Strings 2013, Seoul

Global F-Theory Compactifications with Higher Rank Abelian Symmetries

Mirjam Cvetič

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arXiv:1303.6970 [hep-th]: M. C. Denis Klevers, Hernan Piragua
arXiv:1306.0236 [hep-th]: M. C. A. Grassi, D.Klevers, H. Piragua
arXiv:130n.nnnn [hep-th] (UPR-1251-T): M.C., D.Klevers, H. Piragua

Also:Xiv:1210.6034 [hep-th]: M. C., Thomas W. Grimm, D. Klevers

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F-theory Compactifications with additional U(1)’s

MOTIVATION

Why F-theory Compactification?

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Domain of string theory landscape with promising particle physics

- Focus D=4 N=1 SUSY GUT's [SU(5), SO(10)] w/chiral matter, Yukawa couplings 10 10 5,...

⇒ GUT-model building in F-theory

- Moduli stabilization (fluxes) [Gukov,Vafa,Witten],...

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Local:[Donagi,Wijnholt;Beasley,Heckman,Vafa; ... Review: Heckman,...]

Global: [Marsano,Saulina, Schäfer-Nameki; Blumenhagen,Grimm, Jurke,Weigand;... M.C.,Halverson Garcia-Etxebarria;...]

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M.C.,Halverson Garcia-Etxebarria;...]

Conceptual: geometric description at finite string coupling

[Vafa; Witten;...]

- F-theory via finite coupling Type IIB string theory:
  Consistent set-up of back-reacted seven-branes
  Non-perturbative coupling regions on non-Calabi-Yau geometry
- F-theory via Geometry:
  Globally defined elliptically fibered Calabi-Yau manifold

Why Abelian Symmetries in F-theory?

Particle physics: important ingredient of Beyond Standard Model Physics

- Light U(1) gauge bosons: Z'-physics, NMSSM, $U(1)_{PQ}$, ...
- Massive (Stückelberg) U(1) gauge bosons: low energy global symmetry

⇒ selection rules (proton decay; R-parity violation; neutrino masses...)

Multiple U(1)'s desirable

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Conceptual: new types of elliptic fibrations

- Related to Abelian Mordell-Weil group of elliptic fibrations

  ➡ torsion part studied

  w/free part less understood (global issues)

  Torsion part: [Morrison,Vafa; Aspinwall,Morrison;…]

  For toric K3: [Grassi,Perduca]

- Few systematic studies in contrast to non-Abelian groups

  Non-Abelian: [Kodaira;Tate; Morrison,Vafa;Bershadsky et al.;…]

Outline & Summary of the talk

I. Construction of elliptically fibered Calabi-Yau manifolds w/
   rank 2 Mordell-Weil (MW) group

II. Determination of matter representations and multiplicity in D=6 and D=4

III. First construction of G_4-fluxes on Calabi-Yau four-folds with rk=2 MW-group

IV. Construction of explicit U(1) x U(1) and SU(5)xU(1)xU(1) w/spectra & chirality

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Two-fold advances: Geometry & M-theory/F-theory duality

I. Geometry: in D=6 determines all matter representations and multiplicity

in D=4 G_4 fluxes & some of matter surfaces identified→ some chiralities

II. M-theory/F-theory duality in D=3: Constaints on G_4 w/Chern-Simons terms determine

- all chiral indices (tested against geom. calc.)

- confirm cancellation of all anomalies

The Type IIB perspective

F-THEORY HIGHLIGHTS

The Type IIB perspective

F-THEORY HIGHLIGHTS

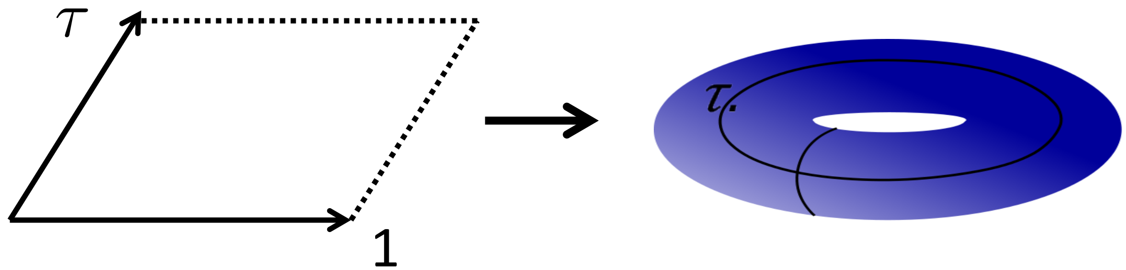
[At Strings'12: D-instantons in F-theory; Heterotic M-theory perspective]

[M.C.,Donagi,Halverson,Marsano]

F-theory via Type IIB: basic ingredients

- F-theory is a geometric, SL(2, **Z**) invariant formulation of Type IIB string theory: invariant geometric object is two-torus $T^2(\tau)$

[Vafa]



- Modular parameter τ of $T^2(\tau)$: $\tau \equiv C_0 + i g_s^{-1}$ Type IIB axion-dilaton (SL(2, **Z**) = S-duality)

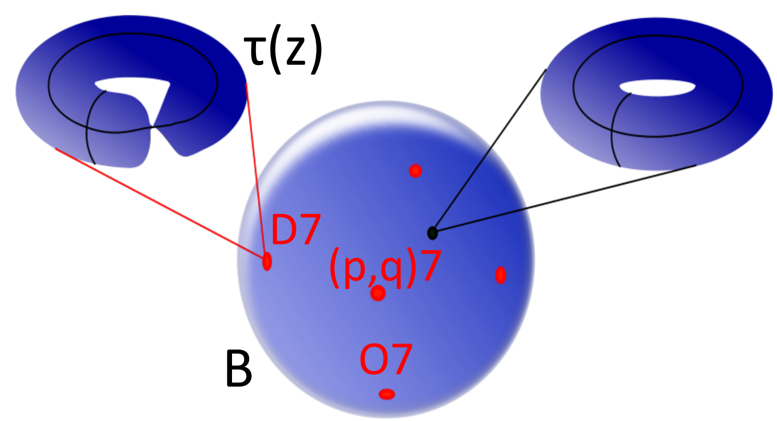
- $T^2(\tau)$-fibration over a base space B:

Weierstrass parameterization:

$$y^2 = x^3 + fxz^4 + gz^6$$

f, g- function fields on B

[z:x:y] homog. coords on $\mathbf{P}^2(1,2,3)$



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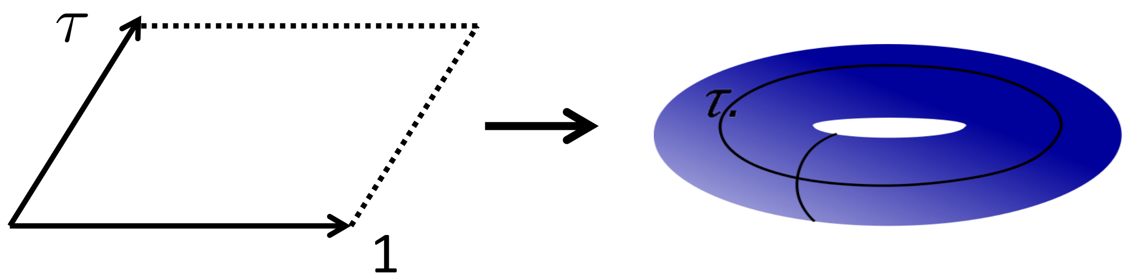

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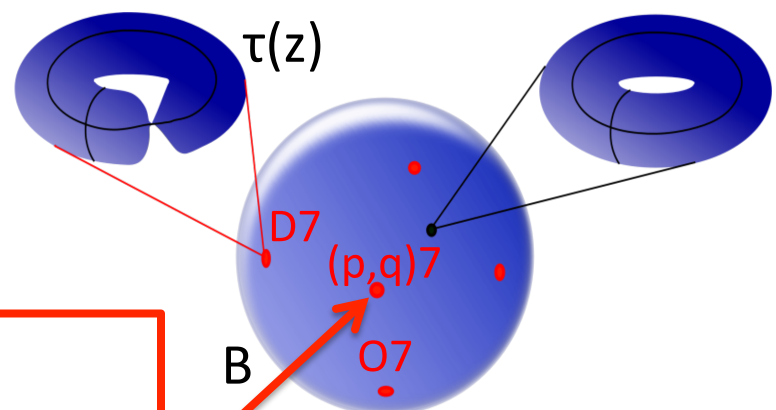


7-branes

non-perturbative regime:

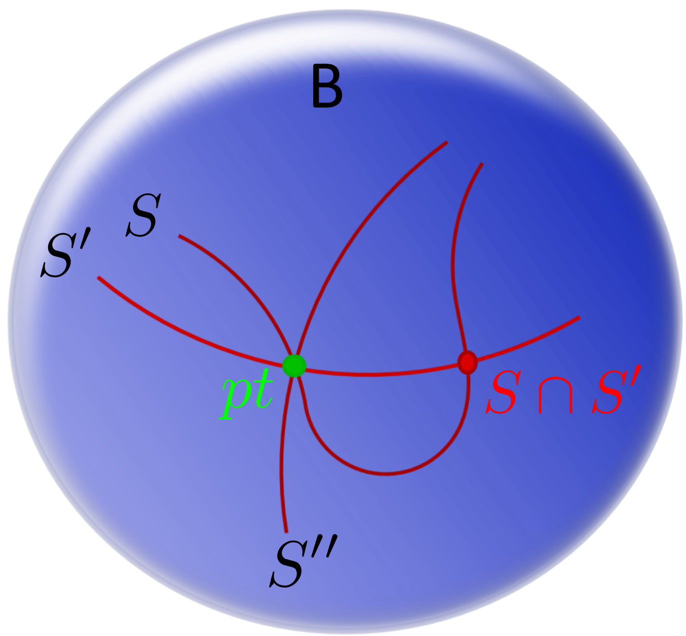
$g_s \rightarrow \infty$ ⟺ singular $T^2(\tau)$

F-theory via Type IIB: basic ingredients

- Total space of $T^2(\tau)$-fibration: singular elliptic Calabi-Yau manifold X

  D=4, N=1 vacua: fourfold X_4 [D=6, N=1 vacua: threefold X_3]

- X-singularities encode complicated set-up of intersecting 7-branes:



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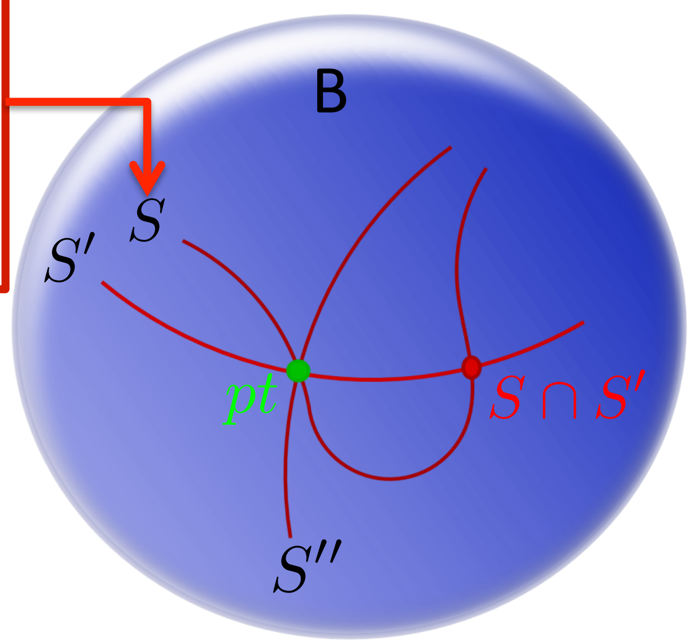
Non-Abelian gauge theory (ADE…) in codim. one in B S (7-branes)



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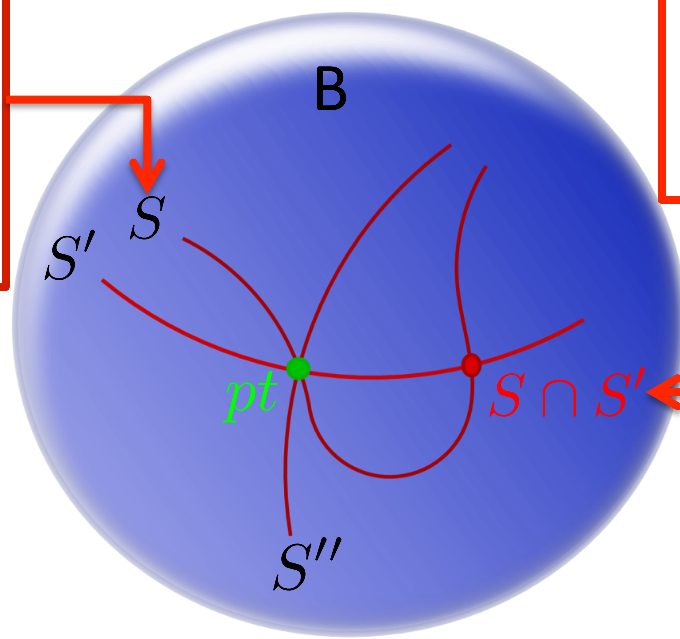

Matter in codim. two in B [Katz,Vafa] (intersecting 7-branes)

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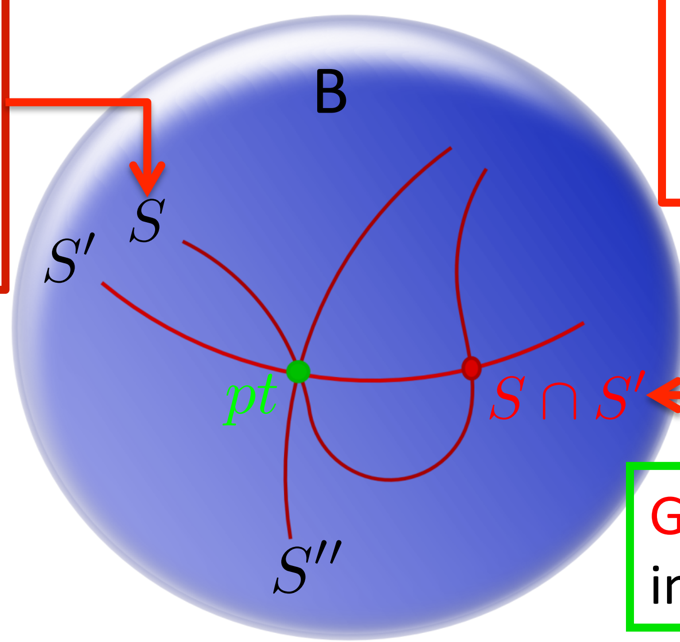

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G_4-flux at intersection induces chiral 4D matter

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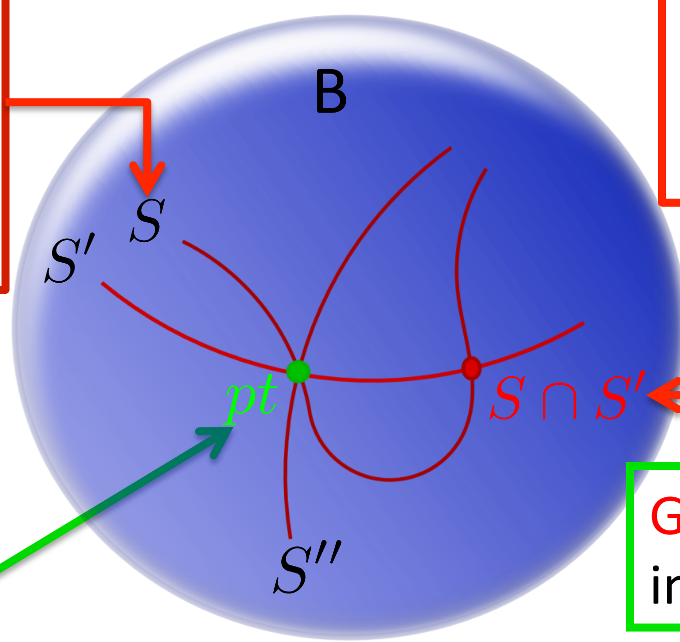

Matter in codim. two in B [Katz,Vafa] (intersecting 7-branes)

Yukawa couplings in codim. three in B $pt = S \cap S' \cap S''$

G_4-flux at intersection induces chiral 4D matter

Constructing elliptic fibrations with rank two Mordell-Weil groups

U(1)XU(1) SYMMETRY IN F-THEORY

MW-group of rational sections & U(1)'s

4D Abelian gauge fields arise from classical Kaluza-Klein-reduction of C_3

$$C_3 = A^B \omega_B \supset A^i \omega_i + A^m \omega_m$$

(1,1)-forms on X

Cartans of non-Abelian group

U(1)-gauge fields

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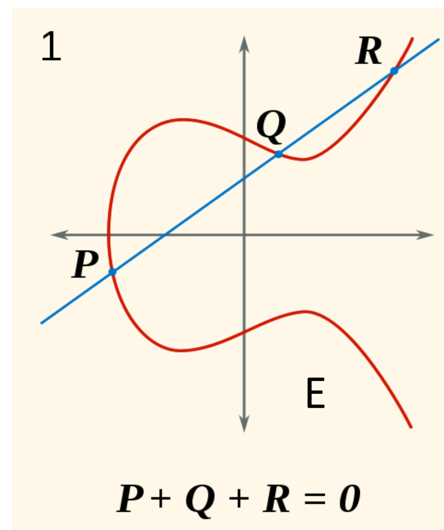
Construction of (1,1)-form ω_m via rational sections

1. Rational point Q on elliptic curve E with zero point P
   - is solution $[z_Q : x_Q : y_Q]$ in field K of Weierstrass form

$$y^2 = x^3 + f x z^4 + g z^6$$

   - Rational points form group (addition) on E

Mordell-Weil group of rational points



[wikipedia.org]

2. Q induces rational section $\hat{s}_Q : B \to X$ of the fibration

   (1,1)-form ω_m Poincaré dual to divisor class S_Q (related to $\hat{s}_Q$ via Shioda map)

Construction of elliptic curve with rk(MW)=2

[M.C., Klevers,Piragua]

Elliptic curve E with two rational points Q, R

related work:[Borchman, Mayrhofer,Weigand]

rk[MW]=1: [Morrison,Park; Mayrhofer,Palti,Weigand]

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Elliptic curve E with two rational points Q, R

Consider line bundle M=O(P+Q+R) of degree 3 on E (non-generic cubic in $\mathbf{P}^2$)

➡ natural representation as hypersurface p=0 in del Pezzo dP_2

$$p = u(s_1 u^2 e_1^2 e_2^2 + s_2 u v e_1 e_2^2 + s_3 v^2 e_2^2 + s_5 u w e_1^2 e_2 + s_6 v w e_1 e_2 + s_8 w^2 e_1^2) + s_7 v^2 w e_2 + s_9 v w^2 e_1$$

$[u:v:w:e_1:e_2]$ –homogeneous coordinates of dP_2

(blow-up of $\mathbf{P}^2$ w/ [u':v':w'] at 2 points: $u'=ue_1e_2$, $v'=ve_2$, $w'=we_1$)

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$$\begin{array}{llllll} & & u & v & w & e_1 \quad e_2 \\ P: & E_2 \cap p = [-s_9 : s_8 : 1 : 1 : 0]\,, \\ Q: & E_1 \cap p = [-s_7 : 1 : s_3 : 0 : 1]\,, \\ R: & D_u \cap p = [0 : 1 : 1 : -s_7 : s_9]\,. \end{array}$$

Points represented by intersections of different divisors in dP_2 with p

Classification of dP_2 elliptic fibrations

[M.C., Klevers, Piragua; M.C.,Grassi,Klevers,Piragua]

$$dP_2 \longrightarrow dP_2^B(\mathcal{S}_7, \mathcal{S}_9) \qquad dP_2^B(\mathcal{S}_7, \mathcal{S}_9) \longrightarrow B$$

I. Ambient space:

– dP_2 fibration determined by two divisors $\mathcal{S}_7$ and $\mathcal{S}_9$ (loci of s_7=0,s_9=0)

$$\hat{E} \subset dP_2 \longrightarrow X \qquad X \longrightarrow B$$

sections $\hat{s}_P, \hat{s}_Q, \hat{s}_R$ ($B \to X$)

II. Calabi-Yau hypersurface X:

– cuts out E in dP_2

– coefficients s_i in CY-equation get lifted to sections of the base B (only s_7,s_9 independent)

– coordinates [u:v:w:e_1:e_2] lifted to sections

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Birational map to Weierstrass fibration explicitly worked out

Classification of dP_2 elliptic fibrations

Construction of general elliptic fibrations:

section	bundle
u'	$\mathcal{O}(H - E_1 - E_2 + \mathcal{S}_9 + [K_B])$
v'	$\mathcal{O}(H - E_2 + \mathcal{S}_9 - \mathcal{S}_7)$
w'	$\mathcal{O}(H - E_1)$
e_1	$\mathcal{O}(E_1)$
e_2	$\mathcal{O}(E_2)$

section	bundle
s_1	$\mathcal{O}(3[K_B^{-1}] - \mathcal{S}_7 - \mathcal{S}_9)$
s_2	$\mathcal{O}(2[K_B^{-1}] - \mathcal{S}_9)$
s_3	$\mathcal{O}([K_B^{-1}] + \mathcal{S}_7 - \mathcal{S}_9)$
s_5	$\mathcal{O}([2K_B^{-1}] - \mathcal{S}_7)$
s_6	$\mathcal{O}([K_B^{-1}])$
s_7	$\mathcal{O}(\mathcal{S}_7)$
s_8	$\mathcal{O}([K_B^{-1}] + \mathcal{S}_9 - \mathcal{S}_7)$
s_9	$\mathcal{O}(\mathcal{S}_9)$

– CY-condition: $\mathcal{S}_7$ and $\mathcal{S}_9$ fixed

Engineer non-Abelian groups: make s_i non-generic

Can apply to toric cases w/ twoU(1)'s

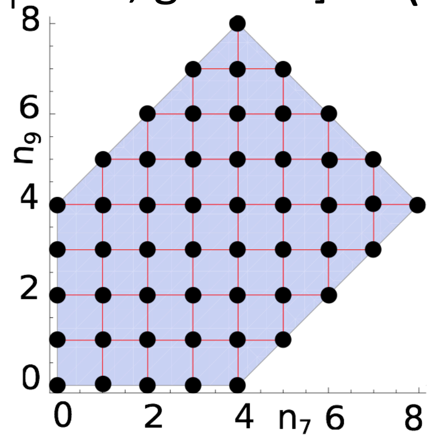
[Borchmann,Mayrhofer,Palti,Weigand; Braun,Grimm,Keitel]

Classification of dP_2 elliptic fibrations

All topologically distinct D=6 & D=4 vacua for fixed base B. Example:

1. B=**P³** , X generic [all s_i exist, generic]: U(1) x U(1)

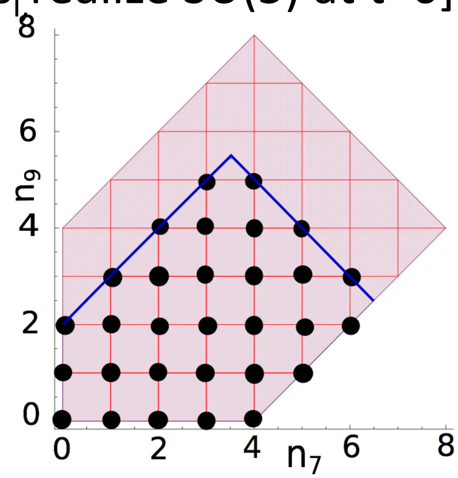
$$\mathcal{S}_7 = n_7 H_{\mathbb{P}^3}$$
$$\mathcal{S}_9 = n_9 H_{\mathbb{P}^3}$$



2. B=**P³** , X non-generic [s_i realize SU(5) at t=0]: SU(5) x U(1) x U(1)

$$s_1 = t^3 s_1'$$
$$s_2 = t^2 s_2'$$
$$s_3 = t^2 s_3'$$
$$s_5 = t s_5'$$

[M.C., Klevers,Piragua]



Can construct and study all these CYs explicitly
(no restriction to toric hypersurfaces seems necessary)

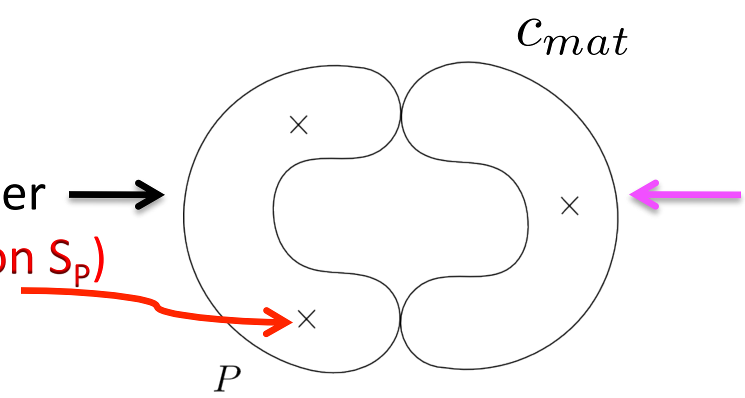
Codimension two singularities of dP_2-elliptic fibrations

MATTER U(1)XU(1) F-THEORY VACUA

Matter representations

[M.C.,Klevers,Piragua]
related work:[Borchman, Mayrhofer,Weigand]

- Matter in F-theory arises from a co-dimension two singularities in B
- Singular fiber resolved into reducible curves $E=c_1+c_{mat}$ w/ $c_1.c_{mat}=2$
  (c_{mat} - M2-branes wrapping isolated **P^1** in reducible fiber)



Advances in higher co-dimension singularities:.[Esole,Yau]..,[Lawrie,Schäfer-Nameki]
Recent advances via deformations of singularities: [Halverson,Grassi,Shaneson]

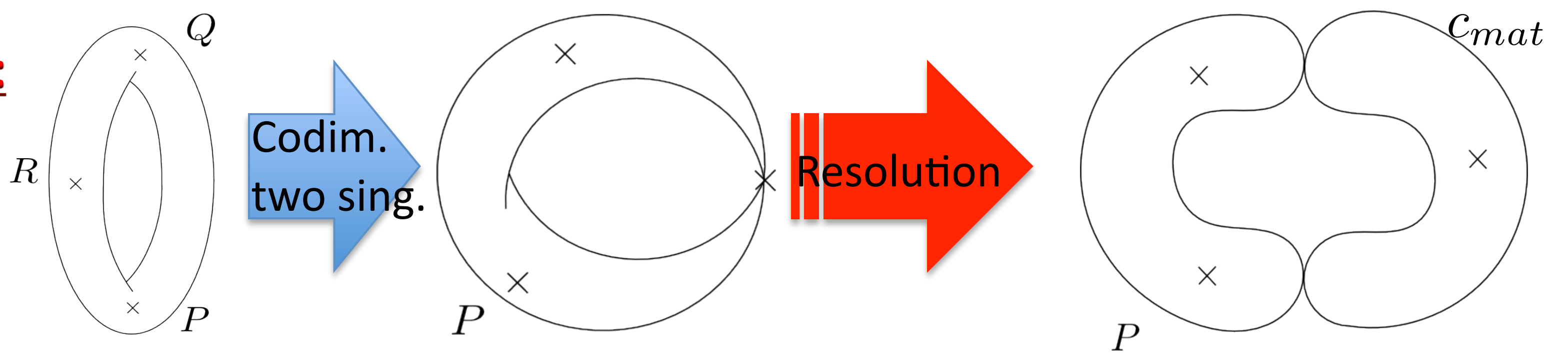
Charged matter of Type I

Charge formula:

$$q_1 = (S_Q - S_P) \cdot c_{mat} \qquad q_2 = (S_R - S_P) \cdot c_{mat}$$

Strategy: look for collisions of rational sections with singularities in Weierstrass fibration (WSF)

WSF:



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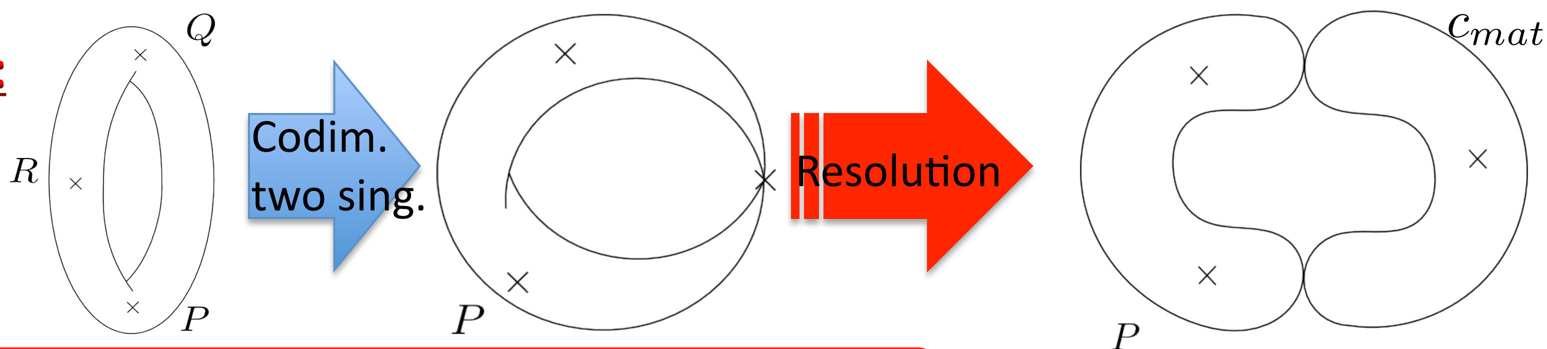
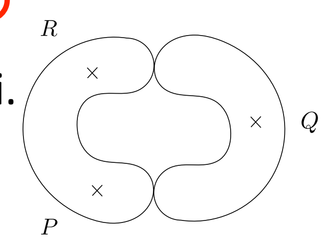


List of charged matter representations

Representation	Collision pattern
i. $(q_1,q_2)=(1,0)$	Q → WS-singularity
ii. $(q_1,q_2)=(0,1)$	R → WS-singularity
iii. $(q_1,q_2)=(1,1)$	Q,R → WS-singularity

i.



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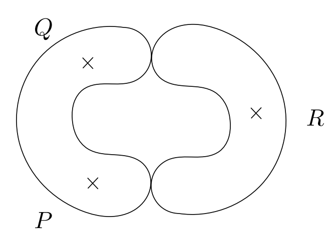

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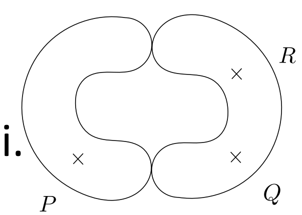

Charged matter of Type II

Strategy: look for loci in B where the sections are ill-defined

$$P: \ E_2 \cap p = [-s_9 : s_8 : 1 : 1 : 0] \, , \quad Q: \ E_1 \cap p = [-s_7 : 1 : s_3 : 0 : 1] \, ,$$
$$R: \ D_u \cap p = [0 : 1 : 1 : -s_7 : s_9] \, .$$

sections no longer points in E, wrap entire **P^1** in smooth X

Charged matter of Type II

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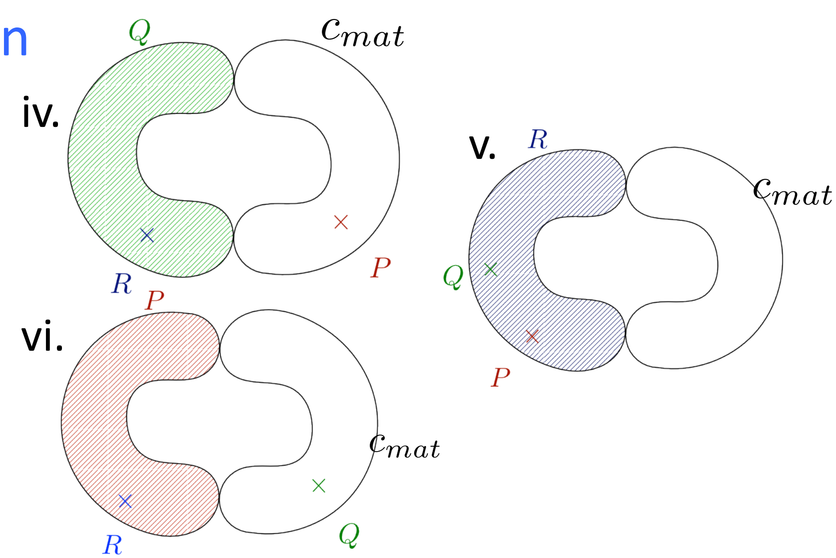
List of charged matter representations

Representation	Ill-defined section
iv. $(q_1,q_2)=(-1,1)$	Q at $s_3=s_7=0$
v. $(q_1,q_2)=(0,-2)$	R at $s_7=s_9=0$
vi. $(q_1,q_2)=(-1,-2)$	zero section P at $s_8=s_9=0$



Summary of Matter Representations

	$U(1) \times U(1)$
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X non-generic → realize SU(5) x U(1) x U(1)

Apply analogous techniques to determine matter representation

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Apply analogous techniques to determine matter representations

Specific example:

$$\begin{aligned} s_1 &= t^3 s_1' \\ s_2 &= t^2 s_2' \\ s_3 &= t^2 s_3' \\ s_5 &= t s_5' \end{aligned}$$

w/ SU(5) at t=0

Summary of Matter Representations

	$U(1) \times U(1)$	$SU(5) \times U(1) \times U(1)$
Type I	$(1,0)\ (0,1)\ (1,-1)$	$(\mathbf{5},-\frac{2}{5},0)\ (\mathbf{5},\frac{3}{5},0)\ (\mathbf{5},-\frac{2}{5},-1)$
Type II	$(-1,1)\ (0,2)\ (-1,-2)$	$(\mathbf{5},-\frac{2}{5},1)\ (\mathbf{5},\frac{3}{5},1)\ (\overline{\mathbf{10}},-\frac{1}{5},0)$

X non-generic → realize SU(5) x U(1) x U(1)

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Specific example:

$$
\begin{aligned}
s_1 &= t^3 s_1' \\
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$$

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Matter multiplicities

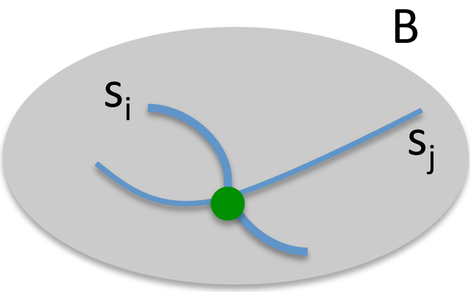
MATTER SPECTRUM IN 6D

6D matter multiplicities

[M.C.,Klevers,Piragua]

Matter multiplicities = number of points in codimension 2 matter loci in B

1. Matter of Type II: simple complete intersection



$s_i = s_j = 0$

Number of points = $\deg(s_i) * \deg(s_j)$

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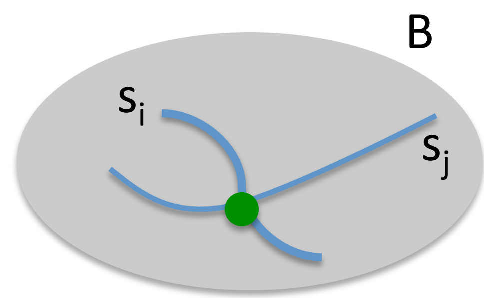

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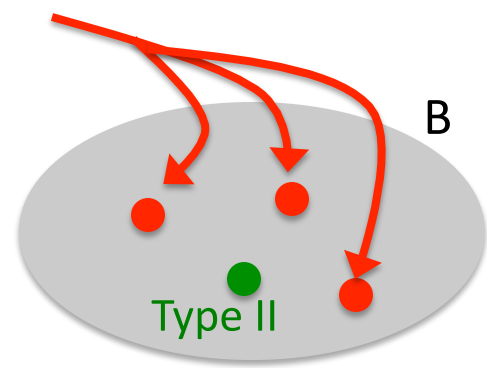
2. Matter of Type I: opposite of complete intersections

Described by prime ideals (8 polynomial equations)

Counting of points via resultant of polynomial system



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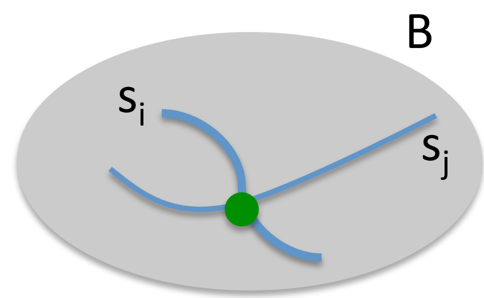

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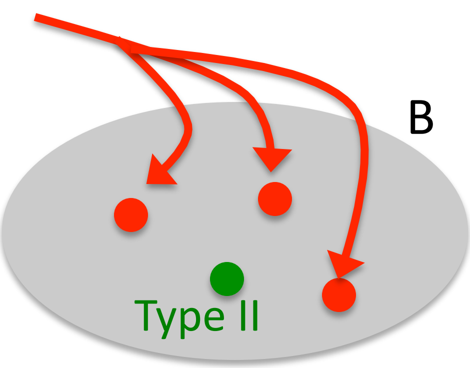

Method general! Can now apply to examples of

rk=1 MW [Morrison,Park;Mayrhofer,Palti,Weigand]

rk=2 MW [Borchman,Mayrhofer,Weigand],...

.

6D matter spectrum & multiplicities

6D matter spectrum and multiplicities can be obtained over any base B

Example: B=**P²** w/U(1) x U(1)

	(q_1, q_2)	Multiplicity
Type I	$(1,0)$	$54 - 15n_9 + n_9^2 + (12 + n_9)\, n_7 - 2n_7^2$
	$(0,1)$	$54 + 2\left(6n_9 - n_9^2 + 6n_7 - n_7^2\right)$
	$(1,1)$	$54 + 12n_9 - 2n_9^2 + (n_9 - 15)\, n_7 + n_7^2$
Type II	$(-1,1)$	$n_7\,(3 - n_9 + n_7)$
	$(0,2)$	$n_9 n_7$
	$(-1,-2)$	$n_9\,(3 + n_9 - n_7)$

Integers n_7, n_9 specify all dP_2-fibration over **P²**

Full spectrum and multiplicities also with SU(5)xU(1)x(1) group

Consistency check: spectrum found to cancel 6D anomalies!

Matter surfaces, G_4-flux & 3D CS-terms

MATTER SPECTRUM IN 4D

4D matter spectrum

[M.C.,Grassi,Klevers,Piragua]

4D-matter representations the same as in 6D (all in the fiber)

4D matter chiralities = codimension two matter loci in B + G_4-flux:

$$\chi(\mathbf{R}) = -\frac{1}{4}\int_{\mathcal{C}_{\mathbf{R}}} G_4$$

Geometry: I.Matter surfaces:

points in B_2 → matter curves $\Sigma_{\mathbf{R}}$ in B_3

$$c_{mat} \longrightarrow \boxed{\mathcal{C}_{\mathbf{R}}} \quad \downarrow \quad \Sigma_{\mathbf{R}}$$

Type II matter surfaces found

Type I matter-hard

II. G_4-flux:

Construction of homology $H_V^{(2,2)}(\hat{X})$

First construction of G_4-flux with non-holomorphic zero-section

4D matter spectrum

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Type I matter-hard

Evaluate integrals

Chiral index

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First construction of G_4-flux with non-holomorphic zero-section

Conditions for G_4-flux in F-theory

G_4-flux in F-theory = G_4-flux in M-theory + extra conditions

M/F-theory duality in D=3

c.f., John Schwarz's talk

F-theory on singular X_4

4D massless chiral matter

Further S^1 reduction

M-theory on resolved X_4

As 3D theory of 11D SUGRA reduced on X_4

D=3, N=2 theory on Coulomb branch

c.f., Nati Seiberg's talk

Match as quantum effective actions in IR

(Integrate out massive states: massive 4D matter, KK-states)

Conditions for G_4-flux in F-theory

I. G_4 in M-theory: 3D Cherns-Simons terms are classical

$$S^{3D}_{\mathrm{CS}} = \int \frac{1}{2}\Theta_{AB} A^A \wedge F^B \qquad \Theta_{AB} = \int_{\hat{X}_4} G_4 \wedge \omega_A \wedge \omega_B$$

D_A= basis of divisors on X_4

II. G_4 in F-theory (3D Coulomb branch):

some classical: 4D gaugings of RR-axions (GS) [Grimm,Kerstan,Palti,Weigand]

some exotic (set to zero) [Grimm,Savelli]

some loop-generated: massive fermions on 3D Coulomb branch +KK-states

$$\Theta^{\mathrm{loop}}_{AB} = \frac{1}{2}\sum_{\mathbf{R}} \chi(\mathbf{R}) \sum_{q\in\mathbf{R}} \sum_{k} q_A q_B \mathrm{sign}(m_{CB} + \frac{k}{r_{\mathrm{KK}}})$$

[Aharony,Hanany,Intriligator,Seiberg,Strassler]

...

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G_4-conditions

Constrain G_4 in M-theory (I.) so that θ_{AB}=0 for CS-terms that in F-theory (II.) are neither classically, nor one-loop generated ⇒

Nonzero θ_{AB} in turn determine all chiralities and all anomaly cancellations!

The full 4D spectrum

Example B=P^3 w/ U(1) x U(1): most general solution for G_4-flux

$$G_4 = a_5 n_9 (4 - n_7 + n_9) H_B^2 + 4a_5 H_B S_P + a_3 H_B \sigma(\hat{s}_Q) + a_4 H_B \sigma(\hat{s}_R) + a_5 S_P^2$$

(q_1, q_2)	4D chiralities
$(1,0)$	$\frac{1}{4}\left[a_5 n_7 n_9 (4 - n_7 + n_9) + a_3 \left(2n_7^2 - (12 - n_9)(8 - n_9) - n_7 (16 + n_9)\right)\right]$
$(0,1)$	$\frac{1}{2}\left[a_5 n_9 (4 - n_7 + n_9)(12 - n_9) - a_4 (n_7 (8 - n_7) + (12 - n_9)(4 + n_9))\right]$
$(1,1)$	$\frac{1}{4}\left[2a_5 n_9 (4 - n_7 + n_9)(12 - n_9) - (a_3 + a_4)(n_7^2 + n_7(n_9 - 20) + 2(12 - n_9)(4 + n_9))\right]$
$(-1,1)$	$\frac{1}{4}(a_3 - a_4) n_7 (4 + n_7 - n_9)$
$(0,2)$	$\frac{1}{4} n_7 n_9 (-2a_4 + a_5(4 - n_7 + n_9))$
$(-1,-2)$	$-\frac{1}{4} n_9 (n_7 - n_9 - 4)(a_3 + 2a_4 + a_5(n_7 - 2n_9))$

All 4D anomalies cancelled;

Chiralities checked against Type II matter geometric chirality calculations

The full 4D spectrum

Example $B=P^3$ w/ $U(1) \times U(1)$: most general solution for G_4-flux

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Same methods for $SU(5) \times U(1) \times U(1)$ applied:

G_4-flux has 7 parameter; all 4D chiralities determined; anomalies checked;

Chirality checked against Type II matter geometric calcultions

Summary

- Systematic construction of elliptic fibrations with rk=2 MW-groups
- D=6: Matter spectrum and multiplicity for general B

  U(1)xU(1) SU(5)xU(1)xU(1) - All Geometry
- D=4 Matter spectrum and chirality

  Geometry: Matter surfaces for Type II matter (Type I matter hard)

  G_4-flux constructed for entire class of vacua (w/fixed base)

  D=3 M/F-theory duality: Thorough formulation of G_4-flux conditions

  (new CS-terms from charged KK-states)

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Outlook

- 4D: Generalization to other bases, SUSY,… → Phenomenology
- More U(1)'s.. →

Announce: Elliptic CY with rk(MW)=3

[M.C.,Klevers, Piragua, Peng Song] to appear

Elliptic curve E with three rational points Q, R, S

Line bundle M=O(P+Q+R+S) of degree 4 on E (non-generic biquadric in **P**^{3})

⇒ Generic E: Calabi-Yau Complete Intersection (defined by two equations) in the blow-up of **P**^{3} at three points

-The birational map to the Weierstrass model worked out

-Elliptic fibration, classification

-Matter, Multiplicities...

} Work in progress