

[Almheiri, XD, Harlow ’14]

Theorem:
Any bulk operator in entanglement wedge of A may be represented as a CFT operator on A.

Proving this conjecture

Quantum error correction	RT with bulk quantum corrections

↓ ✓

CFT relative entropy = bulk relative entropy

↓ ✓

Bulk operator in EW commutes with any $X_{\bar{A}}$

↓ ✓

Reconstruction in entanglement wedge

What We Learned

- Quantum information theory enables us to understand the basic dictionary of quantum gravity.

- Viewing holography as a quantum error correcting code, we can analyze how to "build spacetime from entanglement".

Future Directions

- Simple explicit reconstruction of bulk operators in the entanglement wedge?
- How do we enjoy all of this?

✓ Study black hole interior and information paradox?

✓ Understand better the emergence of spacetime and gravity?