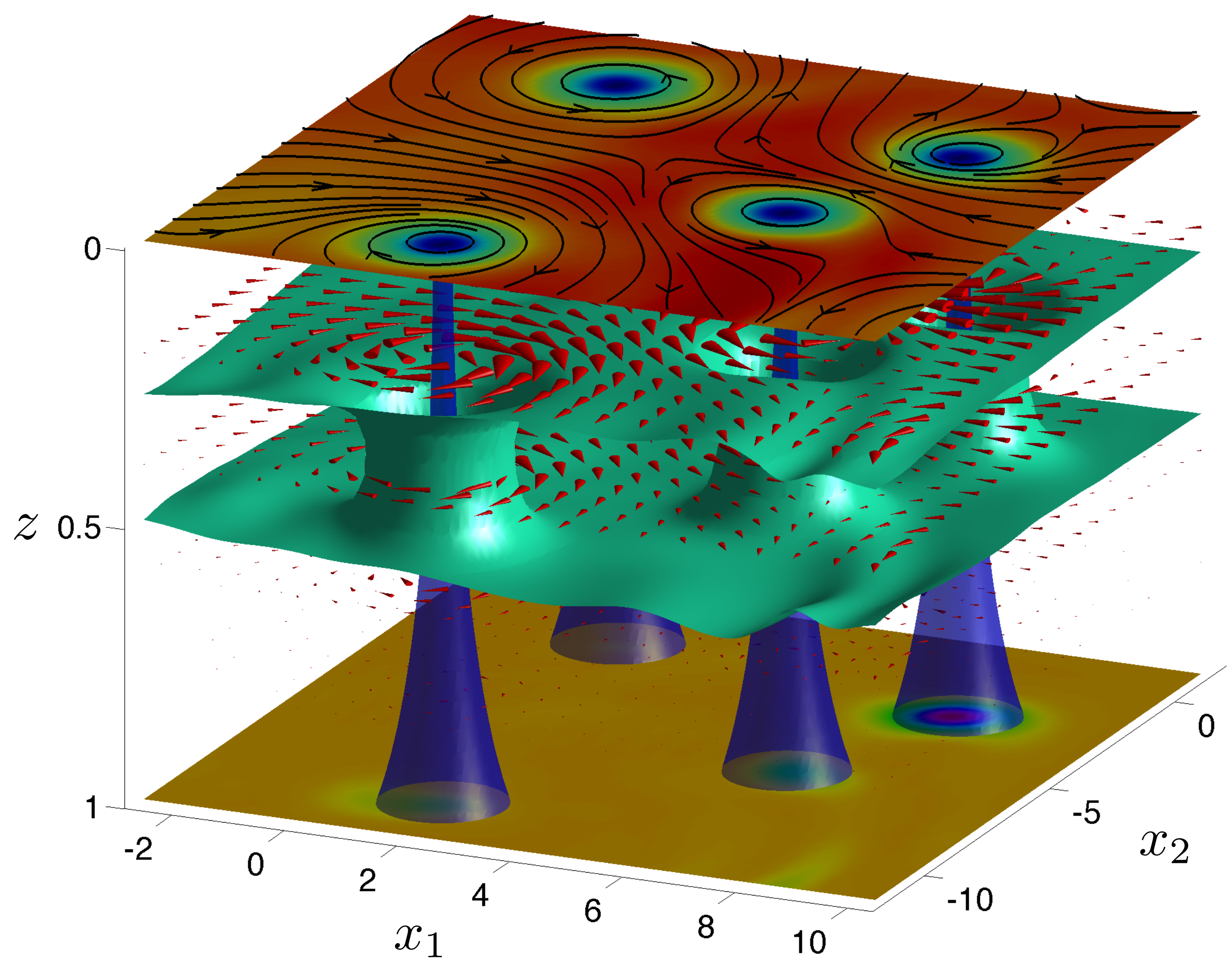
Holographic perspectives on the Kibble-Zurek mechanism

Paul Chesler

Work done with Hong Liu & Antonio Garcia-Garcia

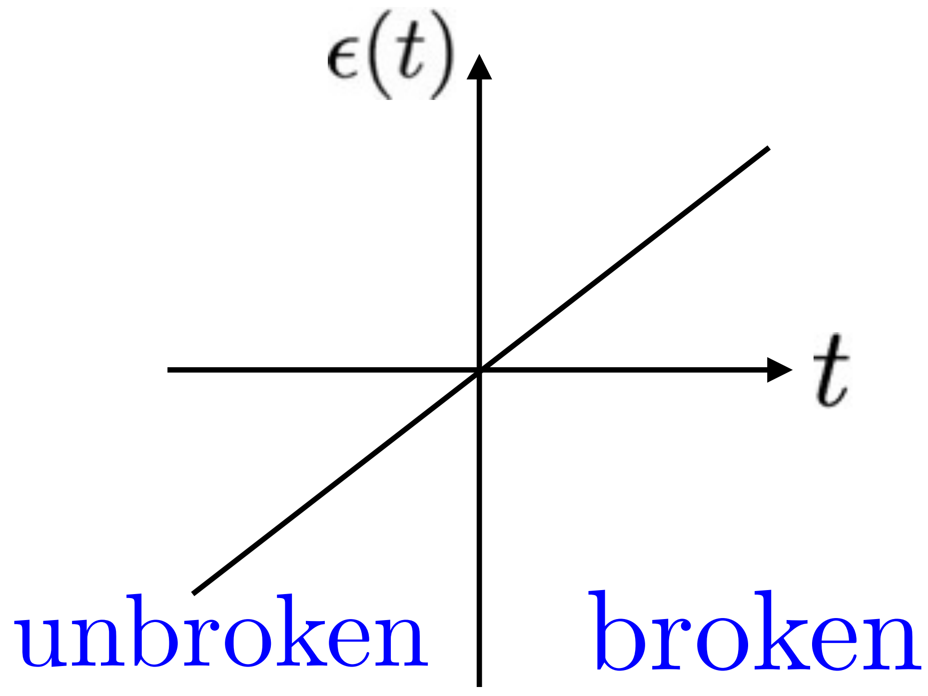


What is the Kibble-Zurek mechanism?

QFT with 2^{nd} order phase transition:

- Example: **superfluid**
- Symmetry group $U(1)$ broken for $T < T_c$.
- Order parameter $\psi \neq 0$ for $T < T_c$.
- **What happens when T is dynamic?**

$$\epsilon(t) \equiv 1 - \frac{T(t)}{T_c}.$$



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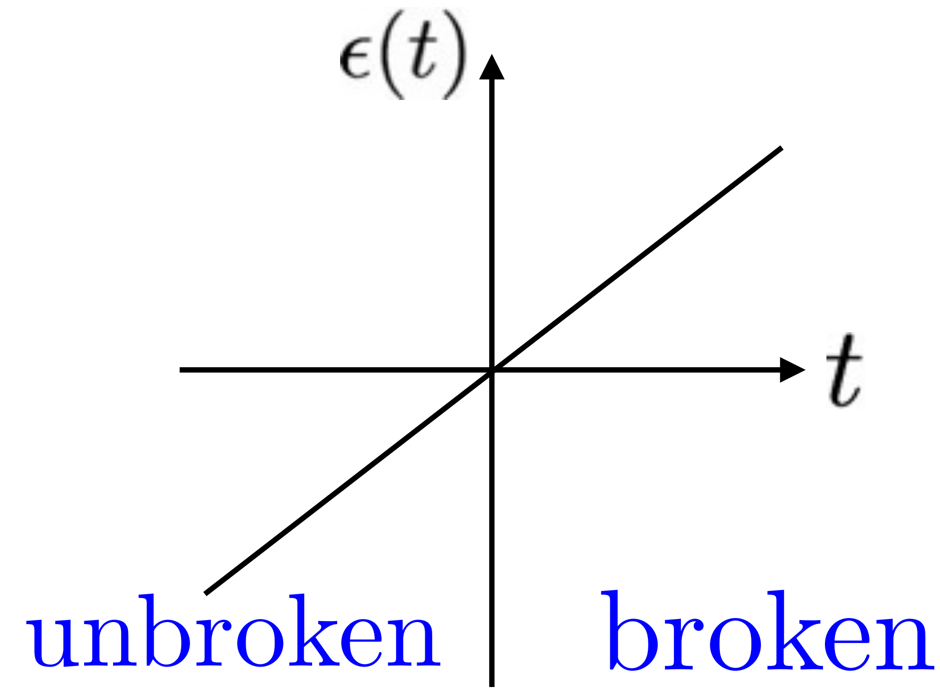

Box of superfluid at $T > T_c$

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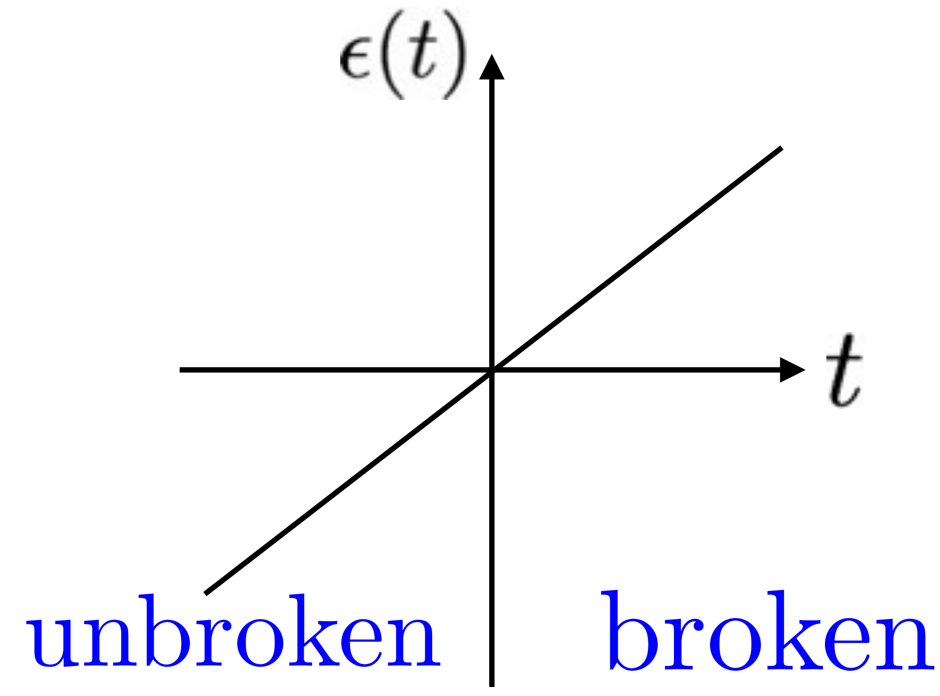

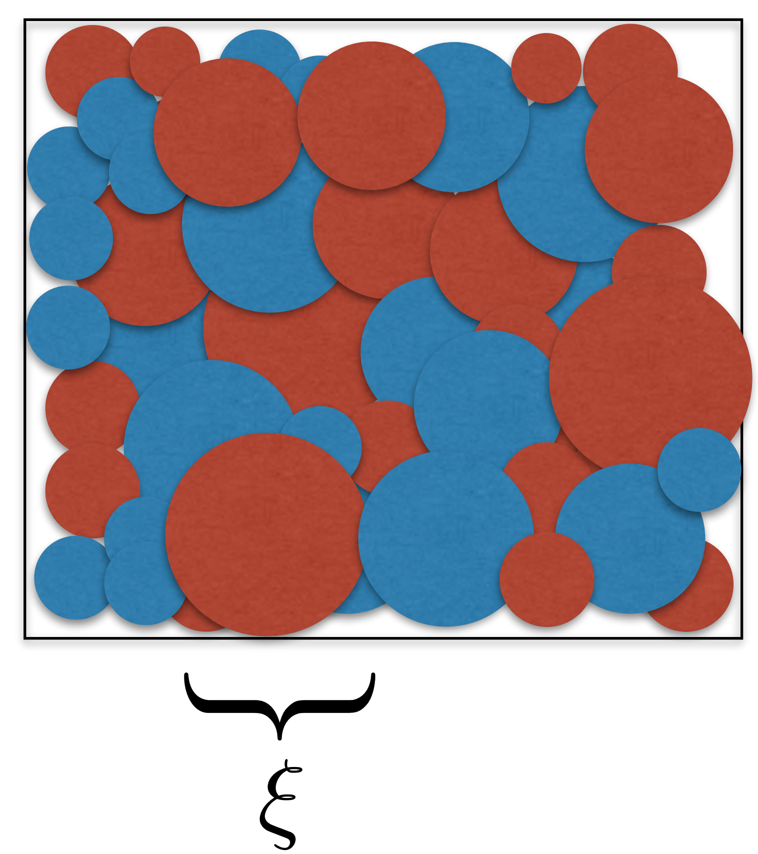

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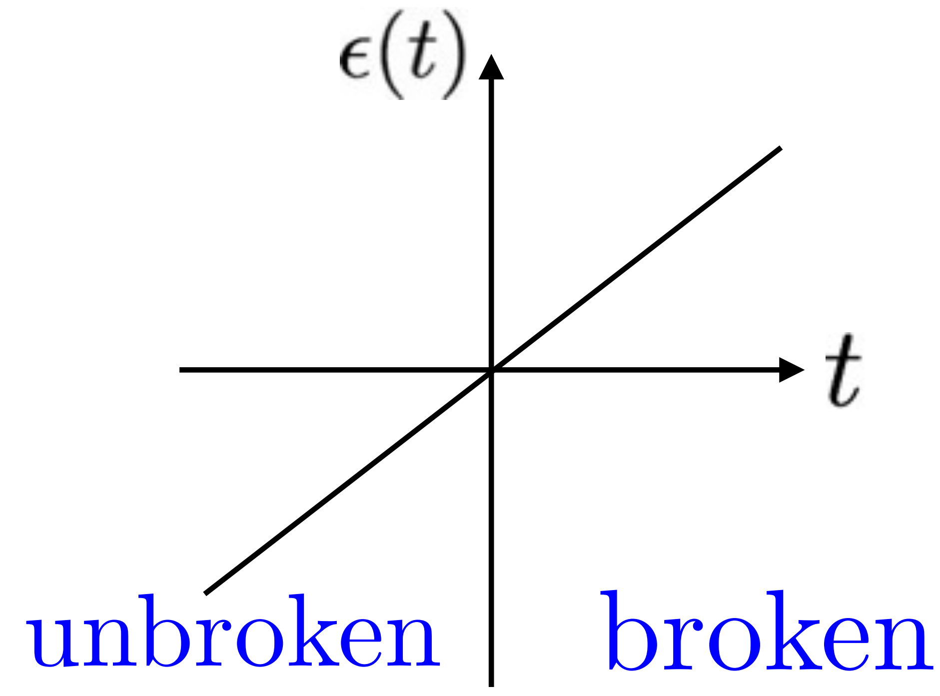
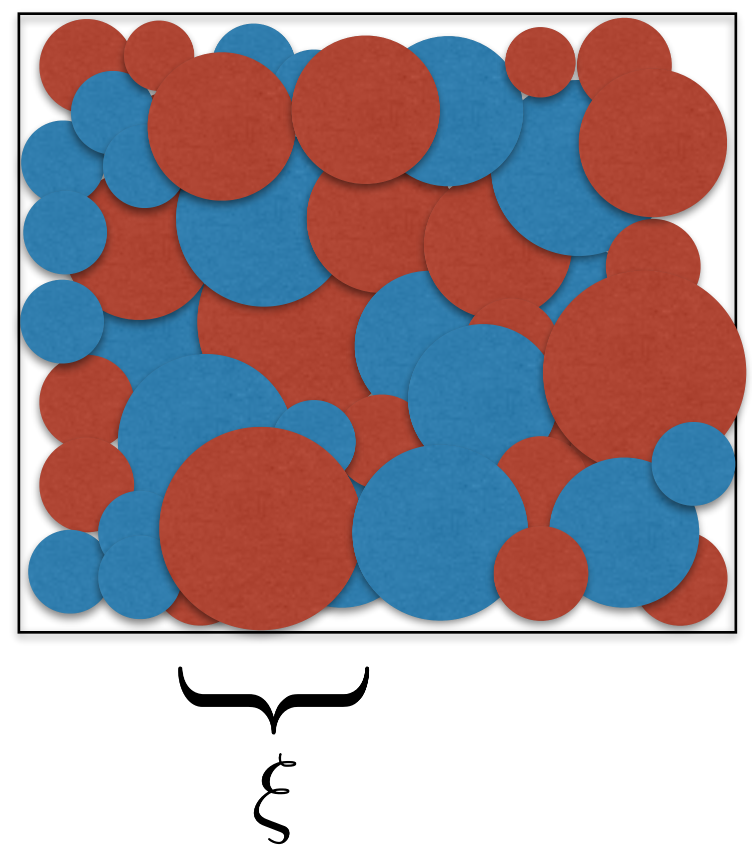
- **Density of defects after quench:**

$$n \sim \xi^{-(d-D)}.$$

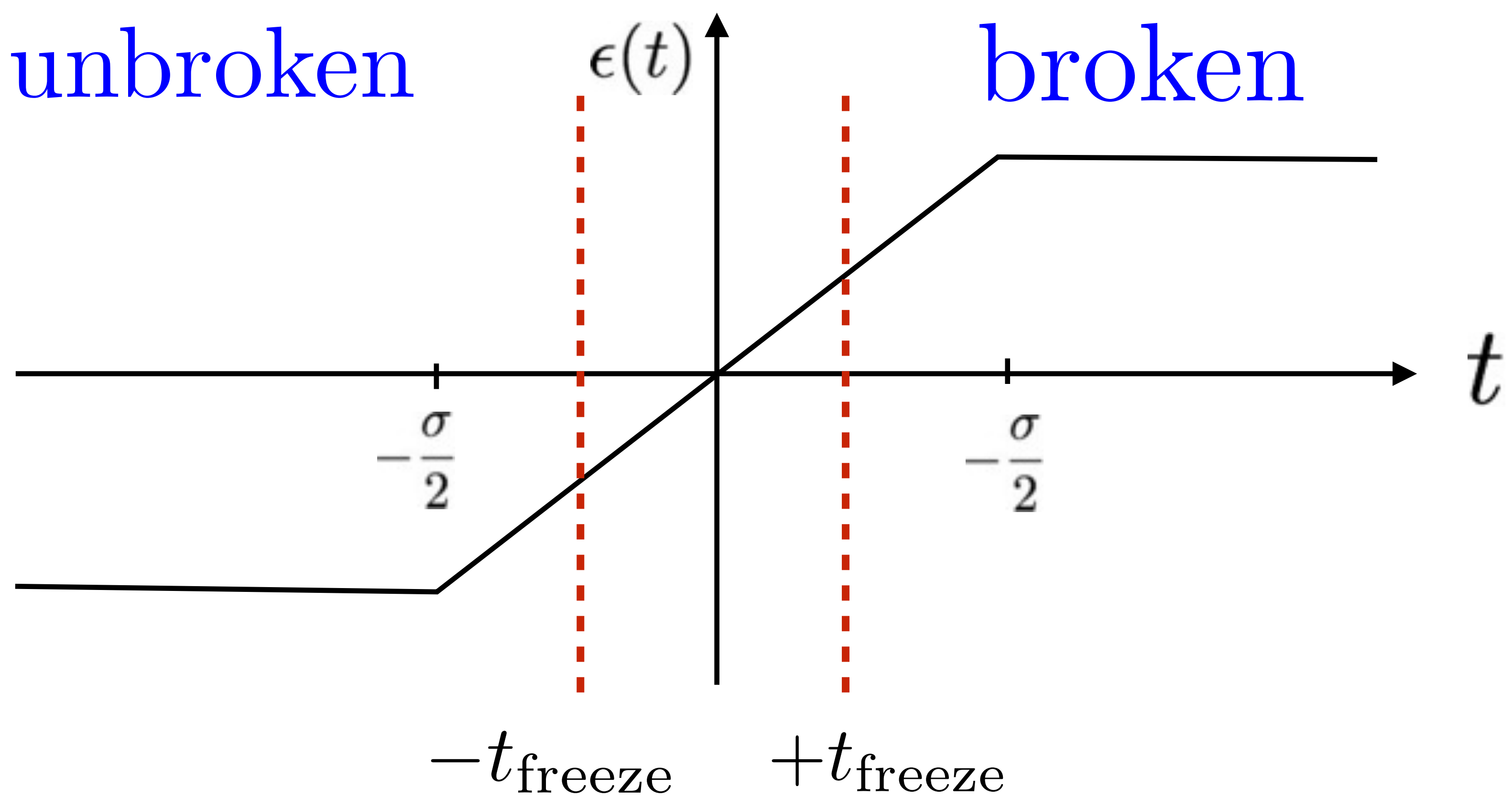




Zurek's estimate of correction length

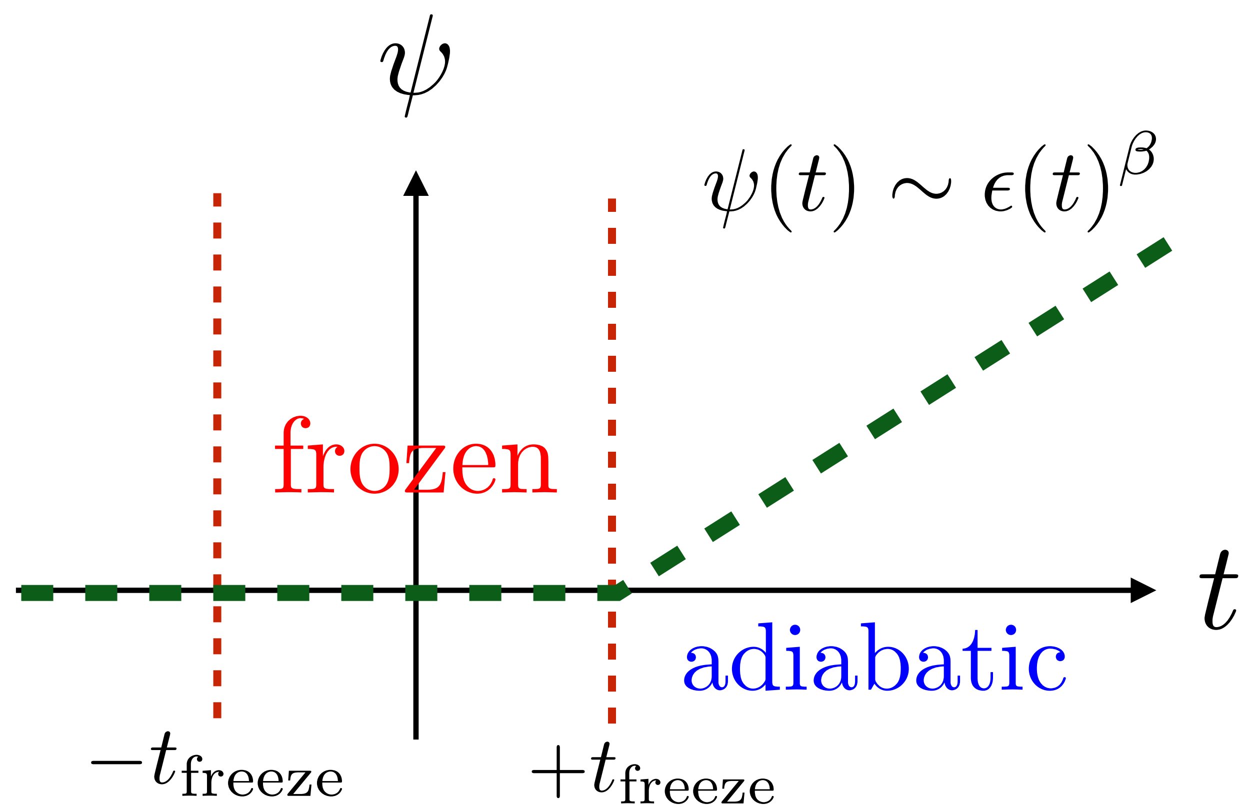
Critical slowing down

- Critical exponents: $\xi_{\rm eq} = \xi_o |\epsilon|^{-\nu}$ and $\tau_{\rm eq} = \tau_o |\epsilon|^{-z\nu}$.
- Inevitable $\exists\ t_{\rm freeze}$ such that $\left.\frac{\partial \tau_{\rm eq}}{\partial t}\right|_{t=t_{\rm freeze}} \sim 1$.



- Characteristic scale: $\xi_{\rm freeze} \equiv \xi_{\rm eq}(t = t_{\rm freeze})$.

The Kibble-Zurek scaling



- **Assume linear quench:** $\epsilon(t) = t/\tau_Q$.

$$\Rightarrow t_{\text{freeze}} \sim \tau_Q^{\nu z/(1+\nu z)}, \qquad \xi_{\text{freeze}} \sim \tau_Q^{\nu/(1+\nu z)}.$$

- **Density of topological defects when condensate first forms:**

$$n_{KZ} \sim \frac{1}{\xi_{\text{freeze}}^{d-D}} \sim \tau_Q^{-(d-D)\nu/(1+\nu z)}$$

Motivational claims

1. Dynamics after $+t_{\text{freeze}}$ need not be adiabatic.

- Adiabatic evolution only after $t_{\text{eq}} \gg t_{\text{freeze}}$.

2. No well-defined condensate until t_{eq}.

3. **Dynamics after $T < T_c$ responsible for KZ scaling.**

4. $\xi(t_{\text{eq}}) \gg \xi_{\text{freeze}} \Rightarrow$ **far fewer defects formed**

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excerpt from [Del Campo & Zurek]

served in numerics. A better estimate is obtained by using a factor f, to multiply $\hat{\xi}$ in the above equations, where $f \approx 5-10$ depends on the specific model.[29,31–35] Thus, while KZM provides an order-of-magnitude estimate

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Employ holographic duality

First holographic study:
[Sonner, del Campo, Zurek: two weeks ago]

A holographic model of a charged superfluid

Action: [Hartnoll, Herzog & Horowitz: 0803.3295]

$$S_{\text{grav}} = \frac{1}{16\pi G_{\text{N}}} \int d^4x \sqrt{-G} \left[R + \Lambda + \frac{1}{q^2} \left(-F^2 - |D\Phi|^2 - m^2 |\Phi|^2 \right) \right],$$

where $\Lambda = -3$ and $m^2 = -2$.

- Near-boundary asymptotics of Φ encodes QFT condensate $\langle \psi \rangle$.
- Spontaneous symmetry breaking:
  - Black-brane solutions with $T > T_c$ have $\Phi = 0$.
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- Cool black brane through T_c.
- Watch Φ and $\langle \psi \rangle$ form.

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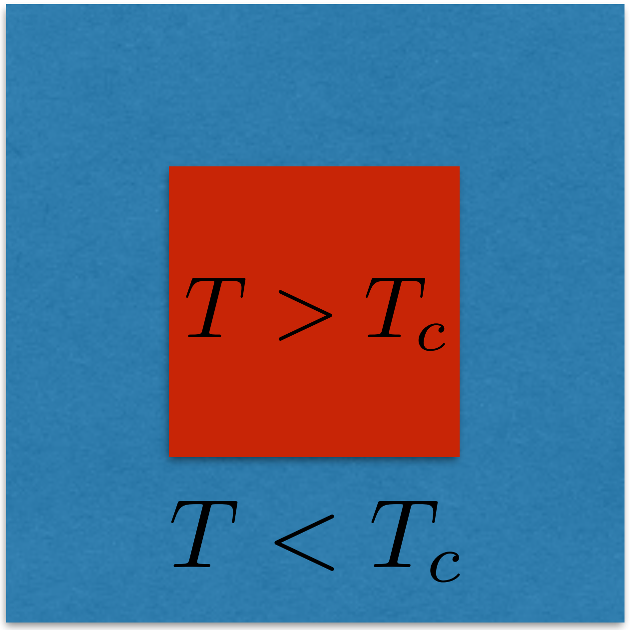
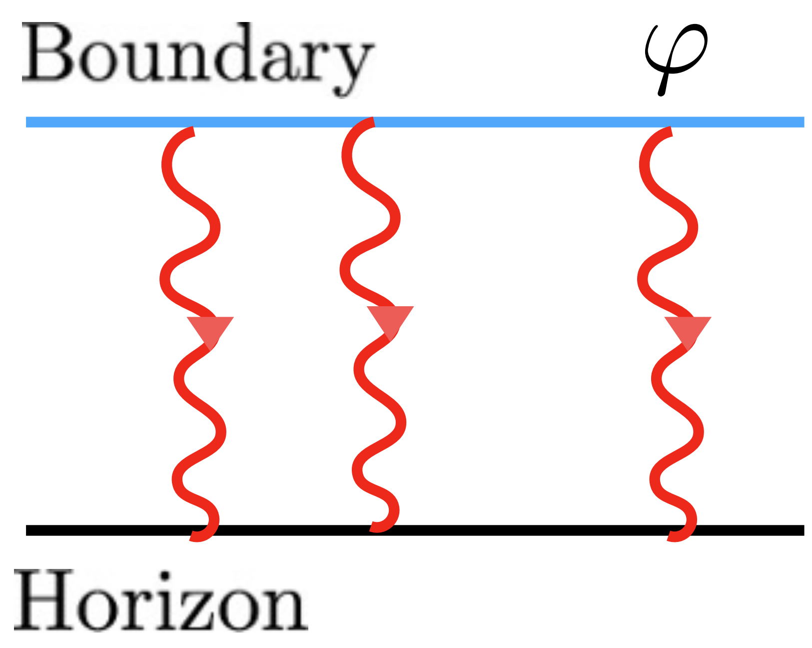


Stochastic driving

1. **Stochastic processes** choose different vacua at different $\mathbf{x}$.
2. Boundary conditions $\lim_{u\to 0} A_\nu = \mu\delta_{\nu 0}$, $\lim_{u\to 0} \partial_u \Phi = \varphi$.
3. Statistics $\langle \varphi^*(t,\mathbf{x})\varphi(t',\mathbf{x}')\rangle = \zeta\delta(t-t')\delta^2(\mathbf{x}-\mathbf{x}')$.
4. Mimics backreaction of G_N suppressed **Hawking radiation.**

$$\zeta \sim 1/N^2$$



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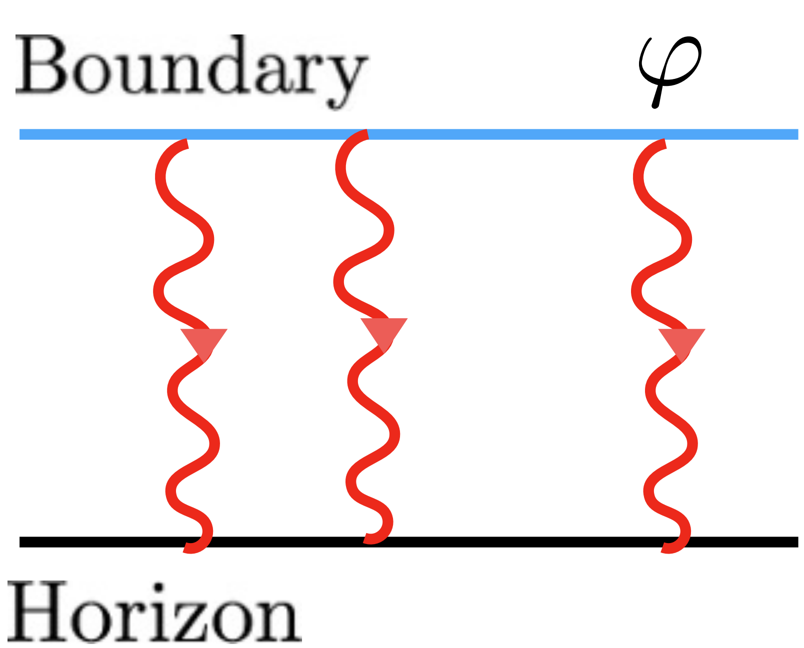
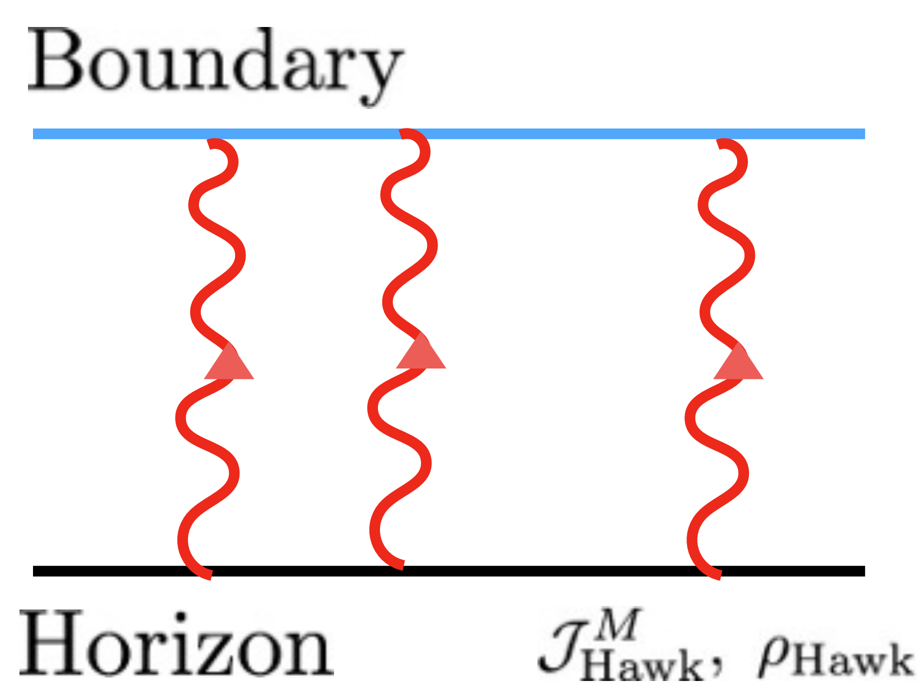
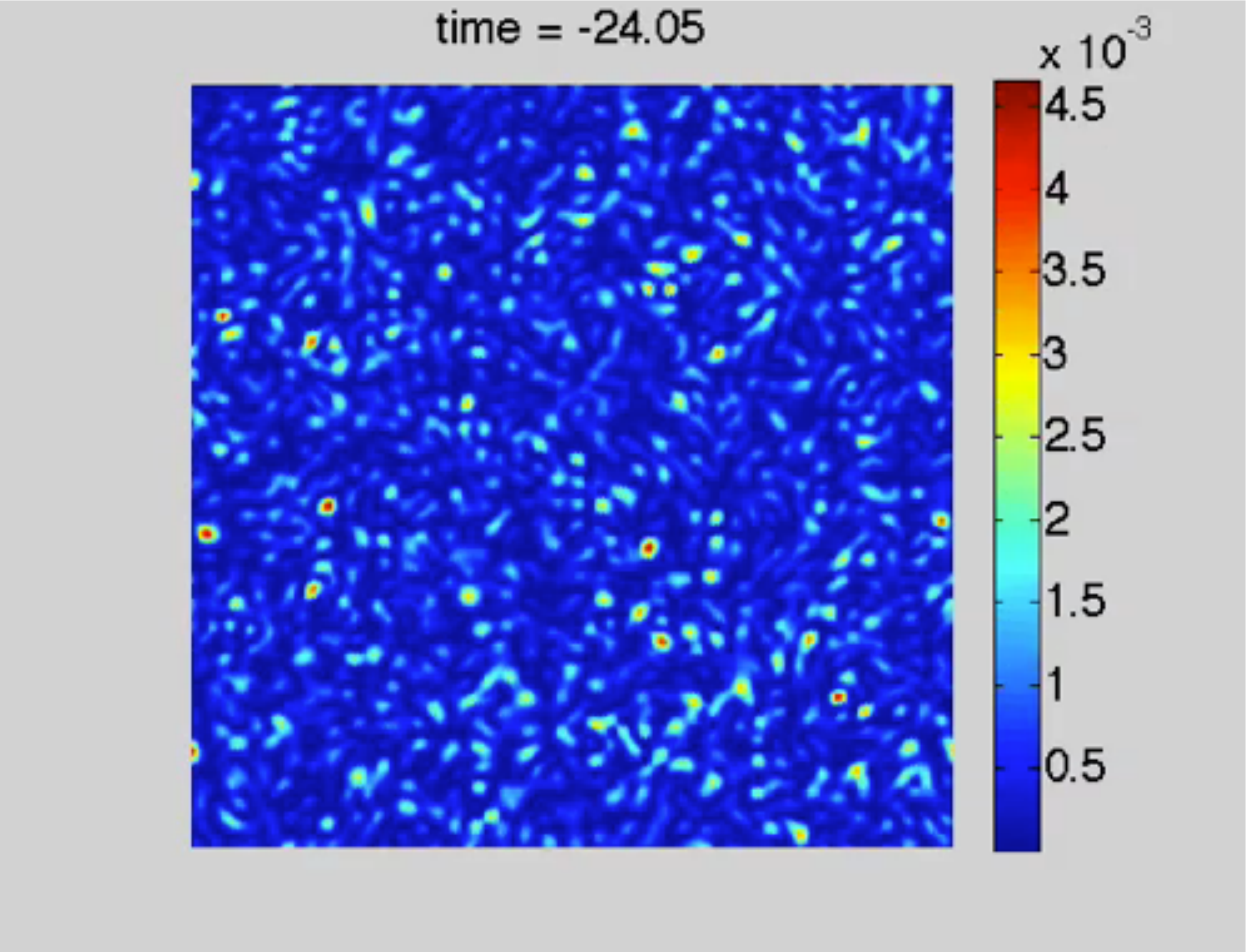




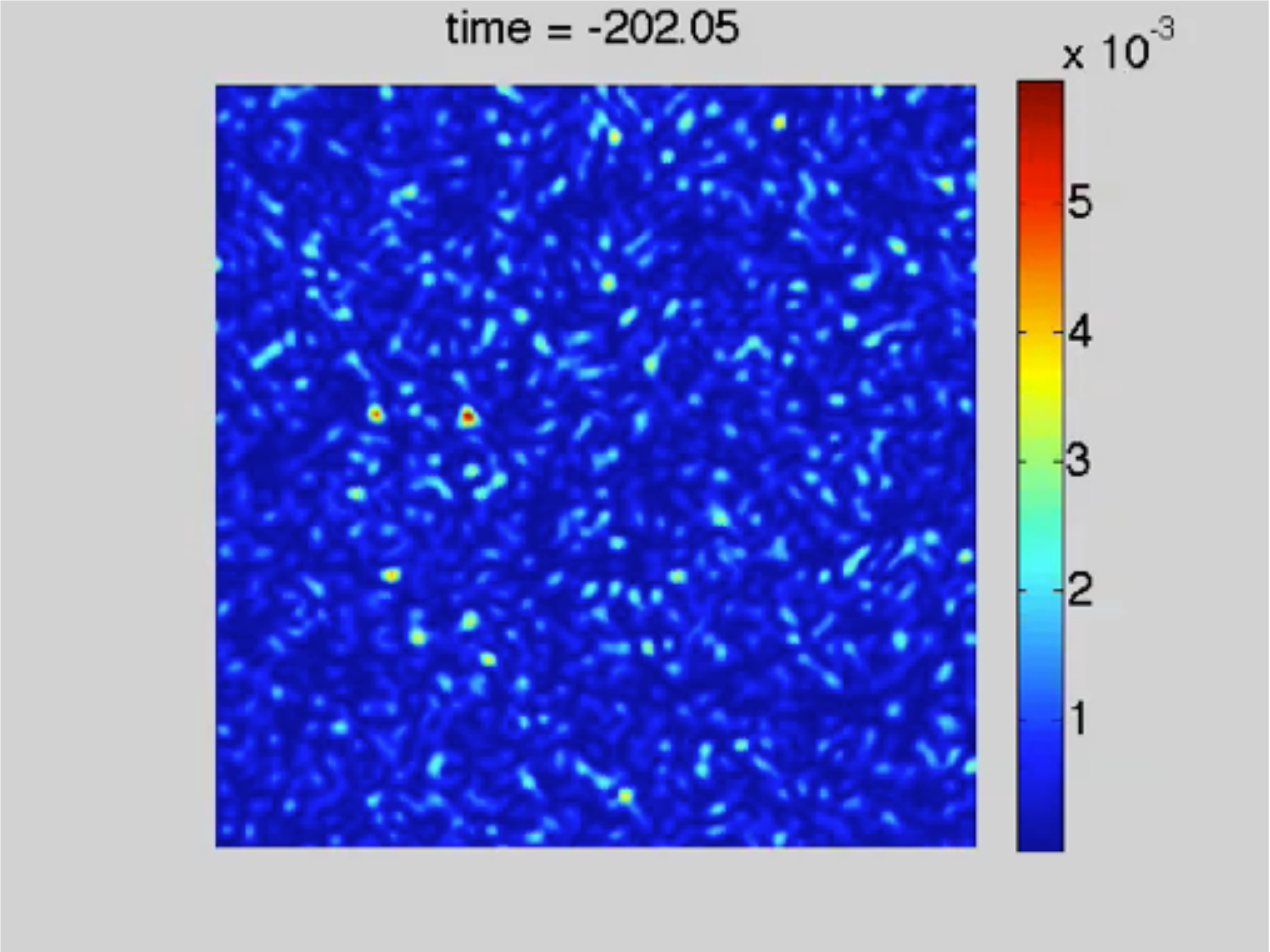
Results illustrated

Movies show $|\langle\psi(t,\mathbf{x})\rangle|^2$

Fast quench



Slow quench



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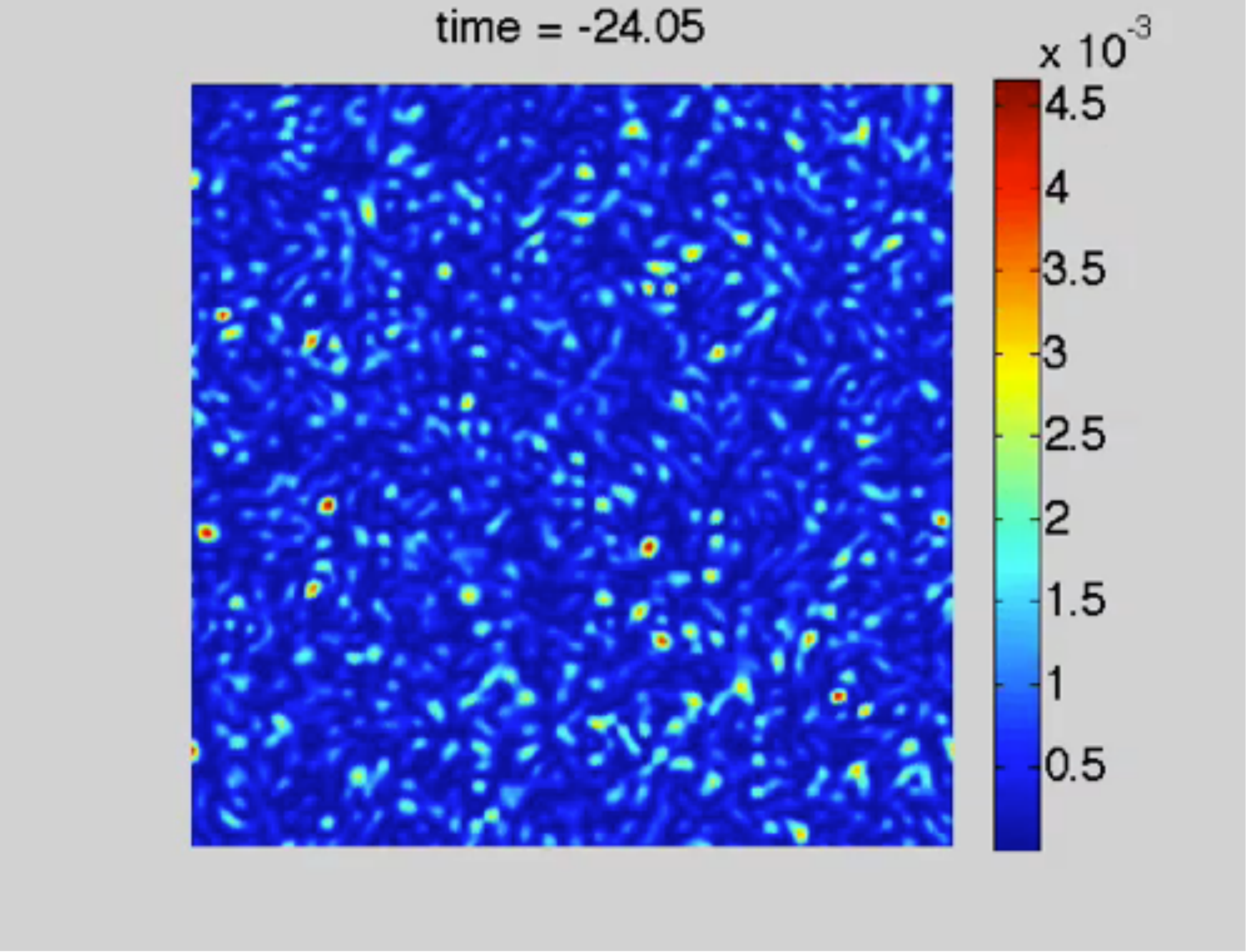

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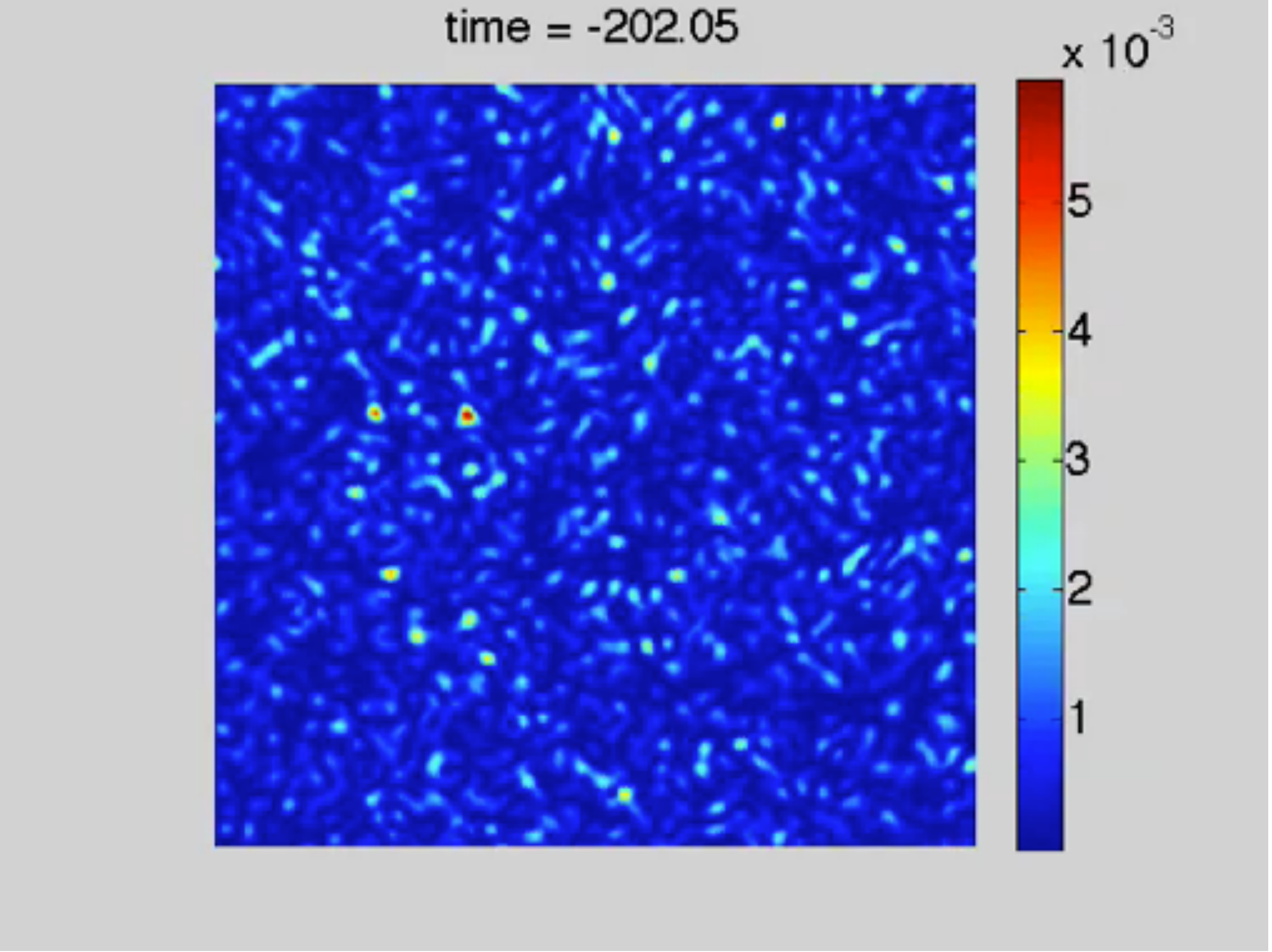

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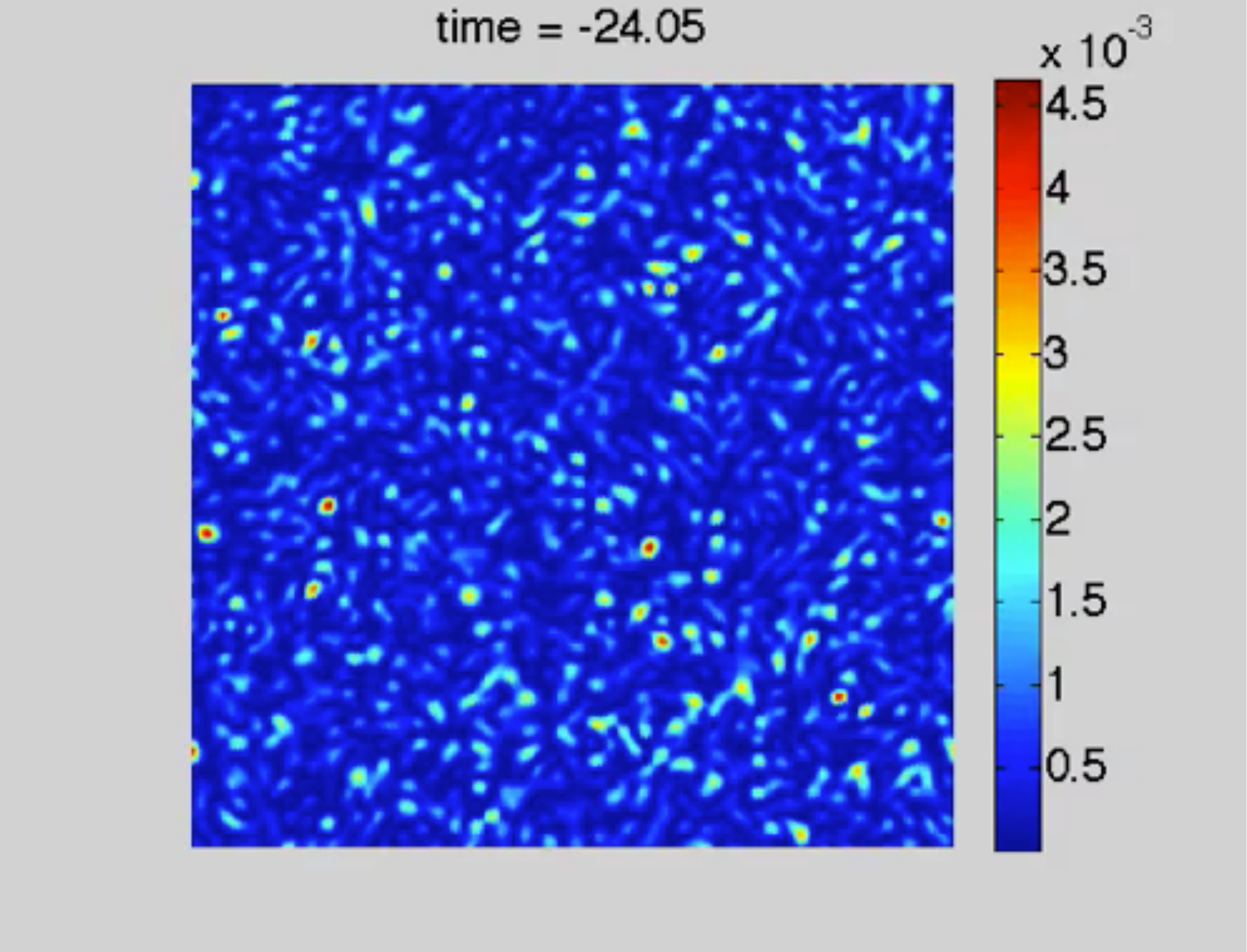

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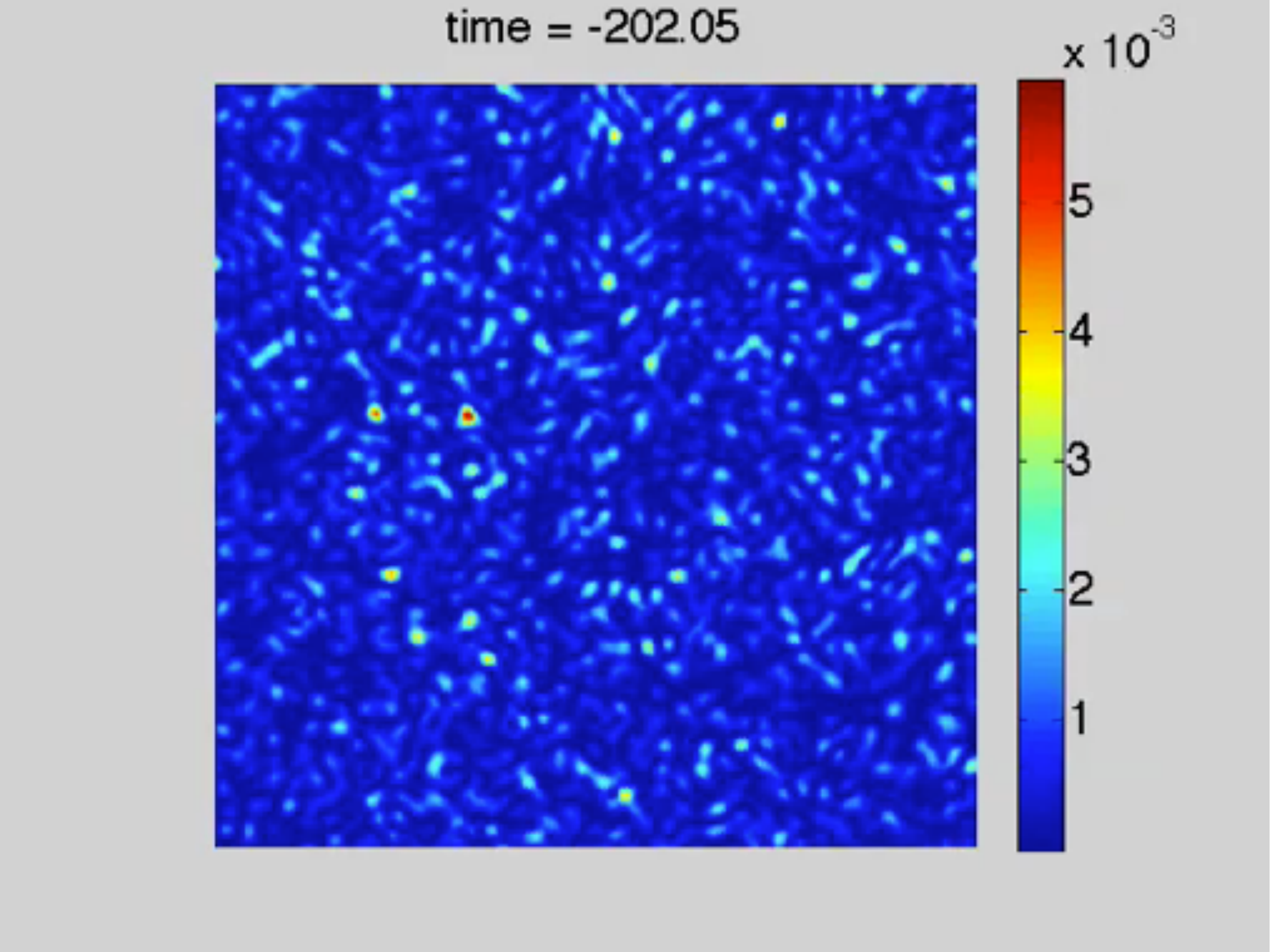
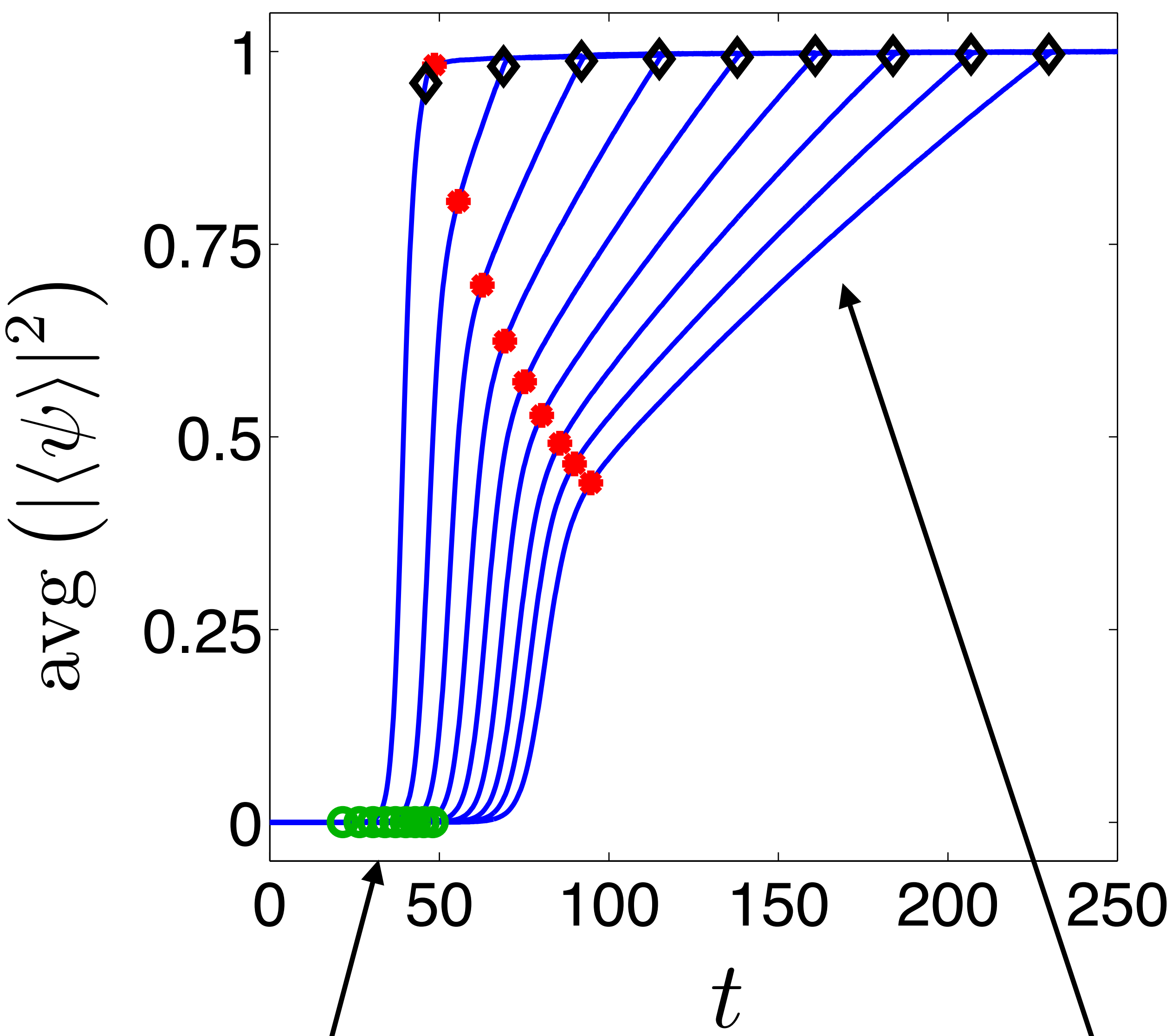


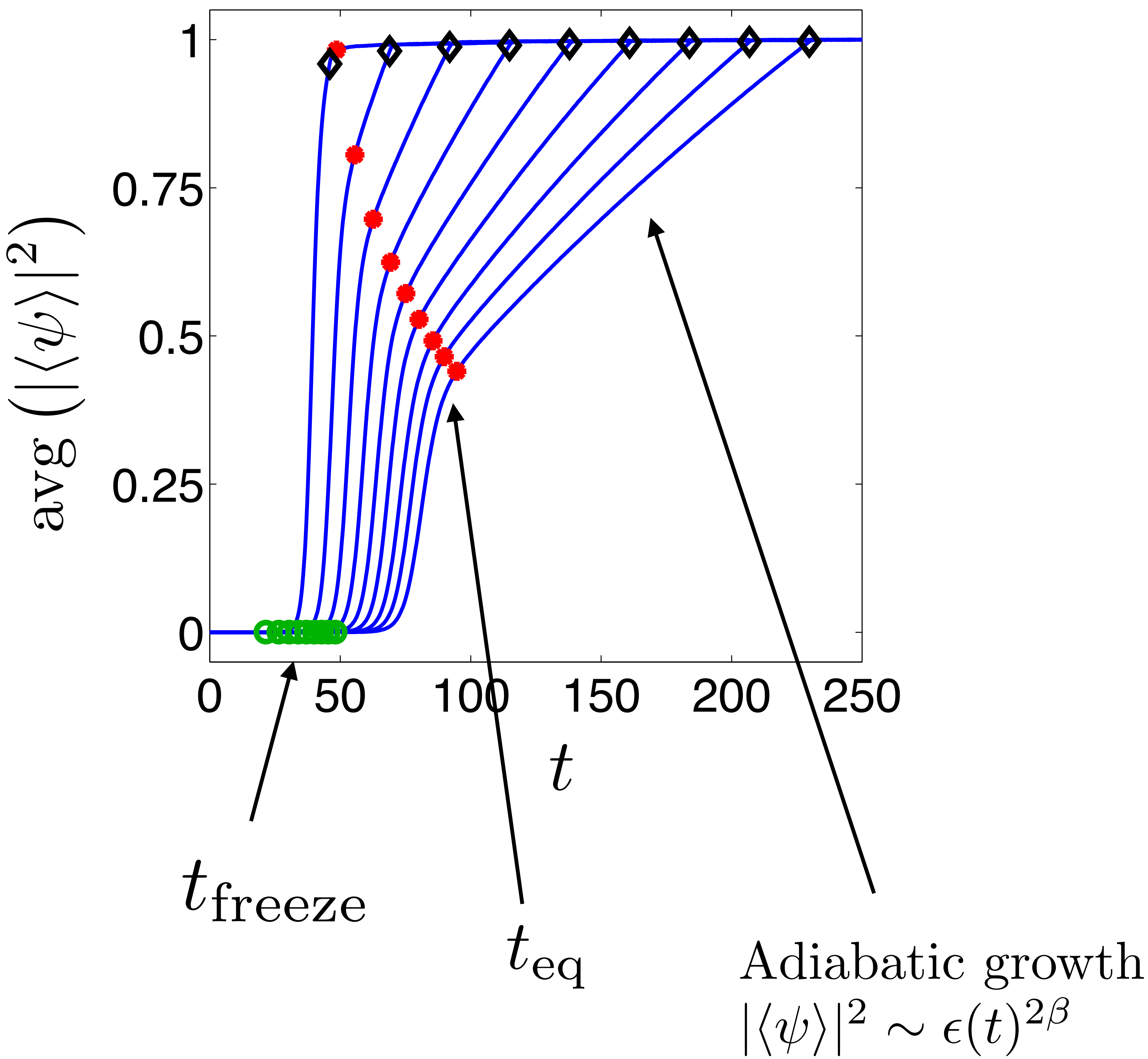
Condensate growth



t_{freeze}

Adiabatic growth
$|\langle\psi\rangle|^2 \sim \epsilon(t)^{2\beta}$

Condensate growth



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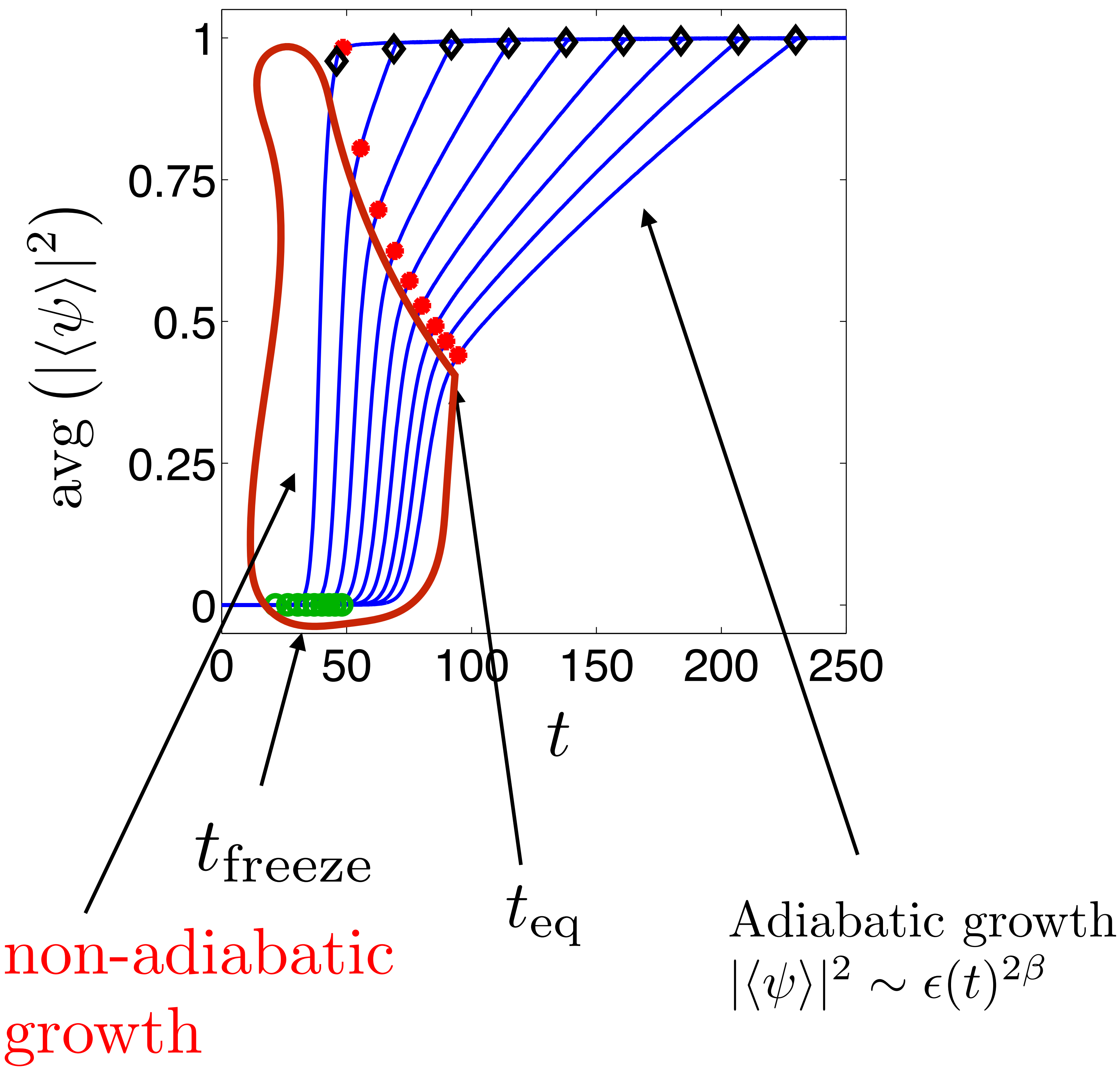

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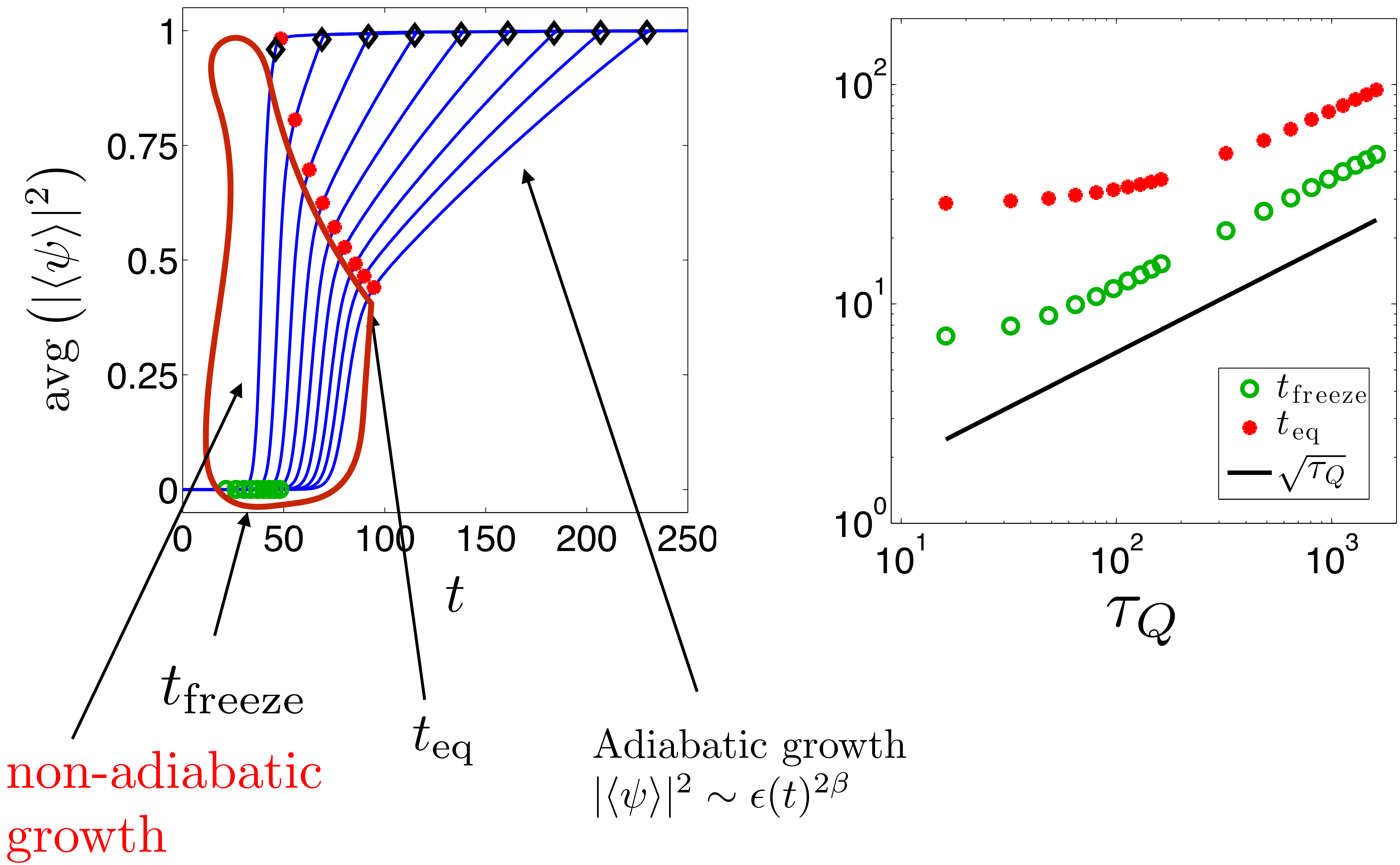

Non-adiabatic condensate growth

- Correlation function $C(t, r) \equiv \langle \psi^*(t, \mathbf{x} + \mathbf{r})\psi(t, \mathbf{x})\rangle$.
- Linear response

$$C(t, q) = \zeta \int dt \, |G_R(t, t', q)|^2.$$

- Relation to black brane quasinormal modes

$$G_R(t, t', q) = \theta(t - t')H(q)e^{-i \int_t^{\prime t} dt'' \omega_o(\epsilon(t''), q)}$$

where ω_o is $\epsilon < 0$ quasinormal mode analytically continued to $\epsilon > 0$

- Instability for $\epsilon > 0$

$$\text{Im}\, \omega_o = b\epsilon^{z\nu} - a\epsilon^{(z-2)\nu} q^2 + O(q^4) > 0.$$

- **Modes with $q < q_{\text{max}}$ with $q_{\text{max}} \sim \epsilon(t)^\nu$ form condensate.**

Non-adiabatic condensate growth (II)

At $t > t_{\rm freeze}$,

$$C(t,r) \sim C_0(t) e^{-\frac{r^2}{\ell_{\rm co}(t)^2}},$$

where

$$C_0(t) \sim \zeta t_{\rm freeze}\, \ell_{\rm co}(t)^{-d} \exp\left\{ \left(\frac{t}{t_{\rm freeze}} \right)^{1+\nu z} \right\}.$$

and

$$\ell_{\rm co}(t) = \xi_{\rm freeze} \left(\frac{t}{t_{\rm freeze}} \right)^{\frac{1+(z-2)\nu}{2}}.$$

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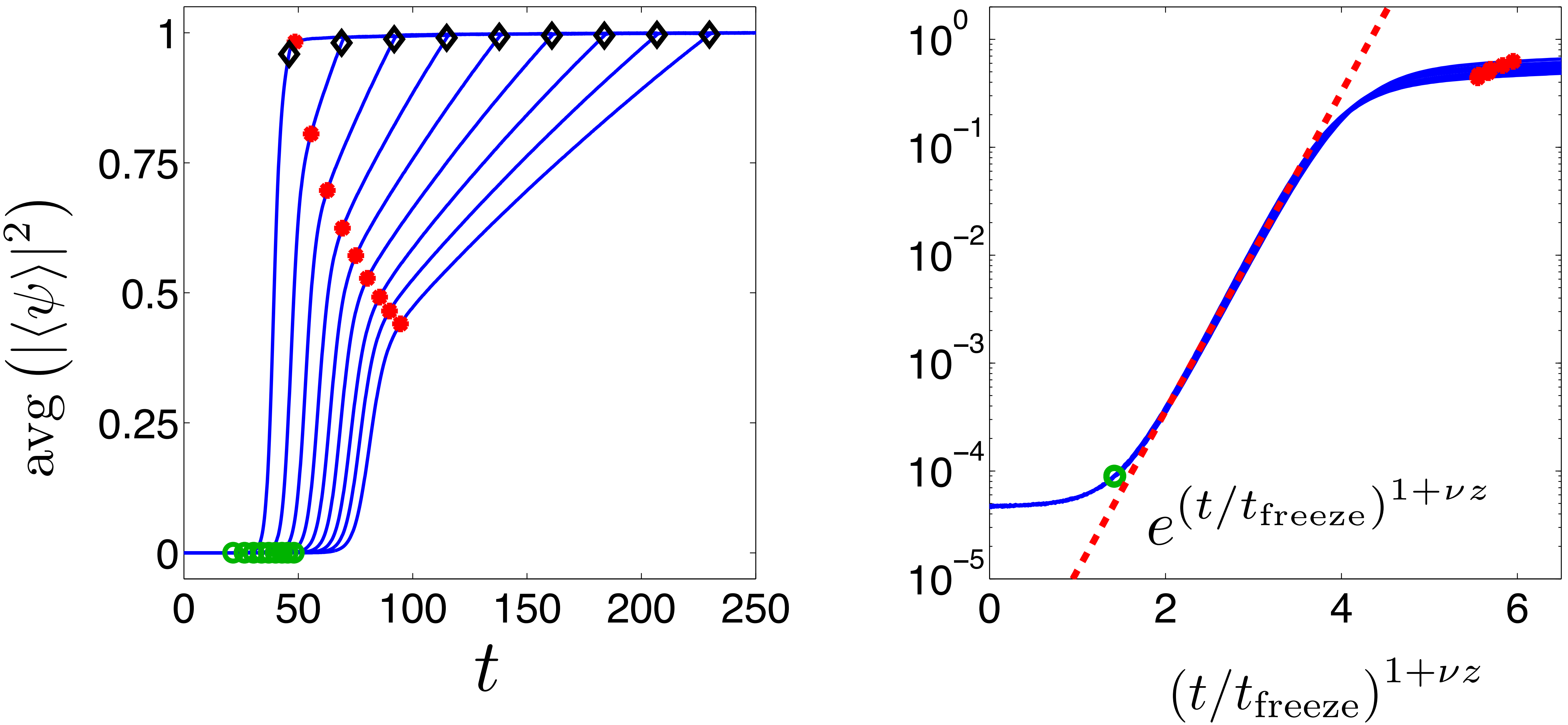
$$\ell_{\mathrm{co}}(t) = \xi_{\mathrm{freeze}} \left(\frac{t}{t_{\mathrm{freeze}}} \right)^{\frac{1+(z-2)\nu}{2}}.$$

Linear response breaks down when $C_0(t) \sim \epsilon(t)^{2\beta}$

$$t_{\mathrm{eq}} \sim [\log R]^{\frac{1}{1+\nu z}} t_{\mathrm{freeze}}, \qquad R \sim \zeta^{-1} \tau_Q^{\frac{(d-z)\nu - 2\beta}{1+\nu z}}.$$

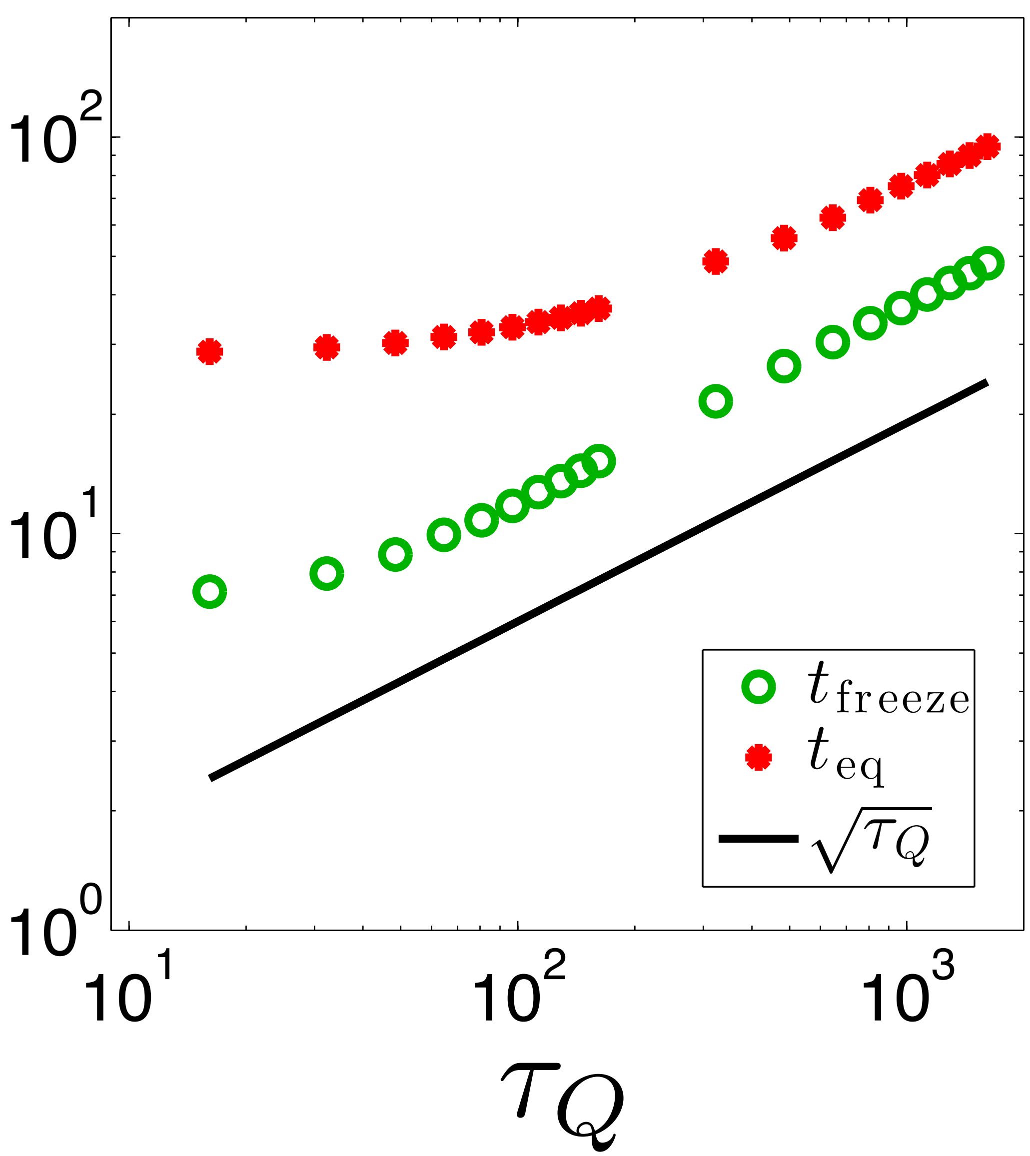
Comparing to unstable mode analysis (I)

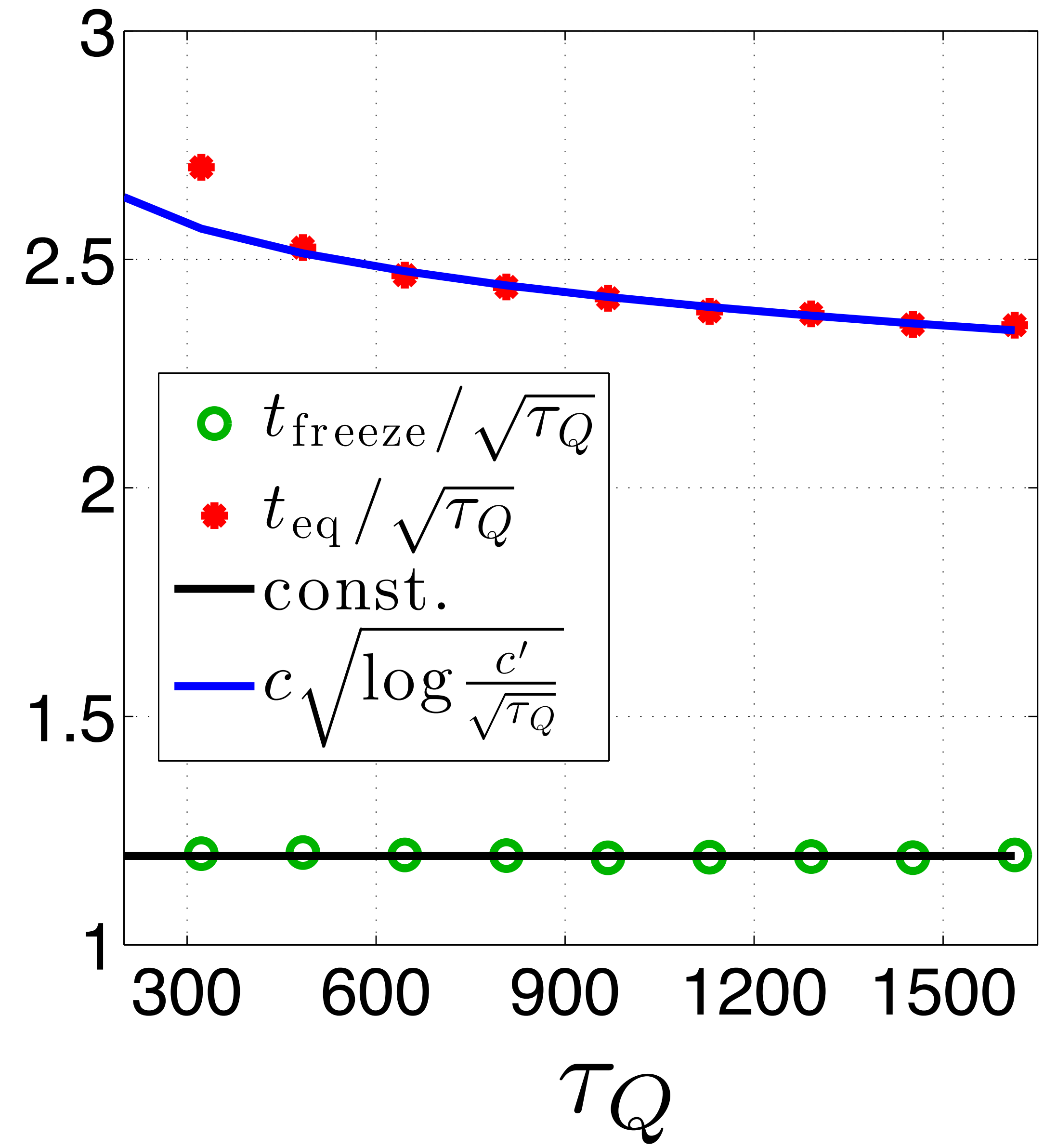


Comparing to unstable mode analysis (II)

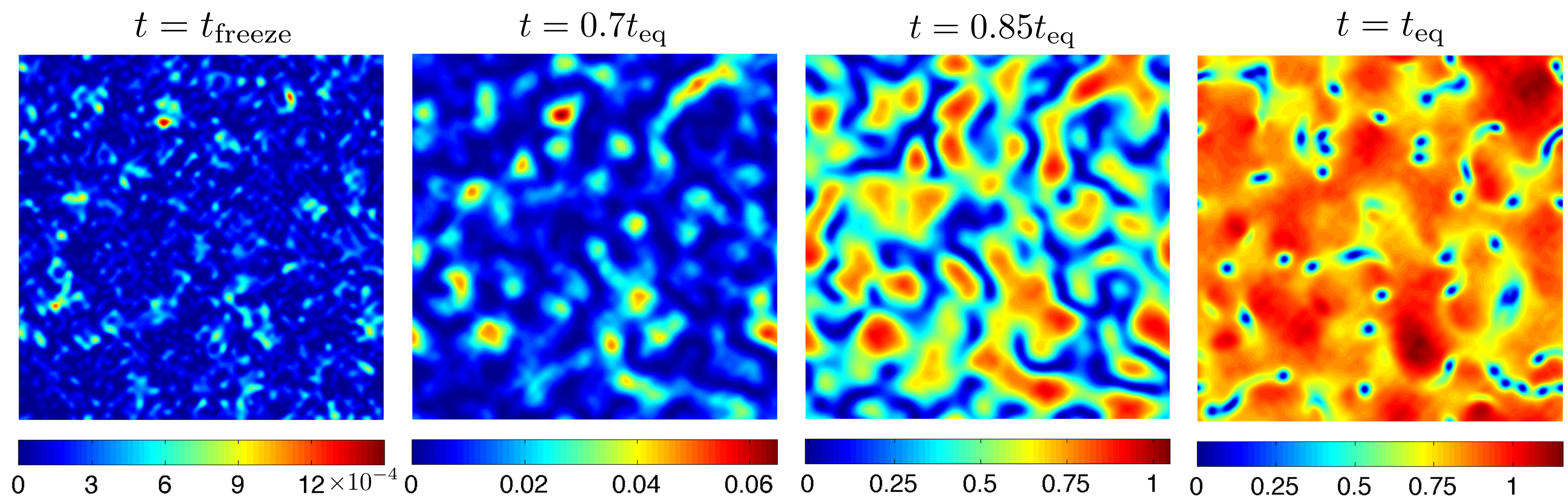
For holography (mean field exponents)

- $R \sim \frac{1}{\zeta\sqrt{\tau_Q}}$ and $t_{\text{freeze}} \sim \sqrt{\tau_Q}$.
- $t_{\text{eq}} \sim \sqrt{\log R}\, t_{\text{freeze}}$





Consequences of extended non-adiabatic growth



If $t_{\mathrm{eq}} \gg t_{\mathrm{freeze}}$ then

- No well-defined vortices form until $t \sim t_{\mathrm{eq}}$.
- $\ell_{\mathrm{co}}(t_{\mathrm{eq}}) = \xi_{\mathrm{freeze}} \left(\frac{t_{\mathrm{eq}}}{t_{\mathrm{freeze}}} \right)^{\frac{1+(z-2)\nu}{2}} \gg \xi_{\mathrm{freeze}}$.
- **Far fewer defects formed than KZ predicts**

$$n/n_{KZ} \sim (t_{\mathrm{eq}}/t_{\mathrm{freeze}})^{-\frac{(d-D)(1-(z-2)\nu)}{2}} .$$

- **State at $t = -t_{\mathrm{freeze}}$ is irrelevant.**

How natural is $t_{\text{eq}} \gg t_{\text{freeze}}$?

1. All holographic theories have $t_{\text{eq}} \gg t_{\text{freeze}}$.

- G_{N} suppressed Hawking $\Rightarrow \zeta \sim 1/N^2$ and

$$t_{\text{eq}} \sim [\log N]^{1/(1+\nu z)} t_{\text{freeze}}.$$

2. Universality classes $(d-z)\nu - 2\beta > 0$ have $t_{\text{eq}} \gg t_{\text{freeze}}$.

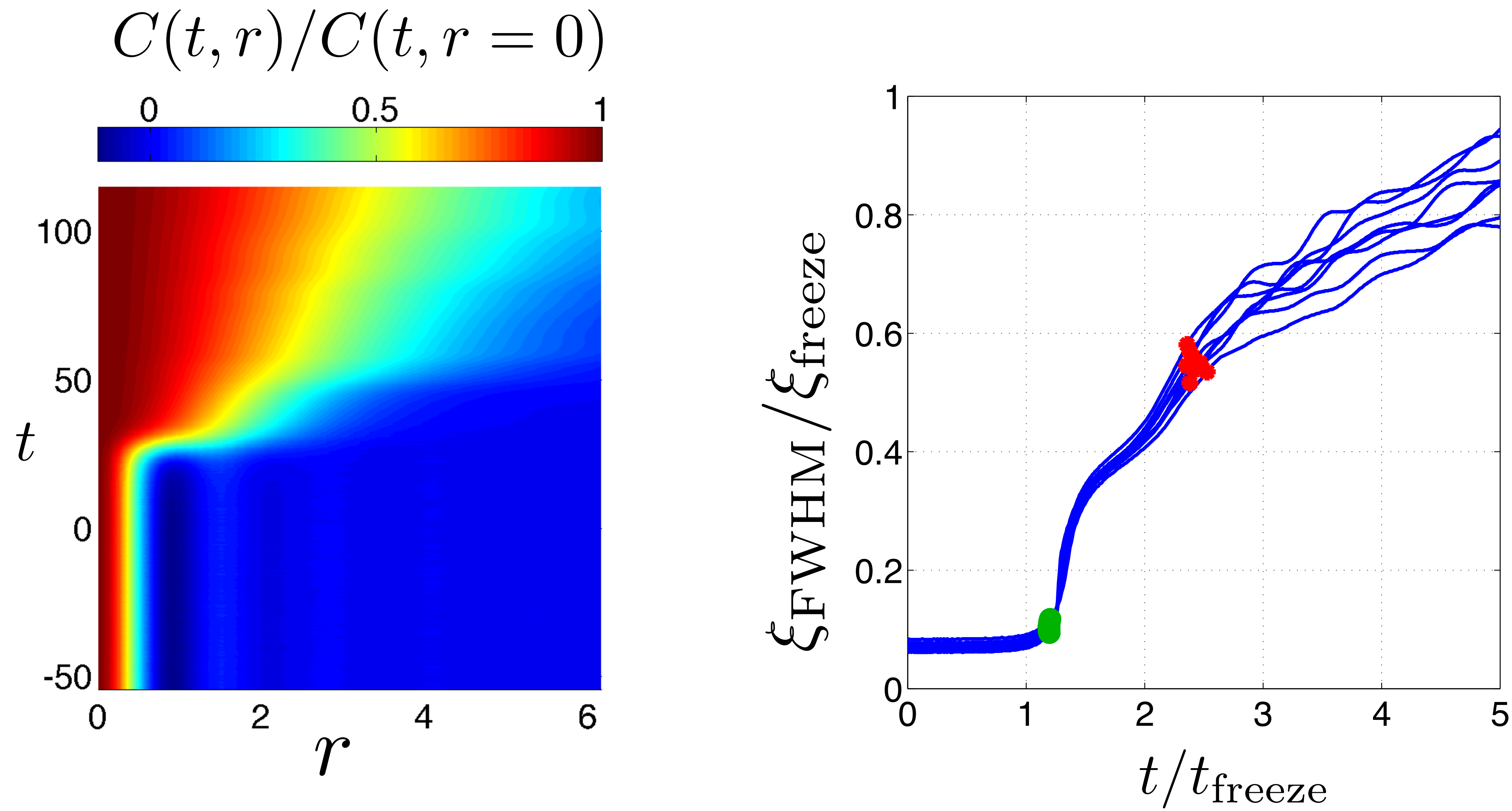
- $t_{\text{eq}} \sim [\log \tau_Q]^{1/(1+\nu z)} t_{\text{freeze}}$.
- Example: superfluid ^{4}He.

$\Rightarrow$ Log correction to density of defects

$$\frac{n}{n_{KZ}} \sim [\log \tau_Q]^{-\frac{(d-D)(1+(z-2)\nu)}{2(1+z\nu}} n_{KZ}.$$

IR coarsening before condensate formation

- Smear over scales $\sim \xi_{\mathrm{freeze}}$.



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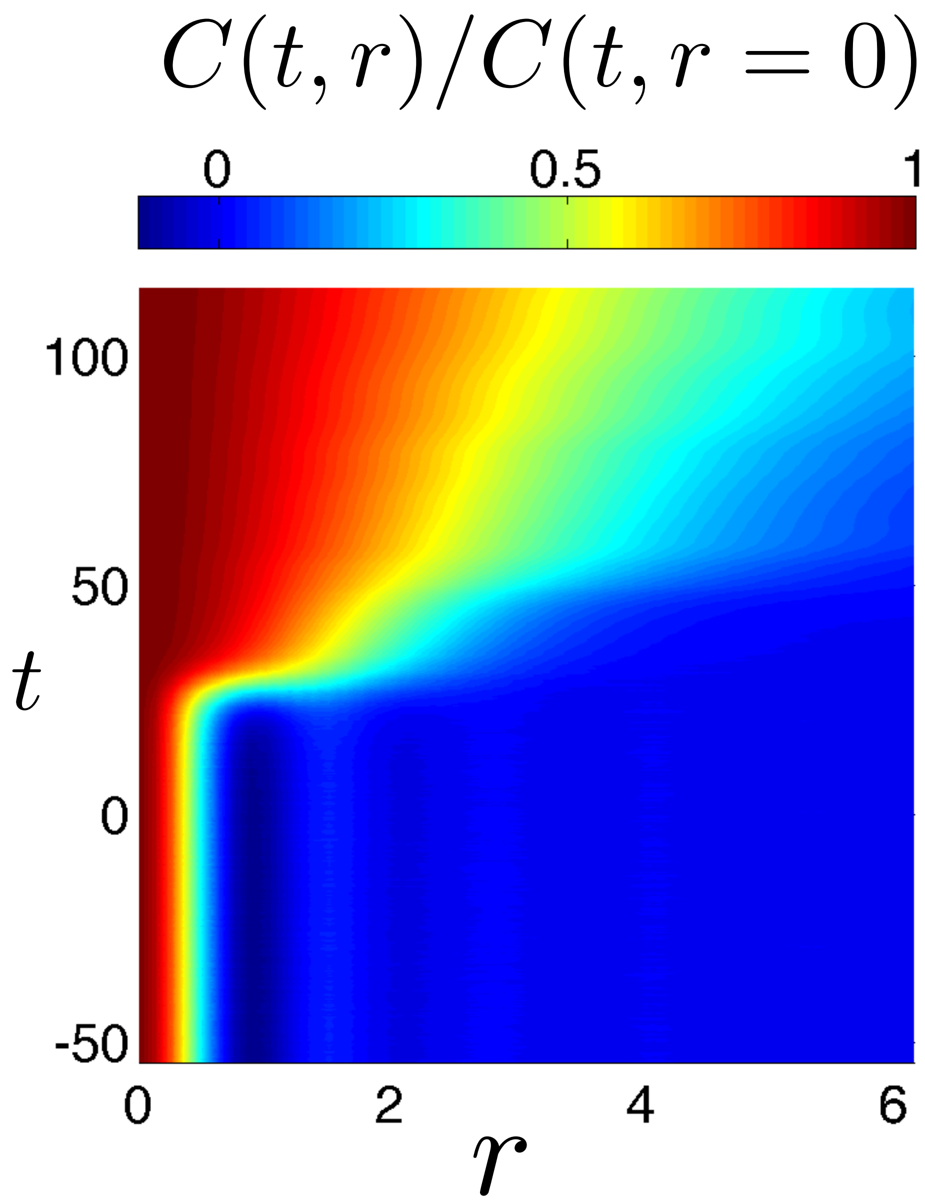

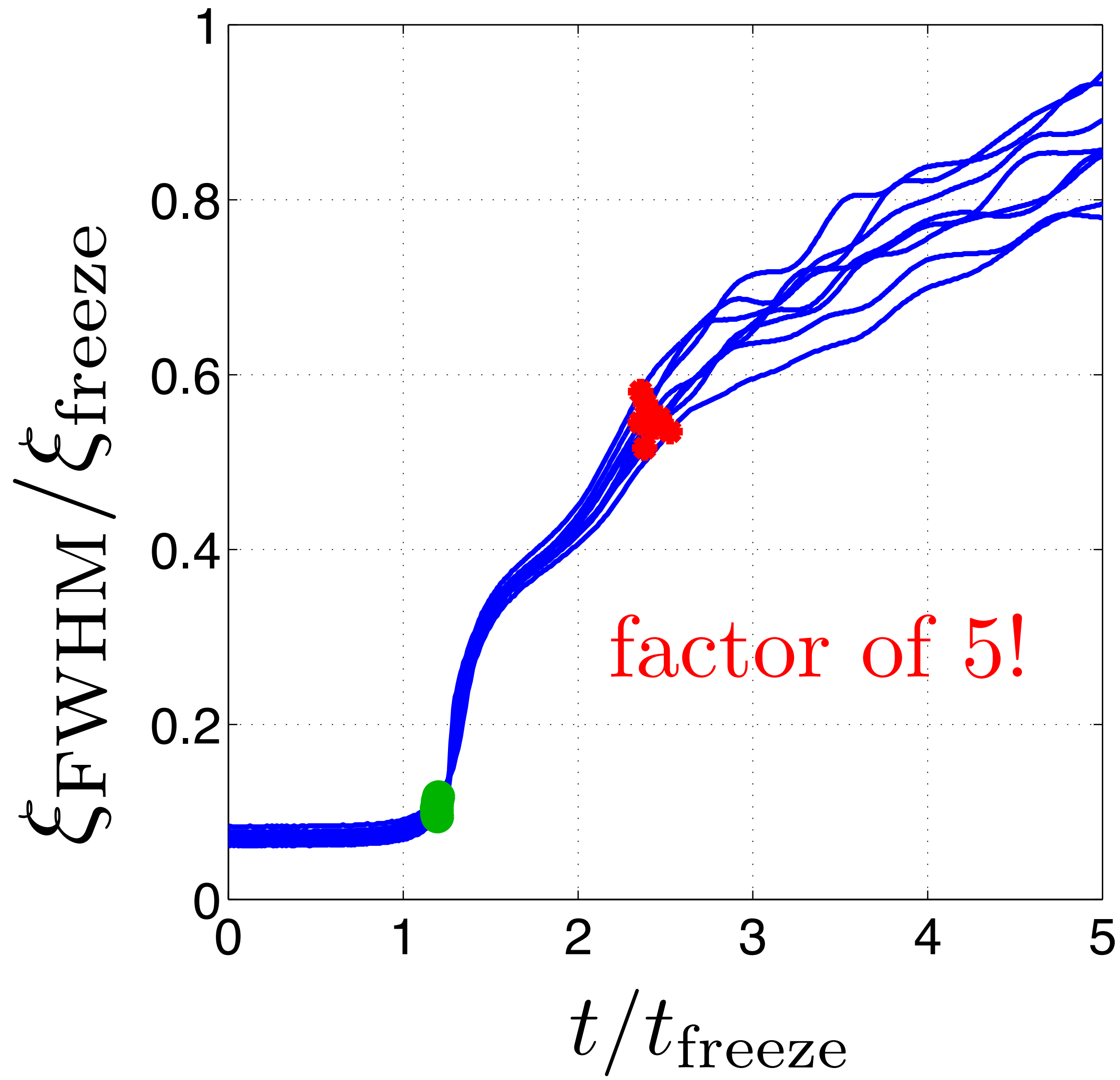

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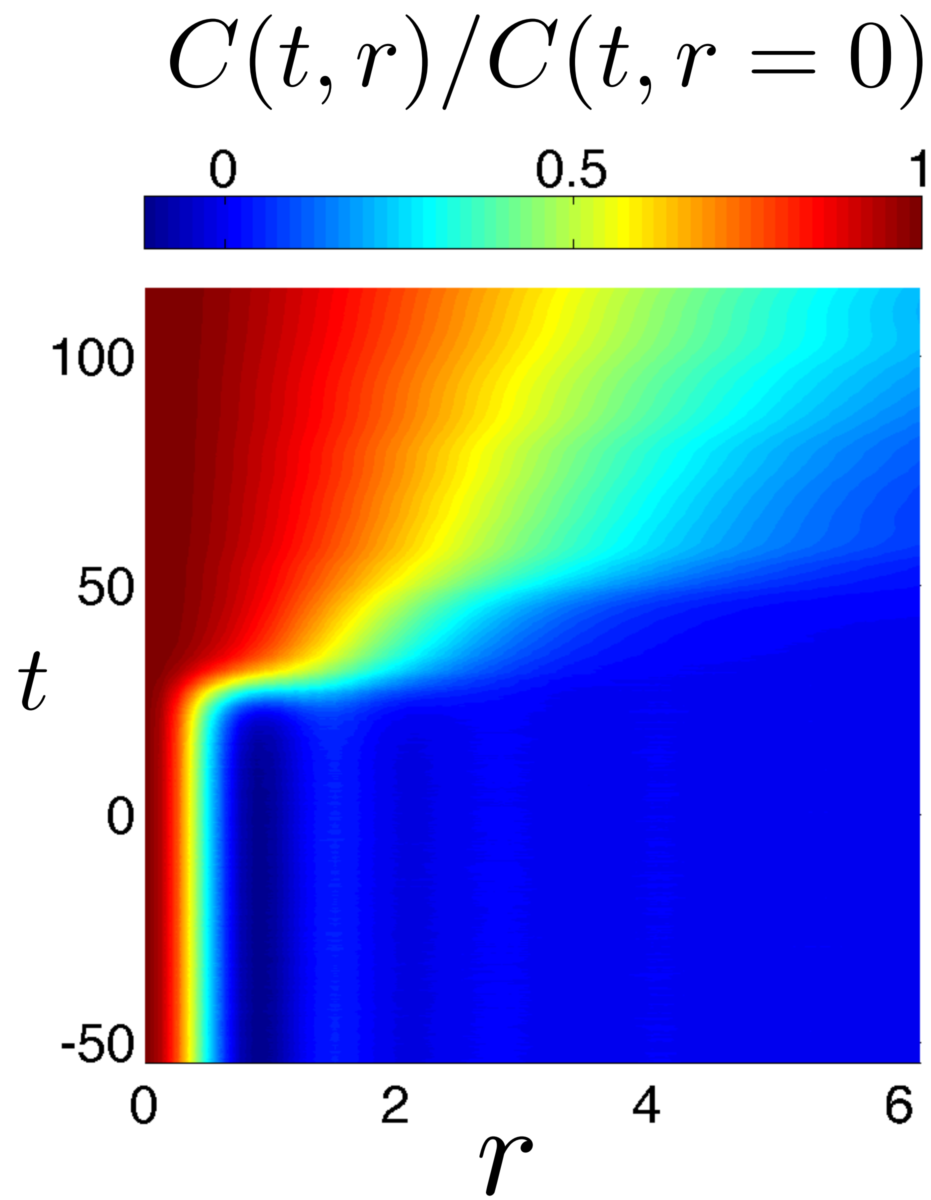
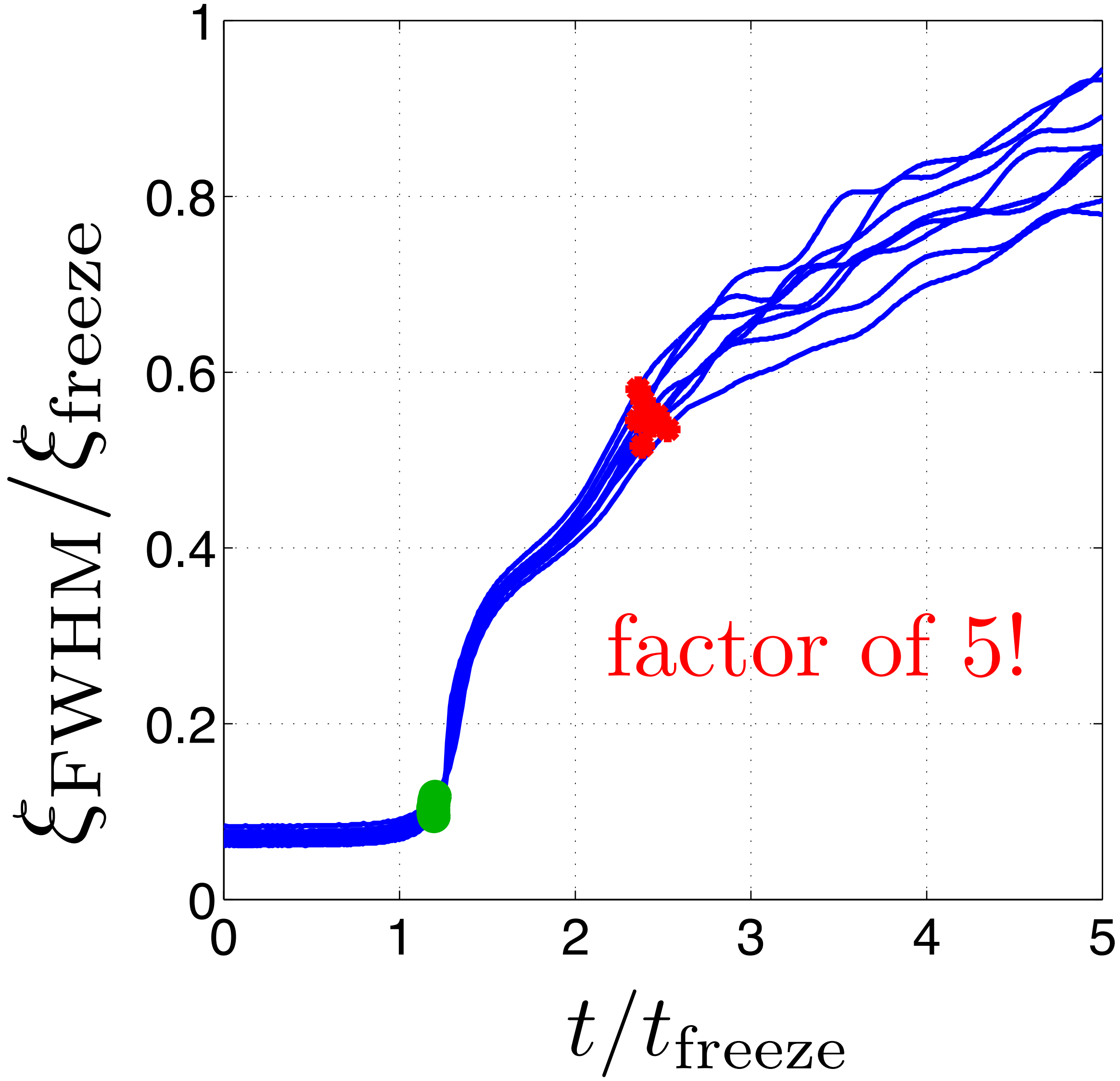
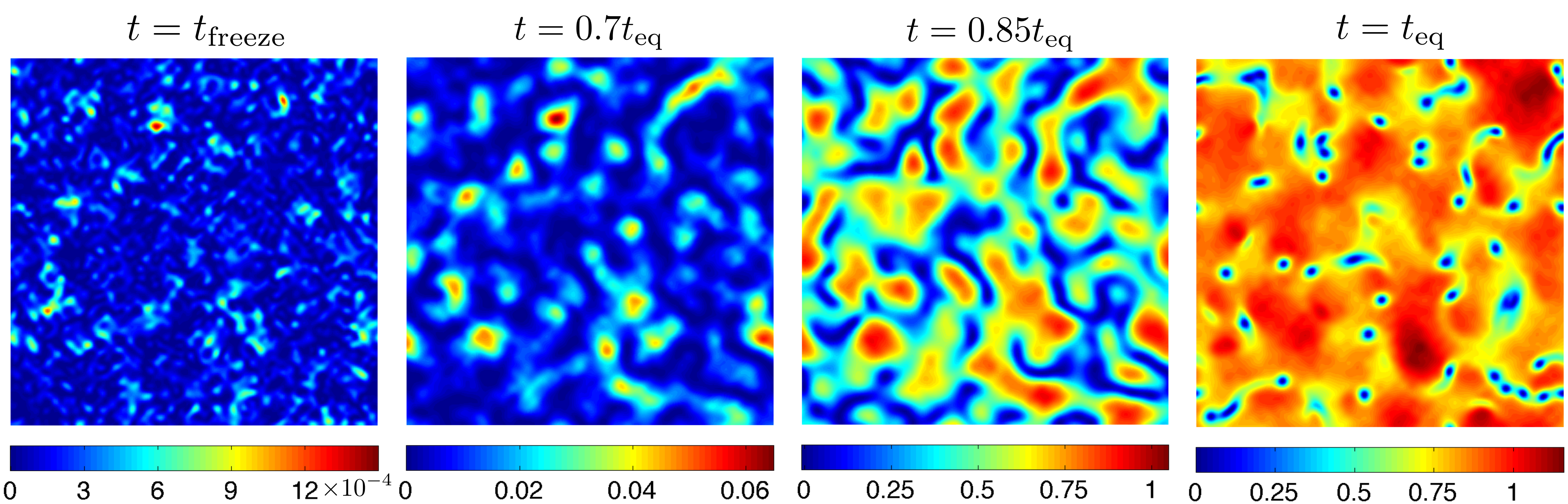




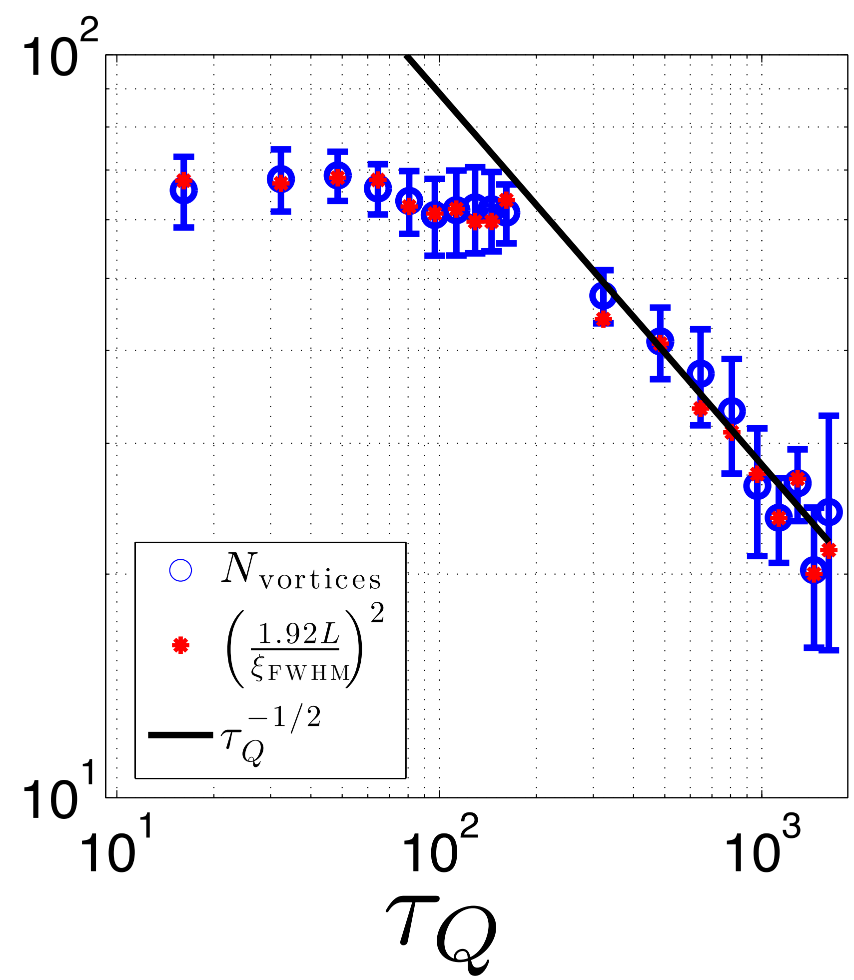
Due to explosive growth of IR modes after $+t_{\text{freeze}}$.

Counting defects



For holography,

$$n \sim \frac{1}{\sqrt{\log N}} \tau_Q^{-1/2}.$$



$O(25)$ fewer vortices than KZ estimate

Summary

- For wide class of theories there exists new scale t_{eq}.
- Exposive growth of IR modes between $t_{\text{freeze}} < t < t_{\text{eq}}$.
- If $t_{\text{eq}} \gg t_{\text{freeze}}$
  - Initial correlation ξ_{freeze} not imprinted on final state.
  - Far fewer defects formed than KZ predicts.
  - Log corrections to KZ scaling law.

Sudden quenches

$$\ell_{\rm co} \sim 1/q_{\rm max} \sim \epsilon_{\rm final}^{-\nu}$$