Probing the structure of quantum phases of matter with holography

matter?

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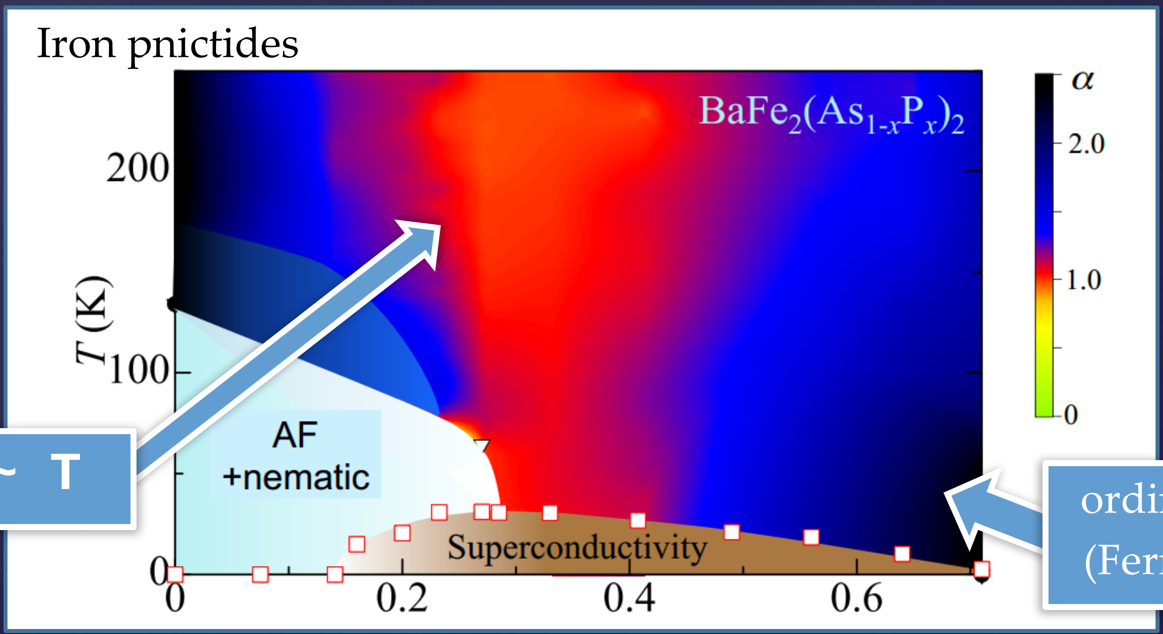
- Traces of holography in many settings – some better understood than others
  → how broad is its range of applicability?

- Recently, applications to a number of condensed matter systems [see talks by Erdmenger, Gauntlett, Liu, Takayanagi,…]

  - materials with unconventional scalings (e.g. `strange metals')
  - new poorly understood phases of matter
  - entangled systems
  - …

scaling of resistivity $\rho \sim T^{\alpha}$



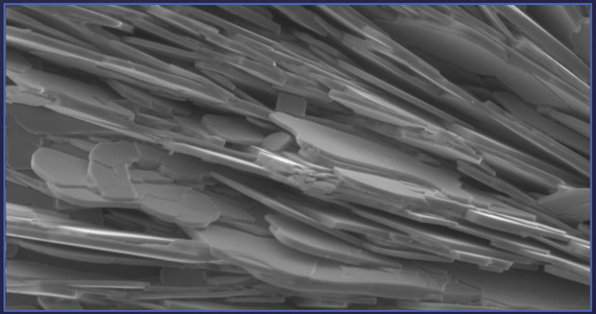
$\rho \sim T$

ordinary metal (Fermi Liquid)

- Common feature: systems whose d.o.f. are not weakly coupled
  → no notion of quasi-particles + Boltzmann/Landau theory does not apply

- Natural setting to use holography
  - a set of analytic tools to probe mechanisms behind such systems

- On the GR side, from the dialogue between the two communities:
  - new classes of (black hole) solutions
  - new types of instabilities
  - ground states with reduced symmetries (broken translations and/or rotations, anisotropic, non-relativistic geometries …)
  - new emergent scaling IR behavior
  - …

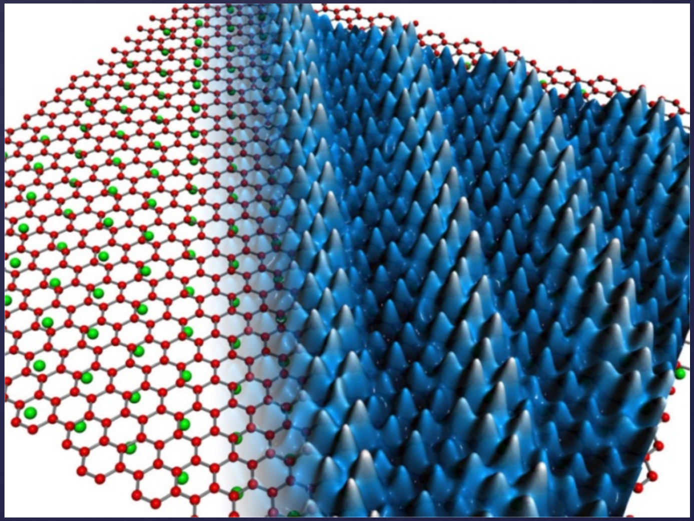


layered structure in cuprate superconductor

My focus today:

- the vacuum structure of some of these novel scaling geometries (Lisfhitz scaling and hyperscaling violation)
- features and questions associated with the rich landscape of IR phases

smectic order in a napoleon



charge density waves on sheets of high T superconductor (CaC_6)

Lifshitz scaling and hyperscaling violation

- **Non-relativistic Lifshitz scaling**

Dynamical critical exponent z → anisotropy between space and time

$$\omega \sim k^z \qquad\qquad x \to \lambda x\,, \quad t \to \lambda^z t$$

Characterizes scaling of thermo quantities $\quad s(T) \sim T^{\frac{d}{z}}$

- **Hyperscaling violation θ** → anomalous scaling of free energy

→ critical excitations do not live in the naïve number of dimensions

$$s(T) \sim T^{\frac{d-\theta}{z}}$$

shifts effective dimensionality of the system $d_{eff} = d - \theta$

d_{eff}=1 of interest for compressible states and systems w/ Fermi surface (S_{ent}~ A log A) [Huijse/Sachdev/Swingle,Takayanagi et al] But FS not easily captured by holography.

How do we geometrize these scalings?

`Minimal' model:

Exact solutions to simple EMD theory (either electric or magnetic field)

$$\mathcal{L}_{d+2} = R - 2(\partial\phi)^2 - e^{2\alpha\phi}F^2 - V_0 e^{-\eta\phi}$$

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z

$$ds^2_{d+2} = \left(-r^{-2z}dt^2 + \frac{dr^2 + d\vec{x}^2}{r^2}\right)$$

$$t \to \lambda^z t\,, \quad \vec{x} \to \lambda\vec{x}\,, \quad r \to \lambda r$$

$$\phi(r) \sim \log(r)$$

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θ

$$ds^2_{d+2} = \left(-r^{-2z}dt^2 + \frac{dr^2 + d\vec{x}^2}{r^2}\right) r^{2\theta/d}$$

$$t \to \lambda^z t\,, \quad \vec{x} \to \lambda\vec{x}\,, \quad r \to \lambda r \qquad ds \to \lambda^{\theta/d} ds$$

$$\phi(r) \sim \log(r)$$

no longer scale invariant

In general, anomalous scaling of gauge field important to understand conductive properties [Gouteraux, Gouteraux/Kiritsis, Karch]

Natural question: IR endpoint of these scaling solutions?

Solutions are supported by a **running dilatonic scalar** **ϕ ~ log r**

→ not expected to be a good description of the geometry in the deep IR

$$\mathcal{L} = R - 2(\partial\phi)^2 - e^{2\alpha\phi}F^2 - V_0 e^{-\eta\phi}$$

Effective gauge coupling of the theory $g \equiv e^{-\alpha\phi}$ drives system to

strong coupling
(magnetic case)

Expect modifications to $g(\phi)$, e.g.

$$\frac{1}{g^2} \to \frac{1}{g^2} + \xi_1 + \xi_2 g^2 + \ldots$$

(toy model for QM corrections)

weak coupling
(electric case)

Expect higher derivative terms no longer negligible

(tree level terms comparable to F^4,…)

Also curvature + tidal singularities [Copsey/Mann, Horowitz/Way, Bao/Dong/Harrison/Silverstein]

IR completion of hyperscaling violation

[arXiv:1208.1752 – J. Bhattacharya, S.C. , A. Sinkovics]

In the Lifshitz case, a toy model for QM corrections generates $AdS_2 \times R^2$ in deep IR [Harrison/Kachru/Wang 1202.6635]

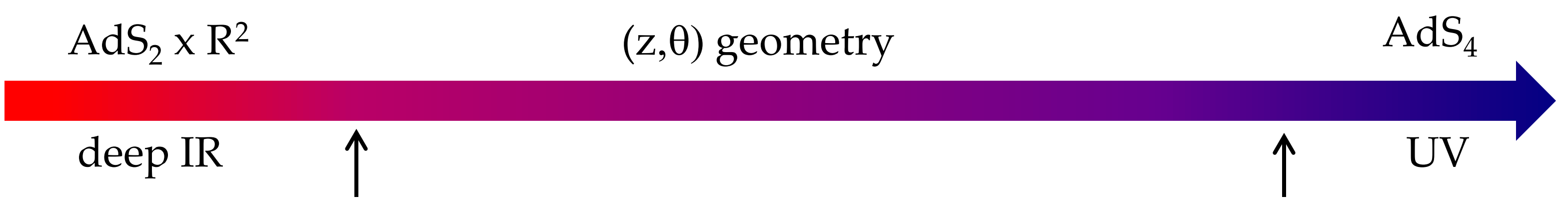
Our starting point:

$$\mathcal{L} = R - 2(\partial\phi)^2 - f(\phi)F^2 - V(\phi)$$

$$f(\phi) = e^{2\alpha\phi}\,, \quad V(\phi) = -V_0 e^{-\eta\phi}$$

Explored **conditions for emergence of $AdS_2 \times R^2$** in deep IR:

- **generic IR modifications to $f(\phi)$ and $V(\phi)$** → whether of classical or `quantum' origin (toy model of QM corrections as baby example)



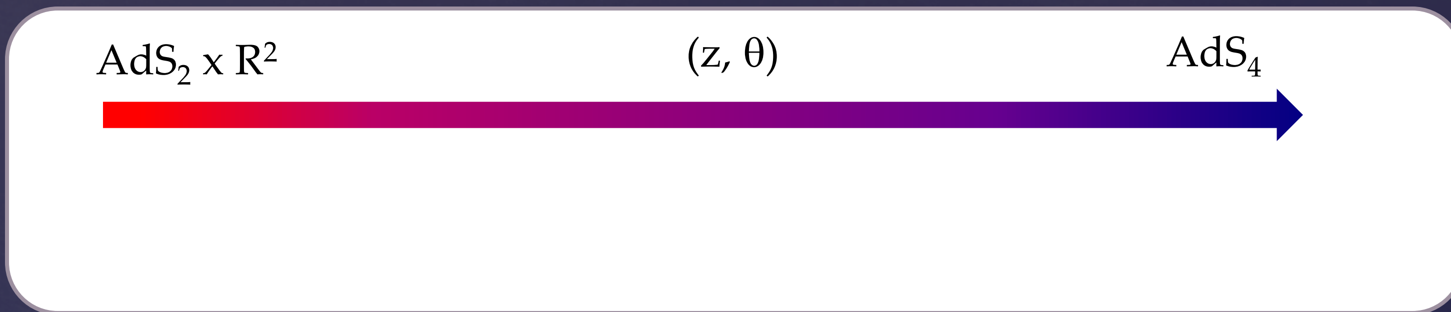
e.g. $\xi_2 g^4 + \xi_1 g^2 \simeq 1$

e.g. $V = -V_0(e^{-\eta\phi} + c_1 e^{\eta_1 \phi})$

Main Message:

[arXiv:1208.1752]

- These scaling geometries should be thought of as intermediate solutions
- In many cases their ` naïve ' **IR completion is AdS_2 x R^2**



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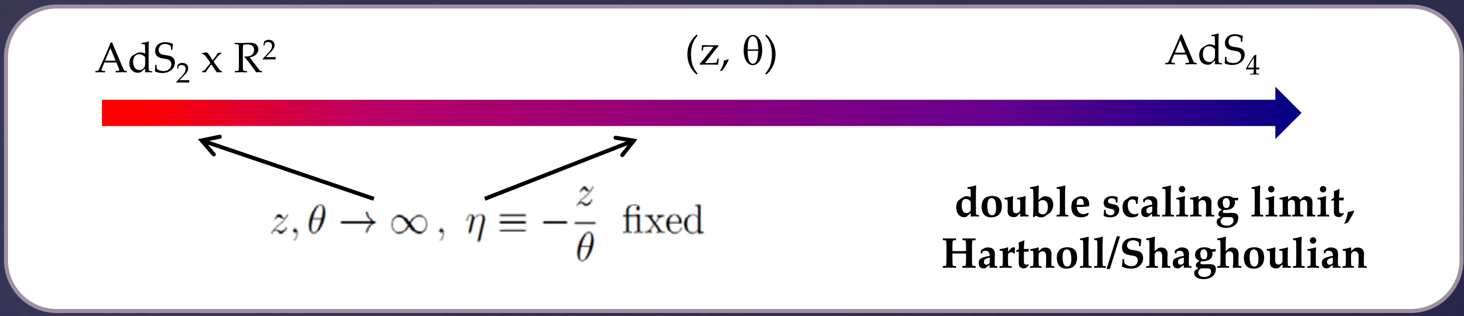

This picture has emerged in a number of setups:

- Dyonic charges yield stabilizing potential for scalar → AdS_2 x R^2 [Trivedi et al, 1208.2008]

- Higher derivative and QM corrections provide stabilization mechanism → AdS_2 x R^2 [Knodel/Liu, Peet et al, Cardoso/Haack et al,…]

- Various SUGRA truncations (sometimes with ` η-geometries' in IR or mid-IR) [Donos/Gauntlett/Pantelidou, Kulaxizi/Parnachev/Schalm,…]

Spatially Modulated Instabilities of (z,θ) geometries
[S.C. arXiv:1310.3279, S.C. and A. Sinkovics arXiv:1212.4172]

- Well-known **extensive ground state entropy of $AdS_2 \times R^2$** in violation of 3rd law (highly degenerate ground state – pathology or feature?)

- New phases expected to emerge → **$AdS_2 \times R^2$ should not be typical ground state**

Spatially Modulated Instabilities of (z,θ) geometries

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- New phases expected to emerge → **$AdS_2 \times R^2$ should not be typical ground state**

- $AdS_2 \times R^2$ suffers from **spatially modulated instabilities** in a variety of setups [Nakamura/Ooguri/Park, Donos/Gauntlett/Pantelidou,…]
  - note also non-linear instability to inhomogeneous horizons Hartnoll/Santos 1403.4612

Our logic:

- use knowledge of instabilities of AdS_2 region to identify (z, θ) geometries which are unstable to spatially modulated phases

  → ubiquitous in CM systems (smectics, spin/charge density waves…)

Spatially modulated instabilities

Simple EMD setup: $\mathcal{L} = R - V(\phi) - 2(\partial\phi)^2 - f(\phi)F_{\mu\nu}F^{\mu\nu}$

Magnetic case 1212.4172
Purely electric in 1310.3279

<u>Strategy:</u>

1) require $f(\phi)$ and $V(\phi)$ to give:

- $AdS_2 \times R^2$ in the deep IR
- an intermediate regime of (z, θ) scaling:

$$f(\phi) = e^{2\alpha\phi} + \ldots$$
$$V(\phi) = V_0 e^{-\eta\phi} + \ldots$$

corrections negligible
in intermediate region

2) identify conditions for existence of IR instabilities **(modes that violate AdS_2 bound)**

→ **instability conditions for generic $f(\phi_h)$ and $V(\phi_h)$**

3) map to conditions on (z, θ) and remaining parameters in the theory

Spatially modulated instabilities

Turn on **spatially modulated fluctuations** about IR $AdS_2 \times R^2$

Purely magnetic case [1212.4172]:

$$\delta g_{tt} = L^2 r^2\, h_{tt}(r)\cos(kx)\,, \quad \delta g_{xx} = L^2 b^2\, h_{xx}(r)\cos(kx)\,, \quad \delta g_{yy} = L^2 b^2\, h_{yy}(r)\cos(kx)$$

$$\delta A_y = a(r)\,\sin(kx)\,, \qquad \delta\phi = w(r)\cos(kx)\,,$$

k=0 case

No unstable modes for $AdS_2 \times R^2$ (Almuhairi/Polchinski, Donos/Gauntlett/Pantelidou)

Spectrum of scaling dimensions:

$$\boxed{\delta_\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 + L^2 V'' + \frac{f''}{f}}} \rightarrow V''_{eff} > 0$$

Possible instabilities controlled by **curvature of effective scalar potential**

→ **none if scalar settles to min of its effective potential** (e.g. $AdS_2 \times R^2$)

Spatially modulated instabilities

Finite k case

- spectrum of scaling dimensions (small k expansion):

$$\delta_{1,2,3,4} = \frac{1}{2} \pm \sqrt{\text{mess}} \qquad V''_{eff} = \frac{f''}{f} + L^2 V''$$

$$\text{mess} = [k = 0 \text{ terms}] \pm k^2 \left[\frac{3}{2} \pm \frac{1}{2} \pm \frac{2}{8 - V''_{eff}} \frac{f'^2}{f^2}\right]$$

AdS_2 x R^2 unstable to spatial modulations (for some k-range) whenever

$$\boxed{8 - 2\left(\frac{f'}{f}\right)^2 < \frac{f''}{f} + L^2 V'' < 8 + \left(\frac{f'}{f}\right)^2}$$

- so far we have a **generic scalar potential and gauge kinetic function** (here constant B field but analogous results for purely electric case)

Connecting with the intermediate regime

Require $f(\phi)$ and $V(\phi)$ to give rise to intermediate scaling regime, e.g.

$$f(\phi) = e^{2\alpha\phi} + \ldots$$
$$V(\phi) = V_0 e^{-\eta\phi} + \ldots$$

corrections negligible in intermediate region

Fully specifying $f(\phi)$ and $V(\phi)$ → values of (z,θ) associated with instabilities

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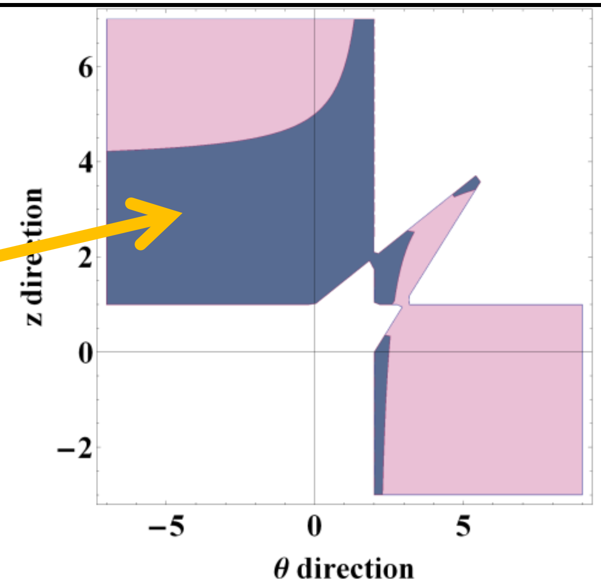
Concrete example [from electric case 1310.3279]

$$f(\phi) = e^{\alpha\phi}\,, \qquad V = V_0 e^{-\eta\phi} + \mathcal{V}(\phi)$$

spatially modulated instabilities provided constraint is obeyed

$$\frac{8}{\theta - 2z + 2} = L^2\left(\mathcal{V}''(\phi_0) - \frac{\theta^2}{(\theta-2)(\theta-2z+2)}\mathcal{V}(\phi_0)\right)$$



Take home message:

- evidence for spatially modulated phases (`stripes') as possible ground states of (certain) (z,θ) geometries

We took a shortcut to identify instabilities and used the naïve IR $AdS_2 \times R^2$

- one should be able to see these unstable modes by analyzing the (z,θ) geometries directly → Iizuka, Maeda 1301.5677

An AdS_4 IR completion of (z,θ) geometries

[Work with J. Bhattacharya and B. Gouteraux, 1407.????]

Natural question: are there other possible ground states?

- not all (z,θ) solutions are unstable to spatial modulations or even approach $AdS_2 \times R^2$

Emergent conformal symmetry in the IR?

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Emergent conformal symmetry in the IR?

Picture we are exploring:

deep IR → UV

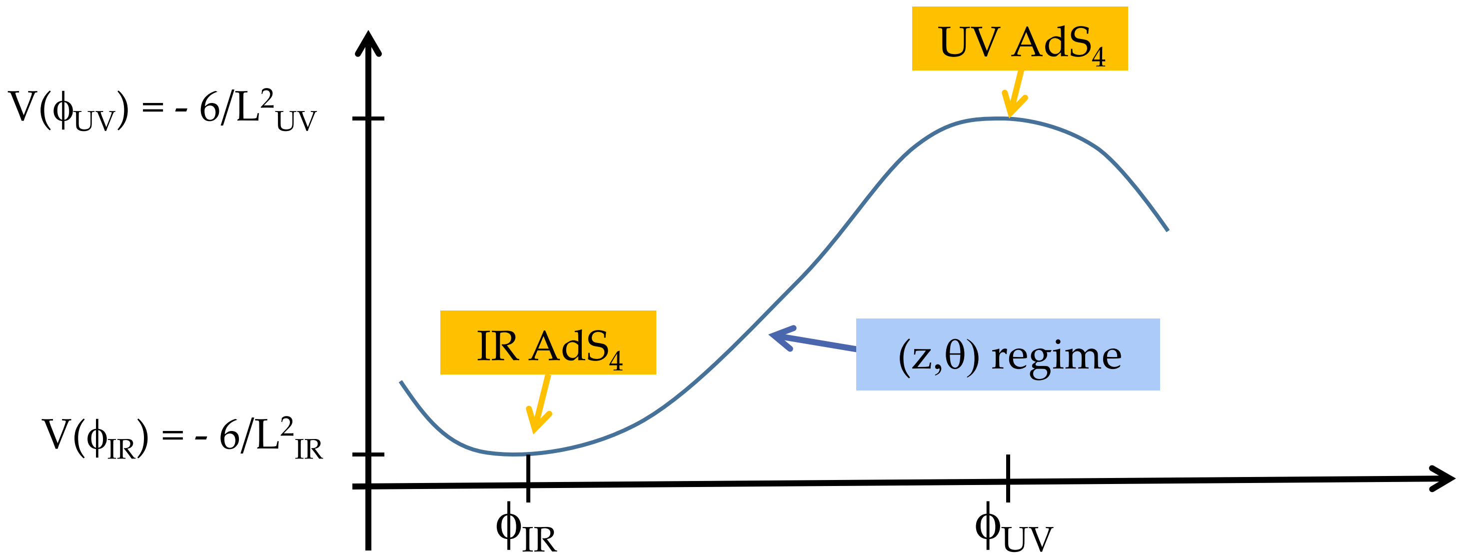
AdS_4 — (z,θ) geometry — (different) AdS_4

Analog of ground state of holographic superconductor but with intermediate hyperscaling violating regime

[Gubser/Rocha 0807.1737, Gubser /Nellore 0908.1972 … , Horowitz/Roberts 0908.3677]

Our toy model $\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - f(\phi)F^2 - W(\phi)A^2 - V(\phi)$

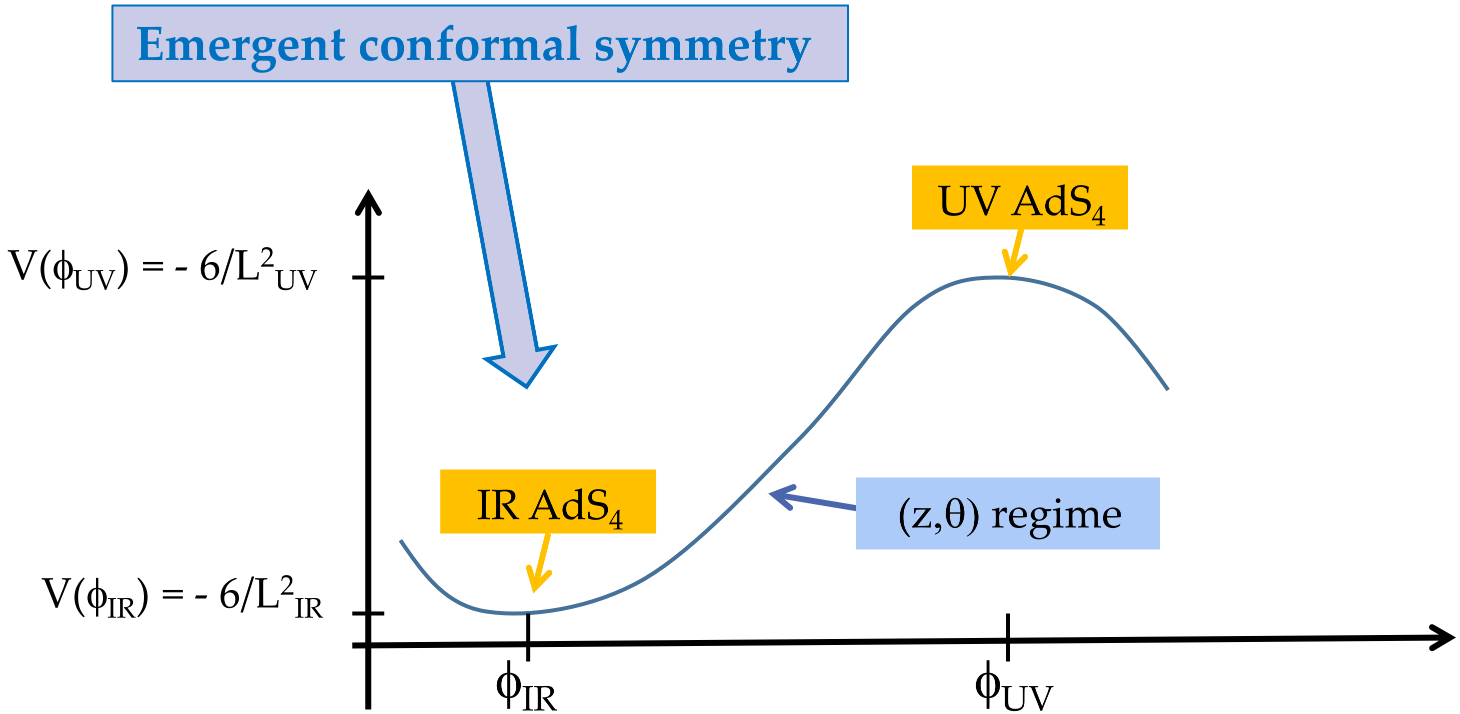
→ broken-symmetry phase of theory w/ U(1) symmetry and charged complex scalar



Note:

- scalar potential must be carefully engineered to get intermediate scaling
- massive gauge field needed to source IR AdS$_4$
- intermediate scaling regime sensitive to where charge density is concentrated

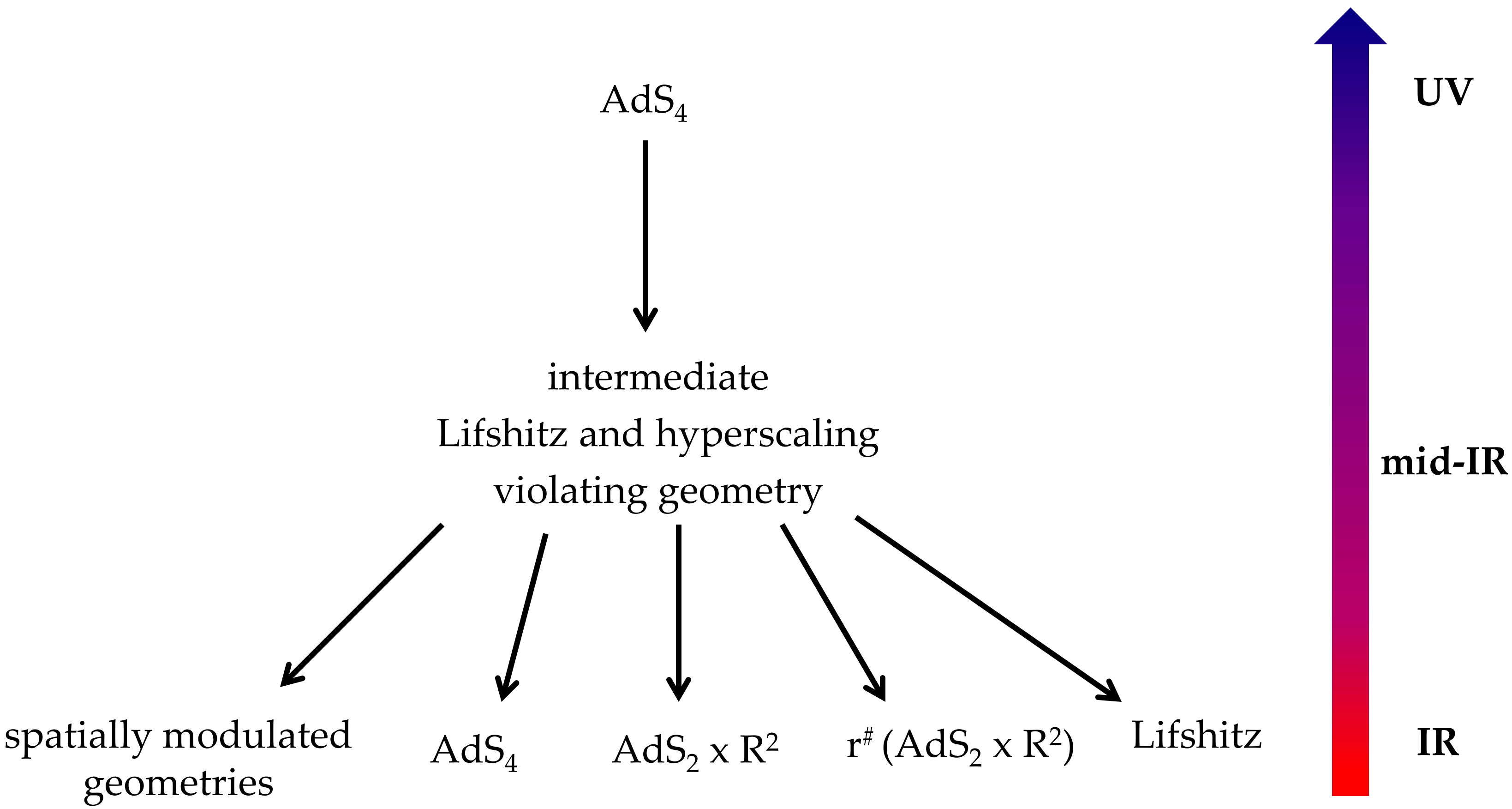
Features



- new stable ground state for scaling solutions w/out extensive entropy issues
- (z,θ) scaling regime in mid-infrared region → tunable knobs
- expect interplay between different scalings at different energy scales

→ Applications to transport?

Rich structure of IR phases



This story falls into the recent efforts to classify IR geometries (Gouteraux + Kiritsis, Iizuka et al, Kachru et al, …)

- scaling IR asymptotics at finite density
- homogeneous Bianchi geometries (e.g. helical structure)
- broken translations (`smectic' order) and/or rotations (`nematic' order)

Breaking of translational invariance crucial for transport
→ lots of work on resulting phenomenology [e.g. talks by Erdmenger and Gauntlett]

IR phases breaking only rotations can also be realized and mirror CM systems

$$ds^2 = -r^{2z}dt^2 + \frac{dr^2}{r^2} + r^{2p}dx^2 + r^{2q}dy^2$$

$$t \to \lambda^z t\,, \quad x \to \lambda^p x\,, \quad y \to \lambda^q y\,, \quad r \to \lambda^{-1} r$$

underlying theme → ground states with reduced symmetries

A number of interesting questions once symmetries are relaxed, e.g.

- Can we geometrize the interplay between different phases and scalings?
  - e.g. nematic phases with no smectic instabilities at weak coupling. At strong coupling? or Isotropic – nematic – smectic transitions?

- Competing orders?
  - competition between possible sources of instabilities and phases [new ω-deformed SO(8) gauged SUGRA theories, work in progress with Y. Pang, C. Pope, J. Rong]

- Holographic RG flows w/out Lorentz invariance: any monotonicity?
  [SC + Xi Dong, arXiv:1311.3307]
  - generically breakdown of monotonicity w/out Lorentz invariance
  - in certain cases, one can still identify criteria on UV geometry that ensure monotonicity (c-function from entanglement entropy of a strip)
  - more fundamental understanding? [e.g. talk by Casini]

To wrap up…

- The structure of phases from gravity is much richer than anticipated, with interesting emergent IR behaviors
- Novel ground states with reduced symmetries
- As the dialogue between gravity and quantum field theories continues, we gain more insight into the mechanisms driving strongly coupled phases of matter

Thank You