BMS Vacua and Black Holes

Geoffrey Compère

Université Libre de Bruxelles
Belgium

Strings 2016,
Beijing, Aug 4th, 2016

Talk based on

- "Vacua of the gravitational field",
  G.Compère, Jiang Long, JHEP, arXiv :1601.04958
- "Classical static final state of collapse with supertranslation memory,
  G.Compère, Jiang Long, CQG, arXiv :1602.05197
- "Symplectic and Killing symmetries of AdS_3 gravity : holographic vs boundary gravitons",
  G.Compère, Pu-Jian Mao, A. Seraj, M.M. Sheikh-Jabbari, JHEP, arXiv :1511.06079

Intuition : AdS_3 (Einstein gravity)

Spectrum (Lorentzian signature) :

- AdS_3, conical defects/excesses and the BTZ black holes
- Holographic gravitons $\leftrightarrow$ Boundary stress-tensor

AdS_3/Defects/BTZ with Virasoro hair :

$$ds^2 = \ell^2 \frac{dr^2}{r^2} - \left(r dx^+ - \ell^2 \frac{L_-(x^-) dx^-}{r} \right) \left(r dx^- - \ell^2 \frac{L_+(x^+) dx^+}{r} \right).$$

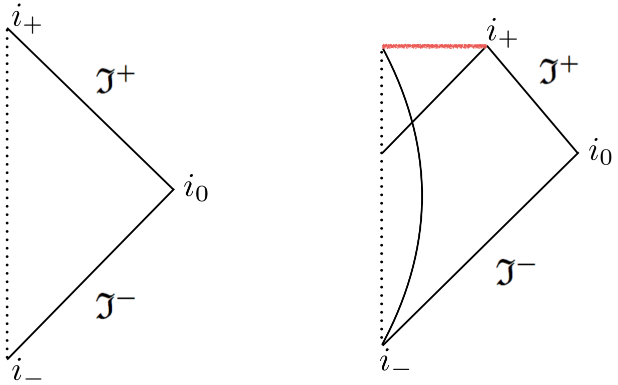
Two not so well-known properties :

- Two Virasoro charges $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$ are conserved everywhere in the bulk (“symplectic symmetry”)
- AdS_3 with Virasoro hair admits a sort of defect which sources the Virasoro charges

Main objective : Describe Minkowski and Schwarzschild with BMS (soft) hair.

The essentials on the BMS group

The BMS group is the Asymptotic Symmetry Group of 4*d* Asymptotically flat spacetimes at past and future null infinity.



The BMS group contains the Poincaré group as a subgroup. It contains an infinite set of commuting *supertranslations*.

[Bondi, van der Burg, Metzner, 1962] [Sachs, 1962]

The BMS group is the group of diffeomorphisms which

- preserve physically motivated boundary conditions written in a convenient gauge
- are associated with non-trivial canonical charges

Physically motivated boundary conditions mean :

- admitting the Kerr metric and gravitational waves
- with finite and well-defined canonical charges
- with positive energy
- allowing to describe memory effects [Zeldovich, Polnarev, 1974] [Christodoulou, 1991]
- allowing for small perturbations to decay (non-linear stability) [Christodoulou, Klainerman, 1993]
- allowing to describe a semi-classical S-matrix which obeys all known theorems [Weinberg, 1965] [Cachazo, Strominger, 2014]

The BMS algebra and its representation

$$BMS \simeq so(3,1) \oplus \text{Supertranslations}$$

The supertranslation symmetry vectors read as

$$T(\theta,\phi)\partial_u + \dots$$

The lowest spherical harmonics of $T(\theta,\phi)$ correspond to translations, e.g. around Minkowski

$$\partial_z = -\cos\theta\partial_u + \cos\theta\partial_r - \frac{1}{r}\sin\theta\partial_\theta.$$

The higher spherical harmonics lead to supertranslations.

BMS invariance is spontaneously broken. The boundary conditions lead to an order parameter : the supertranslation field $C(\theta,\phi)$ which shifts under supertranslations as

$$\delta_T C(\theta,\phi) = T(\theta,\phi).$$

[He, Lysov, Mitra, Strominger, 2014]

The memory effects

The constraints of Einstein's equations imply at $\mathfrak{I}^+$: [Frauendiener, 1992] [Geroch, Winicour, 1981] [Strominger, Zhiboedov, 2014]

$$-\frac{1}{4}\nabla^2(\nabla^2+2)(C|_{u_2}-C|_{u_1}) = m|_{u_2} - m|_{u_1} + \int_{u_2}^{u_1} du T_{uu},$$

$$T_{uu} \equiv \frac{1}{4}N_{zz}N^{zz} + 4\pi G \lim_{r\to\infty}[r^2 T_{uu}^{matter}].$$

The supertranslation field $C(\theta,\phi)$ is effectively shifted by a supertranslation after the passage of radiation.
This conservation law leads to the memory effects : the geodesic deviation vector s^A, $A=\theta,\phi$ gets shifted

$$s^A|_{u_2} - s^A|_{u_1} \sim \frac{1}{r}\partial^A\partial_B(C|_{u_2}-C|_{u_1})s^B$$

and leads to a finite relative displacement and a finite time shift of inertial detectors. [Zeldovich, Polnarev, 1974] [Christodoulou, 1991].

The extended BMS algebra

[Barnich, Troessaert, 2010]

$$Ext\ BMS \simeq \text{Superrotations}^* \oplus \text{Supertranslations}^*$$

where

$$\begin{aligned} \text{Superrotations}^* &\simeq \text{Vir}^* \oplus \text{Vir}^*, \\ \text{Supertranslations}^* &\simeq \text{Regular supert.} \oplus \text{Meromorphic supert.} \end{aligned}$$

The Lorentz subalgebra

$$so(3,1) \simeq sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R}) \subset \text{Vir}^* \oplus \text{Vir}^*$$

is generated by global conformal transformations on the sphere. The rest of the algebra has generators which contain meromorphic functions, $\delta z = R^z(z)$, with poles on S^2.

The extended BMS algebra : comments

The algebra is not realized as asymptotic symmetry algebra, at least in the standard sense :

- The Kerr black hole has infinite meromorphic supertranslation charges. [Barnich, Troessaert, 2010]
- Minkowski with superrotation hair has negative energy. [G.C., Long, 2016]

The superrotations still have a role to play :

- Superrotation charges are finite and can be non-trivial [Barnich, Troessaert, 2011] [Flanagan, Nichols, 2015] [G.C., Long, 2016]
- The subleading soft graviton theorem has been related to the Ward identity of the superrotation algebra [Kapec, Lysov, Pasterski, Strominger, 2014] [Campiglia, Laddha, 2015]

Building BMS vacua/black holes

Algorithm :

- Start with Minkowski/Schwarzschild spacetime.
- Write a generic change of coordinates $x^\mu \to x'^\mu$ which exponentiates the infinitesimal change of coordinates $x'^\mu = x^\mu + \xi^\mu + \dots$ where ξ^μ is a generic supertranslation + superrotation vector field at $\mathfrak{I}^+$
- Solve for the finite change of coordinates at each order in the asymptotic radial expansion such that $g_{\mu\nu} = \frac{\partial x^\beta}{\partial x'^\mu}\eta_{\alpha\beta}\frac{\partial x^\alpha}{\partial x'^\nu}$ fits in BMS gauge.
- Resum the infinite radial expansion.
- Rewrite the final metric in closed and beautiful form.

The result is the BMS orbit of Minkowski/Schwarzschild spacetime. It is the representation of the BMS group on the bulk metric.

The Poincaré vacua of Einstein gravity

The vacuum metric with supertranslation field only is

$$ds^2 = -dt^2 + dx_s^2 + dy_s^2 + dz_s^2 = -dt^2 + d\rho^2 + g_{AB} d\theta^A d\theta^B,$$

where $\theta^A = \theta, \phi$ and

$$\begin{aligned} g_{AB} &= (\rho - C)^2 \gamma_{AB} - 2(\rho - C) D_A D_B C + D_A D_E C D_B D^E C, \\ &= (\rho \gamma_{AC} - D_A D_C C - \gamma_{AC} C) \gamma^{CD} (\rho \gamma_{DB} - D_D D_B C - \gamma_{DB} C) \end{aligned}$$

Under a supertranslation,

$$\delta_T C(\theta, \phi) = T(\theta, \phi).$$

It admits 10 Killing vectors. We checked that the 10 Poincaré charges are zero $\Rightarrow$ Poincaré vacua.

All supertranslation charges are zero.

The Poincaré vacua of Einstein gravity

The vacua are non-trivial in the sense that they admit canonical charges : superrotation charges

$$\mathcal{Q}_R = -\frac{1}{4G}\int_S d^2\Omega R^A \left(\frac{1}{8}D_A(C_{EF}C^{EF}) + \frac{1}{2}C_{AB}D_E C^{EB}\right)$$

where $C_{AB} = -2D_A D_B C + \gamma_{AB} D^2 C$. [Barnich, Troessaert, 2011]

Even thought the superrotation transformation $\delta z = R^z(z)$, $\delta\bar{z} = R^{\bar{z}}(\bar{z})$ admit poles, the superrotation charges are finite.

There is therefore an obstruction in the bulk at shrinking the surface of integration $\Rightarrow$ Bulk defect.

Maybe our universe is patched with such vacua ?
How to characterize the bulk defects ?

Supertranslation horizon

The static coordinates (t, ρ, θ, ϕ) of the vacua

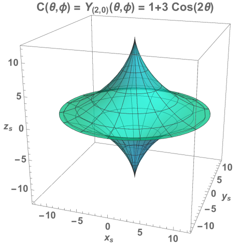
$$ds^2 = -dt^2 + d\rho^2 + g_{AB} d\theta^A d\theta^B,$$

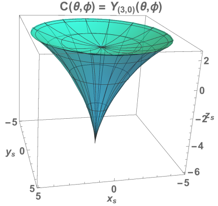
break down where $Det(g_{AB}) = 0$.

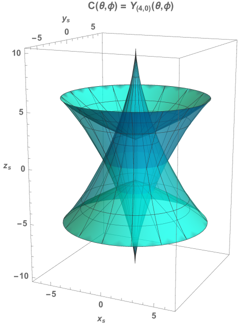
This location $\rho = \rho_{SH}(\theta, \phi)$ defines the *supertranslation horizon*.

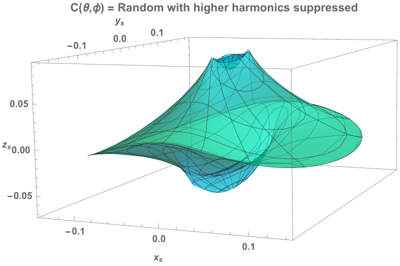
The defect should be at or beyond the supertranslation horizon.

Isometric embedding of the supertranslation horizon









Finite supertranslation diffeomorphism

$$ds^2 = -dt_s^2 + d\rho_s^2 + \gamma_{AB} d\theta_s^A d\theta_s^B = -dt^2 + d\rho^2 + g_{AB} d\theta^A d\theta^B$$

The finite diffeomorphism from Minkowski to the vacua is

$$\begin{aligned} t_s &= t + C_{(0,0)}, & (z \equiv \cot\frac{\theta}{2} e^{i\phi}) \\ \rho_s &= \sqrt{(\rho - C + C_{(0,0)})^2 + D_A C D^A C}, & (Pythagoras'\, rule) \\ z_s &= \frac{(z - \bar{z}^{-1})(\rho - C + C_{(0,0)}) + (z + \bar{z}^{-1})(\rho_s - z\partial_z C - \bar{z}\partial_{\bar{z}} C)}{2(\rho - C + C_{(0,0)}) + (1 + z\bar{z})(\bar{z}\partial_{\bar{z}} C - \bar{z}^{-1}\partial_z C)}. \end{aligned}$$

When C is a combination of the 4 lowest spherical harmonics, it is the change between spherical coordinates to other spherical coordinates at a translated origin.

Supertranslation diffeomorphisms are generalization of "spatially translating the origin of coordinates".

Vacua with superrotation field

The metric with supertranslation and superrotation fields can also be constructed from a combined finite supertranslation and superrotation diffeomorphism from Minkowski with in particular $z \to G(z) + O(r^{-1})$.

The Bondi mass decreases with retarded time u,

$$\partial_u M = -\frac{1}{8} T^{AB} T_{AB}$$

where the traceless, divergence-free tensor is

$$T_{zz} = \frac{\partial_z^3 G}{\partial_z G} - \frac{3(\partial_z^2 G)^2}{2(\partial_z G)^2}$$

$\Rightarrow$ Unbounded negative energy.

$\Rightarrow$ Discard by imposing the Dirichlet boundary condition $T_{zz} = 0$.

$\Rightarrow$ Only Lorentz transformations are asymptotic symmetries

Schwarzschild with BMS hair

The final static ($J = 0$) metric after spherical gravitational collapse, if analytic, is diffeomorphic to the Schwarzschild metric. [No hair theorems]

[Carter, Hawking, Robinson, 1971-1975] [Chrusciel, Costa, 2008] [Alexakis, Ionescu, Klainerman, 2009]

A loophole of no hair theorems is that the diffeomorphism might be singular inside the event horizon, so the black hole can carry superrotation charges which characterize the classical vacuum.

Proposal : there might be a non-trivial supertranslation field $C(\theta, \phi)$ at the end of the collapse.

Two questions :

- What is the metric $g_{\mu\nu}(M, C(\theta, \phi))$?
- How does the supertranslation field $C(\theta, \phi)$ compares to the final mass M ?

The Schwarzschild metric

It admits Weyl conformally flat sections. This is manifest in isotropic coordinates $(t, \rho_s, \theta_s, \phi_s)$:

$$ds^2 = -\frac{\left(1 - \frac{M}{2\rho_s}\right)^2}{\left(1 + \frac{M}{2\rho_s}\right)^2} dt^2 + \left(1 + \frac{M}{2\rho_s}\right)^4 \left(d\rho_s^2 + \gamma_{AB} d\theta^A d\theta^B\right)$$

where

$$\begin{aligned} \gamma_{AB} d\theta^A d\theta^B &= d\theta_s^2 + \sin^2\theta_s d\phi_s^2, \\ \rho_s &= \infty \text{ at spatial infinity} \\ \rho_s &= \frac{M}{2} \text{ at the event horizon} \end{aligned}$$

The Schwarzschild metric embedded in the BMS supertranslation vacuum

$$ds^2 = -\frac{\left(1-\frac{M}{2\rho_s}\right)^2}{\left(1+\frac{M}{2\rho_s}\right)^2}dt^2 + \left(1+\frac{M}{2\rho_s}\right)^4\left(d\rho^2 + g_{AB}d\theta^A d\theta^B\right)$$

where

$$\begin{aligned} g_{AB} &= (\rho\gamma_{AC} - D_A D_C C - \gamma_{AC}C)\gamma^{CD}(\rho\gamma_{DB} - D_D D_B C - \gamma_{DB}C) \\ \rho_s^2 &= (\rho - C)^2 + D_A C D^A C \end{aligned}$$

Remarks :

- When $C = 0$, this is Schwarzschild
- Obtained by finite supertranslation diffeomorphism
- The non-trivial Poincaré charges are just the energy M
- There are superrotation charges quadratic in C

A bound from weak cosmic censorship

This is a competition between the supertranslation horizon and the infinite redshift surface.

$$g_{tt} \sim (1 - \frac{M}{2\rho_s})^2, \qquad \rho_s^2 = (\rho - C)^2 + D_A C D^A C.$$

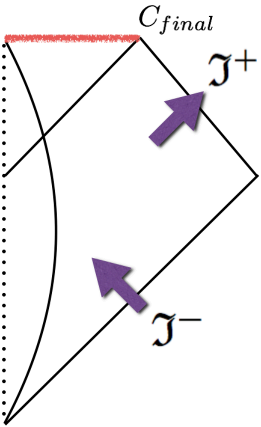
- When $C \ll M$, this is a slightly distorted Schwarzschild black hole
- When $D_A C D^A C > \frac{M^2}{4}$, there is no infinite redshift surface in the coordinate patch.

We argued that weak cosmic censorship requires the bound

$$D_A C D^A C \leqslant \frac{M^2}{4}.$$

How much supertranslation hair ?

What is the final value of $C(\theta, \phi)$?



It depends upon the fluxes and Bondi mass at $\mathfrak{I}^+$ and $\mathfrak{I}^-$ and boundary conditions at spatial infinity.

How much supertranslation hair?

Assuming boundary conditions on radiation [Christodoulou, Klainerman, 1993], Einstein's equations give

$$-\frac{1}{4}\nabla^2(\nabla^2+2)(C|_{final}(\theta,\phi)-\Delta C_\infty - C|_{in}(\pi-\theta,\phi+\pi))$$
$$= m|_{final} - m|_{in} + \int_{-\infty}^{+\infty} du T_{uu}(\theta,\phi) - \int_{-\infty}^{+\infty} dv T_{vv}(\pi-\theta,\phi+\pi)$$

This is the global angle-dependent energy conservation law for asymptotically flat spacetimes. [Geroch, Winicour, 1980] [Strominger, Zhiboedov, 2014]

Assuming $\Delta C_\infty = 0$, spherically symmetric collapse of a null shell $\Rightarrow C|_{final} = 0$ (metric described by Vaidya metric).

How much supertranslation hair ?

Non-spherically symmetric collapse of a null shell is constrained by the null energy condition

$$T_{vv}(\theta, \phi) \geqslant 0.$$

Assuming all matter is incoming at $v = 0$,

$$T_{vv} = \left(\frac{M + M \sum P_{l,m} Y_{l,m}(\theta, \phi)}{4\pi r^2} + O(r^{-3}) \right) \delta(v)$$

we get the constraint

$$\sum P_{l,m} Y_{l,m}(\theta, \phi) \geqslant -1.$$

How much supertranslation hair ?

In the ideal case (no outgoing radiation, no initial mass, only ingoing collapsing radiation) and assuming $\Delta C_\infty = 0$, the solution to the global energy conservation law is

$$C(\theta,\phi) = M \sum_{\ell \geqslant 2, m} \frac{4(-1)^\ell}{(\ell-1)\ell(\ell+1)(\ell+2)} P_{l,m} Y_{l,m}(\theta,\phi) = O(M^1)$$

with the constraint

$$\sum P_{l,m} Y_{l,m}(\theta,\phi) \geqslant -1.$$

This leads to the bound (checked numerically) :

$$D_A C D^A C \leqslant \frac{M^2}{4}.$$

Summary

- We constructed the supertranslation orbit of Minkowski and identified the metrics as Poincaré vacua with a defect carrying superrotation charge which is hidden at/behind the supertranslation horizon.
- Superrotations do not belong to the asymptotic symmetry group since they would lead to unbounded negative energy but superrotation charges are well-defined.
- In the center-of-mass frame, supertranslations are spatial, except the zero mode (i.e. time translation).
- We proposed a new final state of collapse : the Schwarzschild black hole with supertranslation hair. The hair is soft, non-linear, and $O(\hbar^0)$. With one caveat (the boundary conditions at spatial infinity), the final hair is computable from past history of evolution and collapse and is $O(M)$.
- Much physics and maths remains to be understood.