Toda Theory From Six Dimensions

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Daniel Jafferis & C.C.
ArXiv: 1406.XXXX

General Motivation

- Reductions of 6d (2,0) ---> geometric perspective on SQFTs
  - s-dualities, mirror symmetries, ...
- Supersymmetric localization ---> new partition functions
  - instanton sums, sphere partition functions, indices, ...
- Fusion ---> novel interpretations of partition functions for QFTs with 6d parent

Specific Motivation

6d (2,0) on $S^4 \times \Sigma$

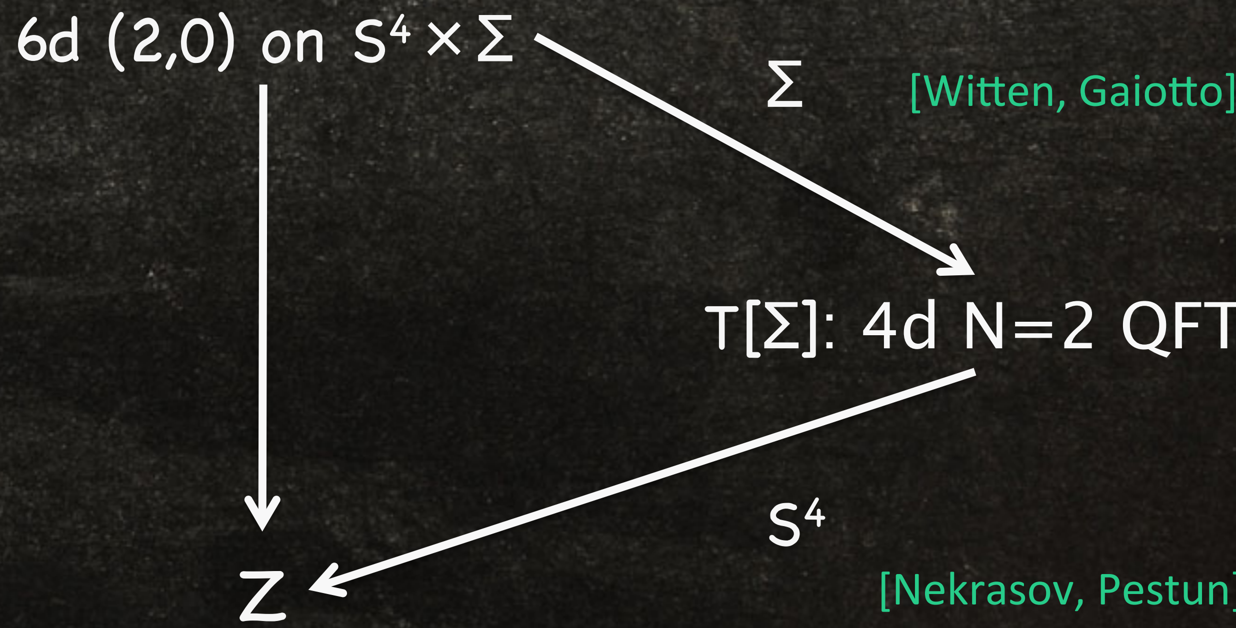
$\downarrow$

Z

- 6d conformal invariance + twisting on Σ + supersymmetry

---> Z independent of vol(Σ) and vol(S^4)

Specific Motivation



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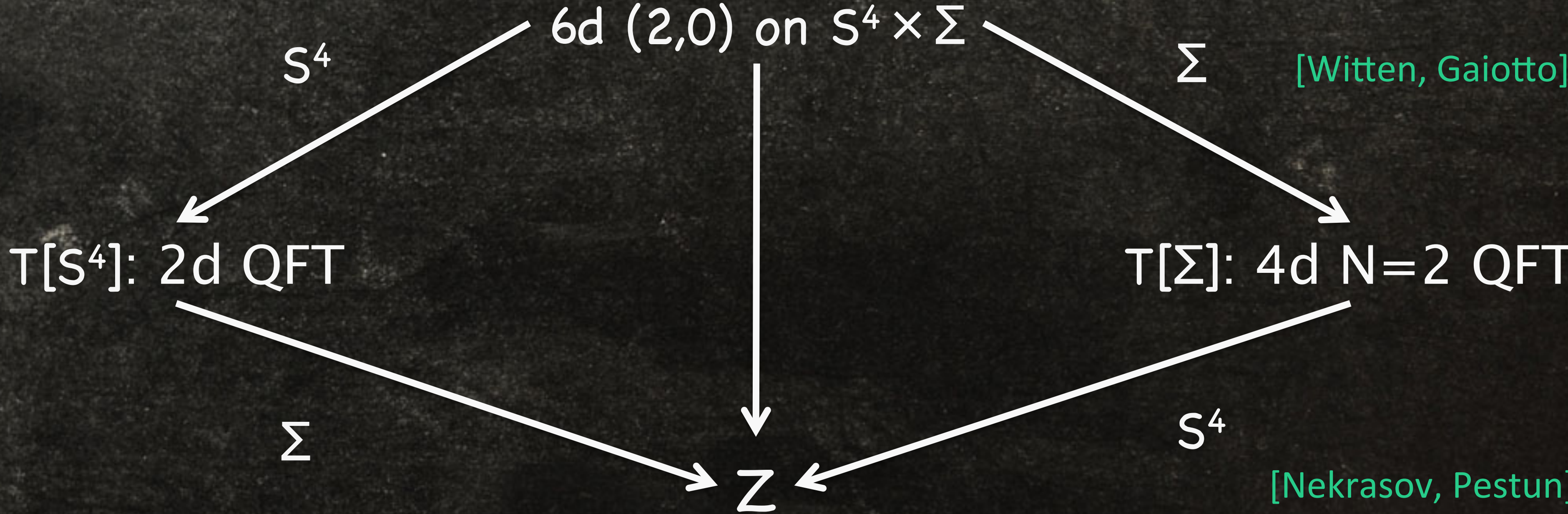

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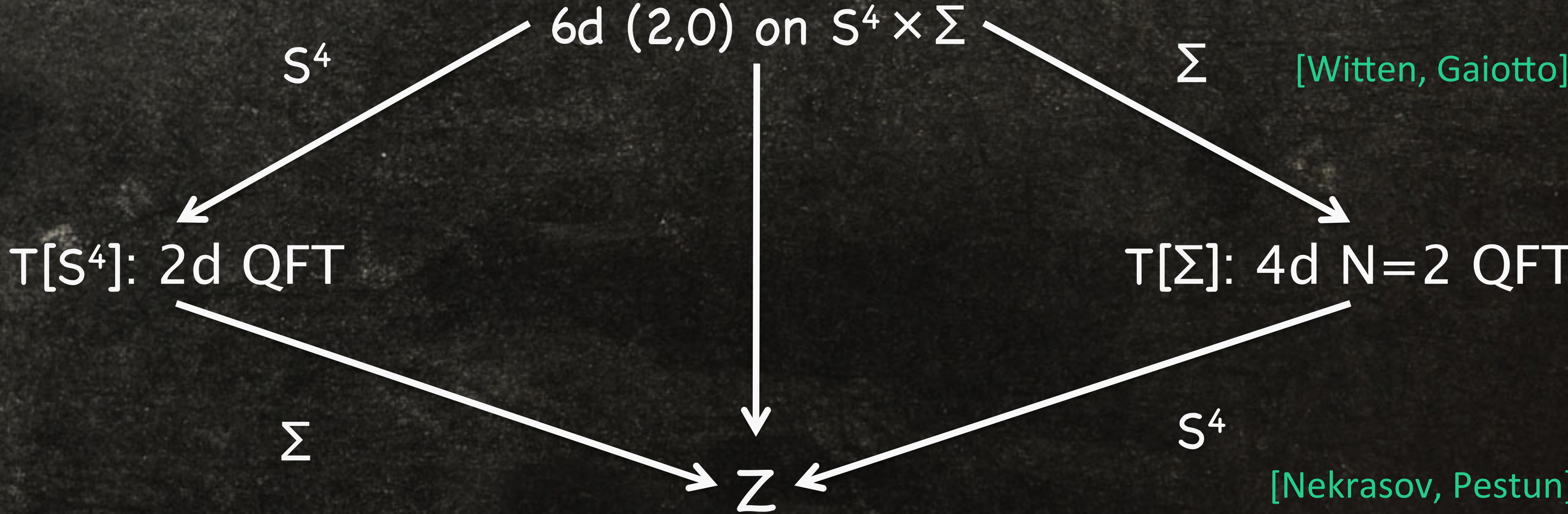

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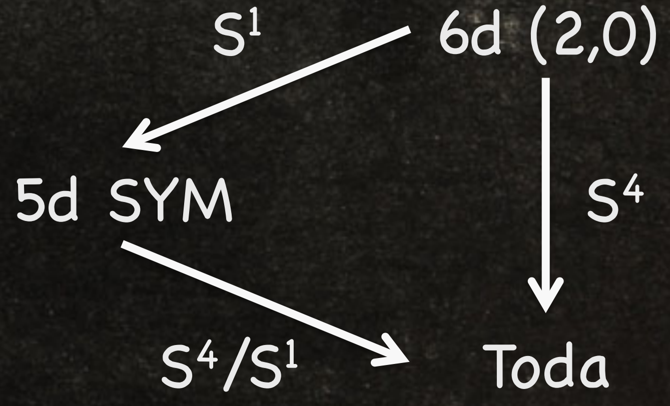
AGT conjecture: T[S^4] = Toda CFT ---> N-1 real scalars Φ^i

$L \sim \Sigma_{ij}\, C_{ij}\, d_\mu\Phi^i d_\mu\Phi^j - \Sigma_i \exp(\, ½\, \Sigma_j C_{ij}\Phi^j)$ (C_{ij} = SU(N) Cartan matrix)

Result & Method

- Result: derivation of Toda theory via reduction from 6d
- Method: factorize reduction:

  [Kim-Kim-Kim, Fukada-Kawana-Matsumiya, Lee-Yamazaki, Jafferis-C.C.]



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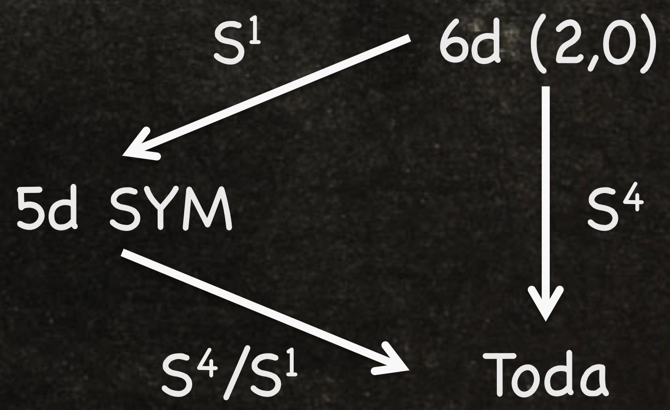

<u>why it works:</u>

- Higher derivative corrections to 5d SYM unimportant
  - suppressed by small $r(S^1)$
  - Q exact
- S^4/S^1 not smooth but at singularity, $g^2_{ym} \sim r(S^1) \to 0$

  ---> understand with weakly coupled 5d physics

S^4 Geometry

- Compactification on $S^4 \times R^{1,1}$ has OSP(2|4) symmetry
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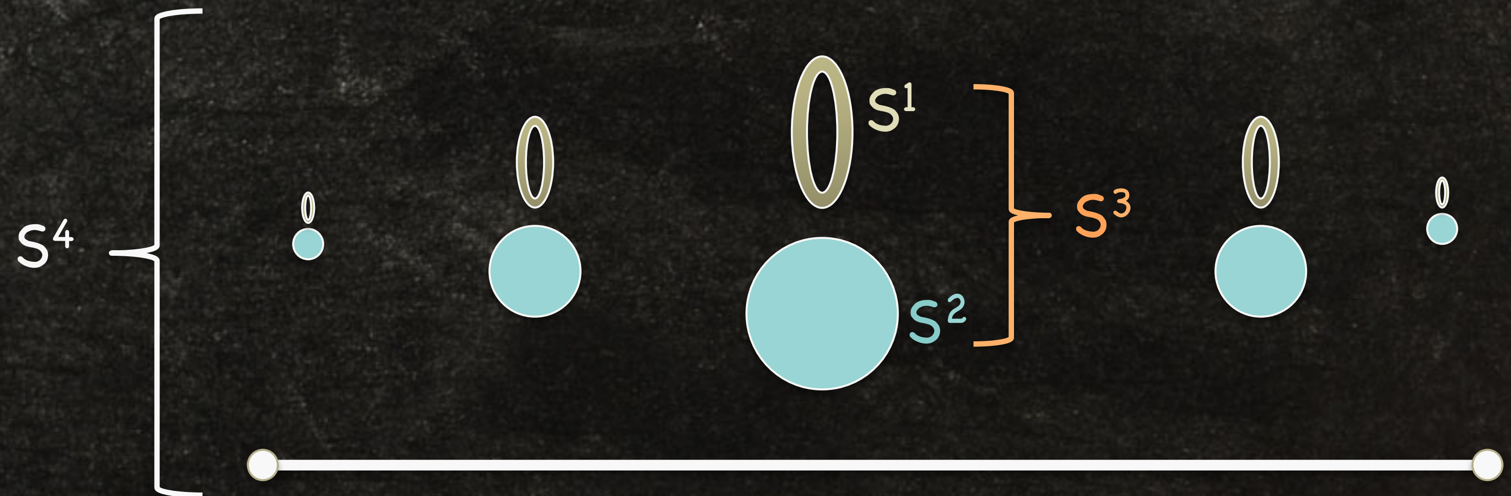

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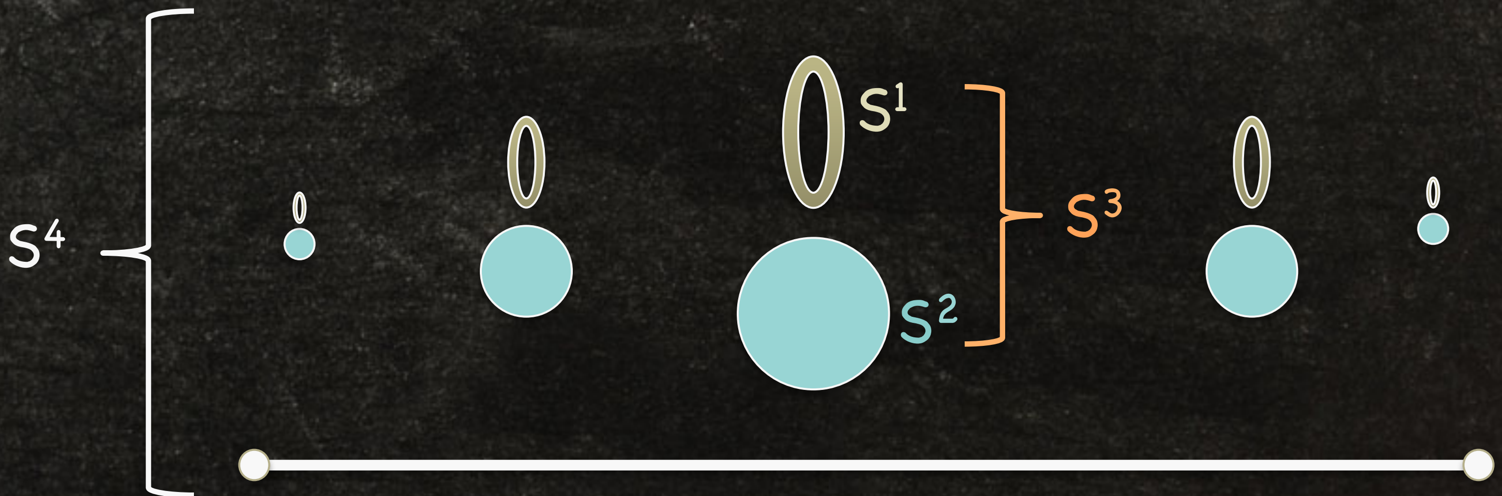

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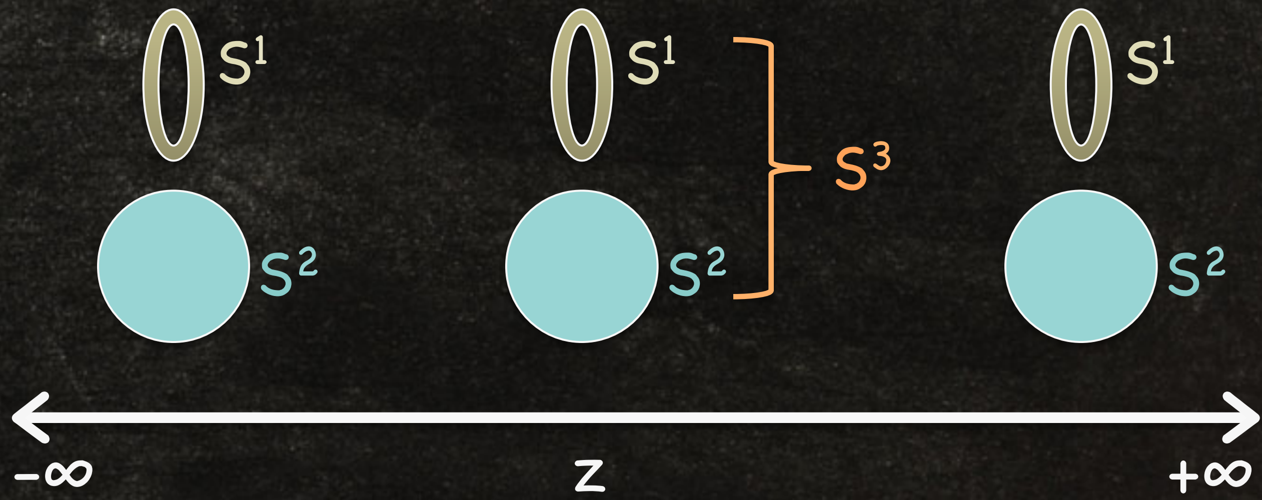

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  ---> S^3 now constant radius

  ---> $ds^2 = (d\Omega_3)^2 + dz^2 + \cosh^2(z/r)(-dt^2+dx^2)$

Plan of Attack

- Reduce from 6d (2,0) to 5d SYM on Hopf circle of S^3
- Reduce 5d SYM on S^2 with one unit of RR-flux
- Place resulting 3d theory on manifold $R^{1,2}$ with non-trivial metric:

$$ds^2 = dz^2 + \cosh^2(z/r)(-dt^2+dx^2)$$

- Add suitable boundary conditions at $|z| = \infty$
- Determine effective boundary theory

Relation To Chern-Simons

- 5d SYM on S^2 with 1 unit of RR-flux ---> complex CS in 3d

[Lee-Yamazaki, Jafferis-C.C.]

  - complex SL(N,C) gauge field $\mathcal{B} = A + i X$
  - $L = {}^{1}/_{8\pi} \left[\mathrm{Tr} \left(\mathcal{B}\, d\mathcal{B} + \tfrac{2}{3} \mathcal{B}^3 \right) + \mathrm{Tr} \left(\bar{\mathcal{B}}\, d\bar{\mathcal{B}} + \tfrac{2}{3} \bar{\mathcal{B}}^3 \right) \right]$

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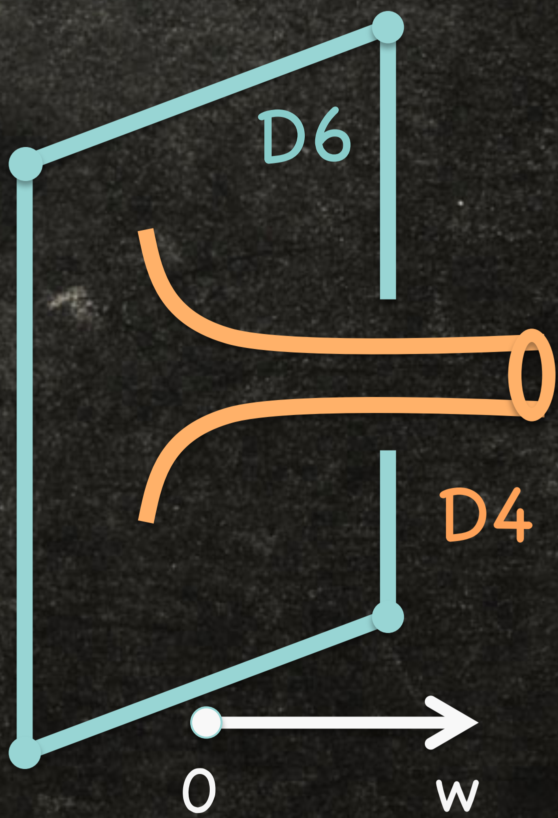
- Puzzle: How does SUSY reduction of SU(N) gauge theory result in bosonic SL(N,C) gauge theory?

- Answer: the two concepts are equivalent!

  - SU(N) covariant Lorenz gauge condition $D^a X_a = 0$ breaks SL(N,C) to SU(N)

  - fermions reinterpreted as Faddeev-Popov ghosts

Boundary Data - One Side



From IIA perspective --> D4 ending on D6

- scalars have a Nahm pole $X_a \sim T_a / w$

[Diaconescu]

- T_a valued in SU(2), $[T_a, T_b] = \varepsilon_{abc} T_c$

- A chosen so that SL(N,C) field, $\mathcal{B}$, is flat

---> $\mathcal{B} = (iT_3)\, dw/w + (T_+)\, dx_+/w$

- Fermions lifted by Dirichlet condition

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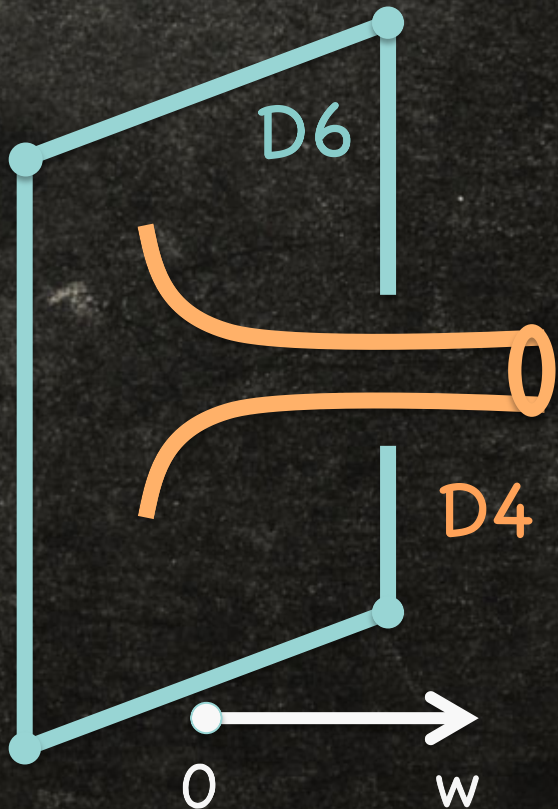

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- The terms ... are less singular in w, and are fluctuating fields. They give rise to a chiral Toda theory

Executive Summary

- Question: What are the Toda fields?

- Answer: The Toda fields are modes of the 5d scalars X_a localized at the poles of the S^4

[Nekrasov-Witten]

Map to Toda

- CS Theory --> boundary theory of currents (WZW-model)

[Witten]

$\mathcal{B} = F^{-1} d F + F^{-1} (H^{-1} dH) F$ (pure gauge HF)

F is background giving Nahm pole, H is dynamical

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Properties of H:

- Gauge $D^a X_a = 0$ --> H is SU(N) valued not SL(N,C) valued

- $H = H(x_+, x_-)$ depends only on boundary coordinates

- Flatness of $H^{-1} dH$ --> $H = H(x_+)$ is chiral

- Regularity of ... --> H fixed by N-1 real scalars = Toda fields

Map to Toda

- More Briefly: Nahm boundary conditions provide constraints on WZW currents which reduce it to Toda
  [Balog-Fehér-Forgács-O'Raifeartaigh-Wipf]
- Each boundary (region near a pole in S^4) gives a chiral half of Toda. Together they form the full non-chiral Toda.

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Central Charge: [Elitzer-Moore-Schwimmer-Seiberg]

- Toda central charge, $c = N-1 + N(N^2-1)(b+b^{-1})^2$. S^4 gives $b = 1$

- Recover b by squashing geometry

Future Directions

- Understand the dictionary between Toda operators, and 6d defect operators
- Use similar techniques to study 6d (2,0) on other geometries. An interesting case is S^6 which should lead to direct information about 6d correlation functions

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Thanks for Listening!