Higher Spin de Sitter Holography

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Motivation

> From the combination of the fundamental constants, G, c, and h it is possible to form a new fundamental unit of length $L_{min} = 7 \times 10^{-28} cm$. It seems to be inevitable that this length must play some role in any complete interpretation of gravitation. [...] In recent years great progress has been made in knowledge of the excessively minute; but until we can appreciate details of structure down to the quadrillionth or quintillionth of a centimetre, the most sublime of all the forces of Nature remains outside the purview of the theories of physics. (Eddington 1918, 36)

[Dean Rickles, Pourparlers for Amalgamation ...]

> **de Sitter is mysterious**
> We will not say much more about de Sitter space in this course. A big reason for this is that we don't have any UV-complete theory of gravity in de Sitter, like we do in anti-de Sitter. We also have no clear answer to the question 'What is the de Sitter entropy?' (Does it count the microstates of something?) Since we live in de Sitter, this seems like a very important question.

[Tom Hartman, Lectures on Quantum Gravity, 2015]

Motivation

String Theory:

- No classical dS solutions [Maldacena-Nuñez,de Wit et al]
- Quantum dS solutions [Kachru-Kallosh-Linde-Trivedi]
- Landscape [Bousso-Polchinski,Susskind,Douglas]

The Great Wall:

- No classical dS = no semiclassical control [Dine-Seiberg]
- No complete theory = no predictions (e.g. [FD-Douglas '04] distribution susy breaking scale: $d\mathcal{N} \sim F^5 dF \Rightarrow$?)
- dS-CFT? [Strominger,Witten,Maldacena]
  Examples? [Silverstein,Anninos-Hartman-Strominger]
  Consensus: troubling [Susskind-Kleban et al]

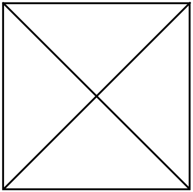
dS-CFT

From AdS to dS:

- Continuation $L \to iL$, $z \to i\eta$:

$$ds^2_{\mathrm{AdS}} = \frac{L^2}{z^2}(dz^2 + dx^2) \quad \to \quad ds^2_{\mathrm{dS}} = \frac{L^2}{\eta^2}(-d\eta^2 + dx^2)\,.$$

- Boundary = future infinity $\eta = -\frac{1}{L}e^{-t/L} = 0$.



- $\lim_{\eta \to 0} \phi(\eta, x) \sim \eta^{\Delta}\,\alpha(x) + \eta^{d-\Delta}\,\beta(x)$

dS-CFT

Quantization:

- $\lim_{\eta \to 0} \phi(\eta, x) \sim \eta^{\Delta}\, \alpha(x) + \eta^{d-\Delta}\, \beta(x)$
- $[\hat{\alpha}(x), \hat{\beta}(y)] \sim i\, \delta^d(x-y)$.
- Dirichlet/Neumann future boundary states: $\hat{\alpha}|D\rangle = 0$, $\hat{\beta}|N\rangle = 0$.
- dS-CFT [Strominger]:

$$\langle D|\hat{\beta}_1 \cdots \hat{\beta}_n|0\rangle = \langle \mathcal{O}_1 \cdots \mathcal{O}_n\rangle_{\mathrm{CFT_D}}\,, \quad \langle N|\hat{\alpha}_1 \cdots \hat{\alpha}_n|0\rangle = \langle \tilde{\mathcal{O}}_1 \cdots \tilde{\mathcal{O}}_n\rangle_{\mathrm{CFT_N}}$$

- dS-CFT 2.0 [Maldacena]:

$$\Psi_{\mathrm{HH_D}}[\alpha] = \langle \alpha|0\rangle = \langle D|e^{i\int \alpha\hat{\beta}}|0\rangle = \langle e^{i\int \alpha\mathcal{O}}\rangle_{\mathrm{CFT_D}} = Z_{\mathrm{CFT_D}}[\alpha]$$

  Note $\hat{\alpha}|\alpha\rangle = \alpha|\alpha\rangle$. Analogous $\Psi_{\mathrm{HH_N}}[\beta]$ = Fourier transform $\Psi_{\mathrm{HH_D}}[\alpha]$.

Challenges for dS-CFT

Unconventional, (very) nonunitary CFT:

- $\langle \mathcal{O}\mathcal{O} \rangle = \partial\partial \log \Psi_{\rm HH}|_0 < 0 \Rightarrow$ CFT inner product not > 0.
- Central charge $C_{\rm AdS_{d+1}} \sim \frac{L^{d-1}}{G} \to C_{\rm dS_{d+1}} \sim i^{d-1} \frac{L^{d-1}}{G}$ not > 0.
- Scalar mass $m^2_{\rm AdS} = \frac{\Delta(\Delta - d)}{L^2} \to m^2_{dS} = -\frac{\Delta(\Delta-d)}{L^2}$.

  $\Rightarrow$ Usual abundance of large Δ operators like $\mathrm{Tr}\, X^n$ would lead to abundance of tachyons + heavy particles have complex Δ: $\Delta = \frac{d}{2} \pm i\sqrt{(mL)^2 - (\frac{d}{2})^2}$.

Note:

- Not inconsistent: Euclidean CFT.
- $\sim$ CFT analog no-go dS brane constructions.
- Examples? $\rightsquigarrow$ Bootstrap? What replaces unitarity constraint?

Challenges for dS-CFT

Objections raised by [(Dyson/Goheer)-Kleban-Susskind] point to trouble for dS-CFT as a complete theory:

- Boltzman brains.
- Eternal dS does not appear to exist in string theory. Connected to entire landscape, ultimately decays into zero or negative cc vacua.
- Tension between features of dS representation theory and finiteness of dS horizon entropy.

Challenges for dS-CFT

Bulk quantum mechanics?

- Time is emergent. Bulk Hilbert space $\neq$ CFT state space. Bulk Hamiltonian $\neq$ CFT Hamiltonian.

- Different CFTs (D/N) for $\hat{\beta}$ and $\hat{\alpha}$ correlators. What about mixed $\hat{\alpha}\hat{\beta}$ correlators? Unified in one QFT? Commutator problem:

$$\langle \hat{\alpha}(x)\, \hat{\beta}(y) \rangle = i\, \delta^d(x-y) = -\langle \hat{\beta}(y)\, \hat{\alpha}(x) \rangle$$

  $\rightsquigarrow$ Different operator orderings in Euclidean QFT?? ×

- CFT supposedly gives "wave function" $\Psi[\varphi] = \langle \phi | 0 \rangle$ but:
  - Computing $\langle 0 | \cdots | 0 \rangle$ correlators requires integrating over φ.
  - Probabilities? Measure? Phase space? Wave function of *what*?
  - Bulk dynamics?

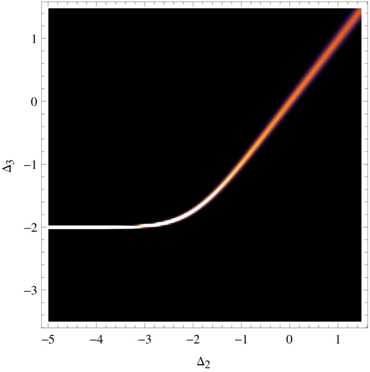
  [ArkaniHamed-Maldacena]

Aside: Cosmic Clustering

Free massless scalar (or metric) in dS:

1. Effective stochastic time evolution: branched diffusion [Starobinsky].
2. Wave function $\Psi[\varphi] \sim e^{-\int d^d k \, k^d \phi_k \phi_{-k}}$

(1) can be cleanly detected in (2) by computing a “phylogenetic” triple overlap distribution introduced in study of spin glasses [Anninos-FD]



Goal of this work

We want to find an exact and complete model for dS holography.

Interesting and useful things can be said at approximate level, e.g. by analytic continuation from AdS-CFT [Skenderis-McFadden-Bzowski,Hartle-Hertog-Hawking,...].

But we want complete, non-perturbative formulation. In exchange we will drop requirement of phenomenological realism.

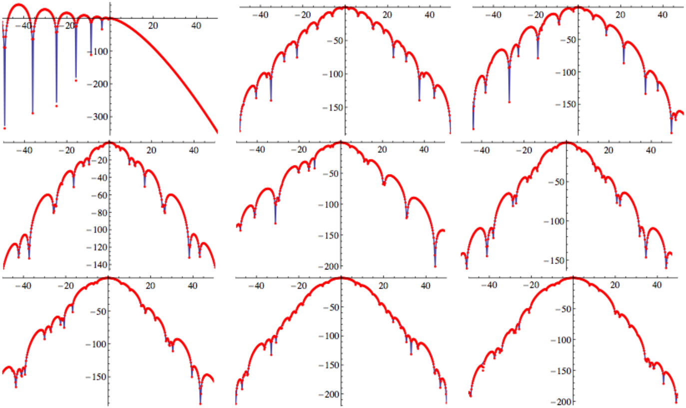
AHS ghost models

- Higher spin AdS_4 - $U(N)$ vector model correspondence [Klebanov-Polyakov,Giombi-Yin] $\rightarrow$ higher spin dS_4 - $U(N)$ vector model correspondence [Anninos-Hartman-Strominger]. Instead of bosonic scalars, fermionic scalars.
- Free model in 3d:

$$S = \int d^3x \left|\partial V^A\right|^2 \qquad (U(N) \text{ singlet sector})\,.$$

  One single trace scalar primary $\mathcal{O} = :V^A \bar{V}^A:$, with $\Delta = 1$.
- Dual to Vasiliev dS_4 containing massless spin s particles for all $s = 0, 1, 2, \ldots$. Scalar of $m^2 = \frac{2}{L^2}$, Neumann future boundary state.
- $m^2 > 0$ ✓, $C = -N$ ✓, $\langle \mathcal{O}\mathcal{O} \rangle < 0$ ✓
- $\Rightarrow \Psi_{\rm HH}[\beta, \ldots] = \det(-\partial^2 + \beta + \ldots)^N$.

Minisuperspace wave functions



$y = \log \Psi[\beta]$ plotted as function of x for $\beta(\omega_3) = x\, Y_\ell^m(\omega_3)$, $\ell = 0, ..., 8$, spherical harmonics on S^3. (Only) the $\ell = 0$ mode leads to a divergent wave function. [Anninos-FD-Harlow,Anninos-FD-Konstantinidis-Shaghoulian]

Other studies: [Banerjee-Belin-Hellerman-LepageJutier-Maloney-Radicevic-Shenker] (divergence with topology), [Hertog-Conti/Van der Woerd] (fluctuations).

Degrees of freedom

- Wave function is function... of *what*?
- Must use correct set of degrees of freedom, measure, etc before drawing conclusions about wave function divergences:
  - $\psi(r) = \frac{e^{-r}}{r}$ diverges on $\mathbb{R}$ but converges on $\mathbb{R}^2$.
  - Gauge invariance
  - Other redundancies (holography!)
- Most general free $U(N)$ invariant Lagrangian:

$$S = \int d^3x\, |\partial V^A|^2 + \int d^3x\, d^3y\, B(x,y)\, V^A(x)\bar{V}^A(y)\,.$$

  $B(x,y)$ bilocal collective field [Jevicki,Das-Jevicki]; can be expanded in primary fields coupling to sources.

- Note mismatch d.o.f. “sources” $B(x,y)$ and “fields” $V^A(x)$.

Toy model illustration

Toy model: Complex bosonic rectangular matrices V_x^A, $A = 1, \ldots, N$, $x = 1, \ldots, K$, coupling to hermitian matrix source B_{xy}:

$$\Psi(B) = \int dV\, e^{-\mathrm{Tr}(V\bar{V}) + i\,\mathrm{Tr}(VB\bar{V})} = \det\left(1 + i\,B\right)^{-N}$$

$$\int dB\, |\Psi(B)|^2 = \int dB\, \det(1 + B^2)^{-N}\,.$$

Under $B \to \lambda B$ with $\lambda \to \infty$:

$$dB \propto \lambda^{K^2}\,, \qquad \det(1 + B^2)^{-N} \propto \lambda^{-2NK}\,.$$

$\Rightarrow \Psi(B)$ normalizable iff $K < 2N$, i.e. source B d.o.f. $<$ field V d.o.f.

Alternatively: Keep in integral form, integrate out B:

$$\int dB\, |\Psi(B)|^2 \propto \int dV\, dV' \prod_{ij} \delta\big((V\bar{V})_{ij} - (V'\bar{V}')_{ij}\big)$$

If $K > 2N$: product of redundant δ-functions.

Degrees of freedom

Extrapolation to original $U(N)$ model with UV cutoff $\rightsquigarrow$ K spatial cells.

$\Rightarrow 2NK$ vector V d.o.f., K^2 source d.o.f.

Continuum limit: $K \to \infty \Rightarrow$ sources vastly redundant.

Even after gauge fixing: 2 d.o.f. per spatial cell for each spin $s \geq 1$, infinite tower of spins.

In addition: $\partial^2 V = 0$, V fermionic $\rightsquigarrow$ further reduction V d.o.f.

Conclusion: Asymmetric parametrization fields/sources inadequate.

Question: Can we do better? Parametrize source as vector bilinears too, part of single dual QFT? Maybe, but recall commutator problem...

Idea: Berezin coherent states $\rightsquigarrow$ convergent integrals over sources.

Berezin coherent states

[Berezin,Das²-Jevicki-Ye]: Quantum mechanics of $U(N)$ vector models = Kähler quantization of dual "source" phase space, generalizing spin N Bloch sphere.

Standard radial quantization $U(N)$ fermionic vector model via mode expansion:

$$\{a_p^A, (a_q^B)^\dagger\} = \delta_p^A \delta_q^B \,, \qquad \{b_p^A, (b_q^B)^\dagger\} = \delta_p^A \delta_q^B \,,$$

where $A, B = 1, \ldots, N$ and $p, q = 1, \ldots, S$ ($\sim$ angular momentum modes $\rightsquigarrow S \sim$ number of points on codim-1 sphere)

Berezin coherent (or squeezed) states:

$$|\bar{Z}) \equiv e^{\frac{1}{\sqrt{N}} \bar{Z}_{pq} b_p^\dagger a_q^\dagger} |0) \,, \qquad (Z|\bar{W}) = \det(1 + \tfrac{1}{N} Z W^\dagger)^N \,.$$

So for normalized states $|Z\rangle \equiv \frac{1}{\sqrt{(Z|\bar{Z})}} |Z)$, and for $N \to \infty$:

$$|\langle Z|\bar{W}\rangle|^2 \approx e^{-\mathrm{Tr}|Z-W|^2} \,.$$

Berezin coherent states

Complex sources Z_{pq} parametrize compact Kähler phase space with Kähler potential $\mathcal{K} = \log\det(1 + \frac{1}{N} ZZ^\dagger)$,

$$ds^2 = \mathrm{Tr}\big[(1 + \tfrac{1}{N} ZZ^\dagger)^{-1} dZ\big]^2 \,.$$

Decomposition of unity: We have $\int dZ \sqrt{G}\, |\bar{Z}\rangle\langle Z| \propto \mathbf{1}$, i.e.

$$\int dZ\ \det\big(1 + \tfrac{1}{N} Z^\dagger Z\big)^{-2S} |\bar{Z}\rangle\langle Z| \propto \mathbf{1}\,.$$

$N \to \infty$ fixed $Z, S \rightsquigarrow$ standard oscillator coherent states.

D.o.f. sources match vectors:

- compact phase space (diag. Z: $(\text{2-sphere})^S$)
- data on codim-1 sphere position space $\rightsquigarrow$ holographic

CFT state space

Noted before: ghost $U(N)$ model inner product *not* > 0.

2-point function $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{c_{\mathcal{O}}}{|x-y|^{2\Delta}}$, $c_{\mathcal{O}} < 0$. Radial quantization: $\mathcal{O}(x)^\dagger = x^{-2\Delta}\mathcal{O}(\frac{x}{x^2})^*$. Norm primary state $\langle \mathcal{O}(0)^\dagger|\mathcal{O}(0)\rangle = c_{\mathcal{O}} < 0$.

Mode expansion V^A gives creation/annihilation operator algebra

$$\{a_p^A, (a_q^B)^\dagger\} = \delta_p^A \delta_q^B \,, \qquad \{b_p^A, (b_q^B)^\dagger\} = -\delta_p^A \delta_q^B \,,$$

with $p = (\ell, m)$ angular momentum quantum numbers. (sign)

$\Rightarrow$ CFT "state" space $\mathbb{Z}_2$ graded with positive/negative norm for even/odd b-number.

Bulk interpretation? Evidently *not* global particle states.

[Anninos,Ng-Strominger,Jafferis-Lupsasca-Lysov-Ng]:

CFT states $\leftrightarrow$ bulk quasinormal modes.

[Note: Berezin construction modified: $(Z|\bar{Z}) = \det(1 - \frac{1}{N} ZZ^\dagger)^N$.]

Bulk reconstruction from CFT

Question: CFT to bulk operator map $\sim$ [Hamilton-Kabat-Lifschytz-Lowe]? (dS: [Sharkar-Xiao])

Approach: AdS group theoretic construction of [Verlinde,Miyaji-Numasawa-Shiba-Takayanagi-Watanabe, Nakayama-Ooguri]

Results: E.g. planar: Most general formula consistent with symmetries:

$$\phi(\eta, x) \sim \sum_{\pm} \eta^{d-\Delta} \int dx' \left((x-x')^2 - \eta^2 \mp i\epsilon\right)^{\Delta-d} \mathcal{O}_{\pm}(x') \,,$$

$\mathcal{O}_{\pm}$ primaries of dim Δ. Has both η^{Δ}, $\eta^{d-\Delta}$ falloffs.

Bulk QM $\Leftrightarrow [\mathcal{O}_+(x), \mathcal{O}_-(y)] \sim (x-y)^{-2\Delta} \rightsquigarrow$ commutator problem.

* Lor. AdS-CFT: inherited from pos./neg. freq. modes in CFT
  $\mathcal{O}_{\pm}(k) \sim \Theta(-k^2)\Theta(\pm k^0)\mathcal{O}(k)$
* dS-CFT? No time in boundary $\rightsquigarrow$ ✕ CFT knows nothing of bulk QM?

Bulk reconstruction from CFT

Not so fast: In static patch dS i.e.

$$ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega^2$$

CFT operator mode expansion in radial/cylinder quantization

$$\hat{\mathcal{O}}(\tau,\Omega) = \sum_{\ell m} \sum_{\kappa} \hat{\mathcal{O}}^{\kappa}_{\ell m}\, e^{\kappa\tau}\; Y^m_\ell(\Omega)\ ,$$

does map (with suitable variant of integration prescription) to:

$$\hat{\phi}(t,r,\Omega) = \sum_{\ell m} \sum_{\kappa_n = \pm(\Delta+\ell+2n)} \hat{\mathcal{O}}^{\kappa_n}_{\ell m}\, e^{-\kappa_n t}\, \psi^n_\ell(r)\; Y^m_\ell(\Omega)$$

= static patch quasinormal mode expansion!

Moreover with $(\mathcal{O}^{\kappa}_{\ell m})^\dagger = (-)^m \mathcal{O}^{-\kappa}_{\ell,-m}$:

$$[\hat{\mathcal{O}}^{\kappa}_{\ell m}, (\hat{\mathcal{O}}^{\kappa}_{\ell m})^\dagger] \sim \pm \delta_{\ell\ell'}\delta_{mm'}\delta_{\kappa\kappa'} + O(\tfrac{1}{N})$$

$N \to \infty \rightsquigarrow$ QM $\sim$ QNM quantization of [Strominger et al].

Conclusions

Way ahead: putting these pieces of the puzzle together

But I'm out of time.

Thank you!