Constrainting effective actions via scattering amplitudes

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Introduction

- Complicated Feynman diagram computation of scattering amplitudes sometimes gives to very simple results.

- Search for new ways of studying scattering amplitudes: New methods, new mathematical structures and new symmetries, new formulations, ...

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$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{c_2}{\Lambda^4}\partial^4\phi^4 + + \frac{c_3}{\Lambda^6}\partial^6\phi^4 + \ldots ,$$

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- For an effective field theory, we are interested in low-energy physics below a certain energy scale Λ. The effective theory can be studied in the expansion of $1/\Lambda$,

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- Low-energy physics are constrained by consistent conditions, symmetries etc.. One efficient way to realize them is using scattering amplitude with some advantages: independent of field redefinition, non-linearity is encoded in factorizations...

Outline

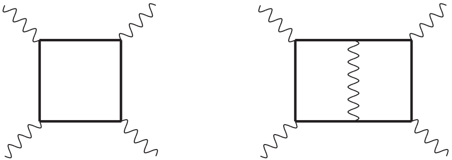
- Supersymmetry constraints on the effective action of $\mathcal{N} = 4$ SYM in the Coulumb branch.
- Constraints from breaking conformal symmetry in the form of soft theorems.
- Scale invariance v.s. conformal symmetry.

Effective action of $\mathcal{N}=4$ SYM in Coloumb branch

- We are interested in $\mathcal{N}=4$ SYM in the Coloumb branch, $U(N+1) \to U(N) \times U(1)$. We focus on the $U(1)$ part.
- Expand in large m_w, the effective action takes the form:

$$S_{\text{eff}} = -\frac{1}{4}F^2 + \frac{c_4^{(0)}}{m_w^4}F^4 + \frac{c_6^{(0)}}{m_w^8}F^6 + \frac{c_4^{(2)}}{m_w^8}\partial^4 F^4 + \frac{c_4^{(3)}}{m_w^{10}}\partial^6 F^4 + \dots .$$

- Example of perturbation contributions:



$$A_4^{1-\text{loop}} = F^4 \int \frac{d^4\ell}{(2\pi)^4} \frac{g^4 N}{(\ell^2+m_w^2)((\ell+p_1)^2+m_w^2)((\ell+p_{12})^2+m_w^2)((\ell-p_4)^2+m_w^2)} .$$

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- F^6 ($F_-^2 F_+^4$ and its conjugate) is not generated at one loop, but appears at two loops, and it was conjectured to be two-loop exact! [Buchbinder, Petrov, Tseytlin (2001)]

non-renormalization theorem from $\mathcal{N} = 4$ SUSY

- This non-renormalization theorem is due to the fact that, with $\mathcal{N} = 4$ SUSY, there is no consistent local superamplitude corresponding to $F_-^2 F_+^{2\ell}$. Thus the contributions (coming from $F_-^2 F_+^{2q}$ with $q \leq \ell$) should sum up to 0.

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- It gives to a recursion relation to relate $F_-^2 F_+^{2\ell}$ to $F_-^2 F_+^2$, while the later is one-loop exact leads to $F_-^2 F_+^{2\ell}$ being ℓ-loop exact,

$$\mathcal{L}_{\mathrm{F_-^2\,F_+^{2q}}} = \sum_{\ell=1}^{\infty} (4)^{\ell-1} \left(-\frac{\lambda}{2(4\pi)^2} \right)^{\ell} \frac{(F_-)^2 (F_+)^{2\ell}}{m_w^{2\ell}} \,,$$

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- Go beyond SUSY: Further constraints from the spontaneously broken symmetries in the form of Soft theorems.

Soft theorems for $\mathcal{N} = 4$ SYM in Coloumb branch

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- Due to the (broken) symmetries, amplitudes with Goldstone boson π satisfy soft theorems:

$$A_n(\phi_1, \cdots, \phi_{n-1}, \pi_n)|_{p_n \to 0} = \sum_{i=1}^{n-1} \langle \phi_1, \cdots, \delta\phi_i, \cdots, \phi_{n-1} \rangle|_{\mathrm{LSZ}}$$

  $\delta\phi_i$ is the infinitesimal transformation of the hard particle ϕ_i under the generators of the symmetry.

Soft theorems from spontaneously breaking conformal symmetry

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- In the case of dilaton of conformal symmetry breaking,

$$\begin{aligned}\delta\phi &= [\mathcal{D},\phi] = i(d + x^\mu\partial_\mu)\phi\,,\\ \delta_\mu\phi &= [\mathcal{K}_\mu,\phi] = i\left((2x_\mu x_\nu - \eta_{\mu\nu}x^2)\partial^\nu + 2d\,x_\mu\right)\phi\end{aligned}$$

- It leads to the soft theorems of dilatons [Boels, Wormsbecher (2015), Huang, C.W. (2015), Di Vecchia, Marotta, Mojaza, Nohle (2015)]

$$v\,A_n\Big|_{p_n\to\tau p_n} \to \left(\cdots + \mathcal{S}_n^{(0)} + \tau\mathcal{S}_n^{(1)}\right)A_{n-1} + \mathcal{O}(\tau^2)\,,$$

with soft factors,

$$\begin{aligned}\mathcal{S}_n^{(0)} &= \sum_{i=1}^{n-1}\left(p_i\cdot\frac{\partial}{\partial p_i} + \frac{d-2}{2}\right) - d \quad \text{From } \mathcal{D}\\ \mathcal{S}_n^{(1)} &= p_n^\mu\sum_{i=1}^{n-1}\left[\frac{1}{2}\left(2\,p_i^\nu\frac{\partial^2}{\partial p_i^\nu\partial p_i^\mu} - p_{i\mu}\frac{\partial^2}{\partial p_{i\,\nu}\partial p_i^\nu}\right) + \frac{d-2}{2}\frac{\partial}{\partial p_i^\mu}\right] \quad \text{From } \mathcal{K}_\mu\,.\end{aligned}$$

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- Standard BCFW cannot apply here due to 'bad' large-z behaviour. We instead use recursion relations based on soft theorems [Cheung, Kampf, Novotny, Sheng, Trnka (2015)][Luo, C.W. (2015)].
- Recursion relations using soft theorems of the (broken) conformal symmetry lead to: all the amplitudes up to the order s^n are fully determined by the k-point amplitude at order s^k for $k \leq n$.

Constraints on pure dilaton effective actions

- Summary:

$s^k \setminus$ # of points	4	5	6	7	$\cdots$
2	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
3	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
4	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
5	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$
6	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$
7	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Table: The $\times$ is to indicate the amplitudes that have to be computed by other means, then all other amplitudes marked with $\checkmark$ are completely determined by the soft theorems.

Constraints on pure dilaton effective actions

- If each scalar mostly carries one derivative ($\mathcal{L}_{\text{eff}}(\phi, \partial\phi)$): at $2n$ or $(2n+1)$ points, amplitudes mostly go as s^n:

s^n \ # of points	4	5	6	7	8	$\cdots$
2	×	✓	✓	✓	✓	✓
3	0	0	✓	✓	✓	✓
4	0	0	0	0	✓	✓
5	0	0	0	0	0	✓
⋮	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

It is uniquely fixed and identical to the conformal DBI.

Constraining effective action with soft theorems and SUSY

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- For order-s^2 amplitudes in $\mathcal{N} = 4$ SYM in the Coulumb branch (or $\partial^4 \phi^n$), we find they are all one-loop exact, and in fact identical to conformal DBI.

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- For order-s^2 amplitudes in $\mathcal{N} = 4$ SYM in the Coulumb branch (or $\partial^4 \phi^n$), we find they are all one-loop exact, and in fact identical to conformal DBI.

- At order s^3 ($\partial^6 \phi^n$), they are all two-loop exact, and identical to conformal DBI.

Constraining effective action with soft theorems and SUSY

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- At order s^4 $(\partial^8\phi^n)$: four-point amplitude has an unique kinematics structure but with a highly non-trivial coefficient

$$\mathcal{A}_4^{(4)} = c_4^{(4)}(g, N) \times (s^2 + t^2 + u^2)\,\mathcal{F}^4\,.$$

$c_4^{(4)}(g, N)$ is expected to receive all-loop and instanton contributions, and should be constrained by SL(2, Z) symmetry.

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- All higher-point amplitudes at order s^4 are completely determined by a single coefficient $c_4^{(4)}(g, N)$ via soft theorems.

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- Up to order s^4, the effective action can be summarised as

$$\sum_{m\leq 8}\mathcal{L}_{\partial^m\phi^n} = c_4^{(4)}(g,N)\mathcal{L}^{(1)}_{\partial^8\phi^n} + \sum_{m\leq 8}\mathcal{L}^{\mathrm{DBI}}_{\partial^m\phi^n}\,.$$

$\mathcal{L}^{(1)}_{\partial^8\phi^n}$ is the one-loop Lagrangian (fixed by soft theorems!).

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  $\mathcal{L}^{(1)}_{\partial^8\phi^n}$ is the one-loop Lagrangian (fixed by soft theorems!).

- Effective action at order s^5 are completely determined by two four-point coefficients:

$$\mathcal{L}_{\partial^{10}\phi^n} = c_4^{(5)}(g,N)\mathcal{L}^{(1)}_{\partial^{10}\phi^n} + c_4^{(4)}(g,N)\mathcal{L}^{(2)}_{\partial^{10}\phi^n} + \mathcal{L}^{\mathrm{DBI}}_{\partial^{10}\phi^n}\,,$$

  $\mathcal{L}^{(2)}_{\partial^{10}\phi^n}$ is the two-loop Lagrangian (fixed by soft theorems!).

Scale invariance v.s. Conformal symmetry

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- In our language, the question can be framed as, to what extent does the subleading soft theorem due to conformal boost follow from the leading behaviour stemming from dilation symmetry?
- We find that if n-point order-s^k amplitude satisfies the leading soft theorem, it automatically obeys the subleading soft theorem, with $n = 5$ or $n > 2k$ and the lower-point amplitudes entering factorizations satisfy both leading and subleading soft theorems.

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A seven-point example of order s^3:

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A seven-point example of order s^3:

- Using both leading and sub-leading soft theorems:

$$
\begin{aligned}
A_6^{(3)} &= -c_5^{(3)}(s_{12}^3 + \mathcal{P}_6) - \left(\frac{c_5^{(3)}}{2} + (c_4^{(2)})^2 \right) (s_{123}^3 + \mathcal{P}_6) \\
&+ (c_4^{(2)})^2 \left((s_{12}^2 + s_{13}^2 + s_{23}^2) \frac{1}{s_{123}} (s_{45}^2 + s_{46}^2 + s_{56}^2) + \mathcal{P}_6 \right) .
\end{aligned}
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\end{aligned}
$$

- Now, the leading soft theorem alone allows us to determine order-s^3 7-point amplitude

$$
A_7^{(3)} = c_5^{(3)}(s_{12}^3 + \mathcal{P}_7) + \left(c_5^{(3)} + 3(c_4^{(2)})^2 \right) (s_{123}^3 + \mathcal{P}_7) - (c_4^{(2)})^2 A_7^{\rm fac} .
$$

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- Using both leading and sub-leading soft theorems:

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- $A_7^{(3)}$ with particular parameters fixed by the leading soft theorem does satisfy the sub-leading soft theorem automatically.

Conclusions and remarks

- The requirement of having consistent S-matrix can impose highly non-trivial constraints on the theory.
- Both SUSY and soft theorems can strongly constrain the effective action of $\mathcal{N} = 4$ SYM (as well as (2,0)) in the Coloumb branch, and lead to new non-renormalization theorems.
- With some conditions, we observe amplitudes determined by leading soft automatically satisfy the subleading soft theorem.
- It would be interesting to explore other possible constraints such as SL(2,Z) symmetry.
- We may eventually require explicit higher-loop as well as multi-instanton data, at least for some lower-point cases.

Thank you!