Rényi Entropy in AdS_3/CFT_2

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Based on the following works:

- B.C., J.-j. Zhang, arXiv:1309.5453 [hep-th].
- B.C., F. -y. Song and J. -j. Zhang, arXiv:1401.0261 [hep-th].
- B.C., J. Long and J.-j. Zhang, arXiv: 1312.5510 [hep-th].

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- B.C., J. Long and J.-j. Zhang, arXiv: 1312.5510 [hep-th].
- B.C., J.-q. Wu, arXiv:1405.6254 [hep-th], arXiv:1412.0761 [hep-th], arXiv:1412.0763 [hep-th], arXiv:1506.03206 [hep-th],arXiv:1509.02062 [hep-th].
- B.C., J.-q. Wu and Z.-c. Zheng, arXiv:1507.00183 [hep-th].

- Many other related works ...

AdS$_3$/CFT$_2$ correspondence

A new window to study AdS/CFT without resorting to string theory

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g}(R + \frac{2}{l^2})$$

3D AdS$_3$ Einstein gravity is special: No locally dynamical d.o.f

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More precisely they found that under appropriate boundary conditions the asymptotic symmetry group (ASG) of AdS$_3$ Einstein gravity is generated by two copies of Virasoro algebra with central charges

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In modern understanding: quantum gravity in AdS$_3$ is dual to a 2D CFT at AdS boundary

AdS$_3$/CFT$_2$: a perfect platform

AdS$_3$ gravity is solvable: all classical solutions are quotients of AdS$_3$ such that a path-integral is possible in principle_{E. Witten (1988) ...}
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2D conformal symmetry is infinitely dimensional so that 2D CFT has been very well studied_{Belavin et.al. (1984) ...}, even though the explicit construction of dual 2D CFT is unknown
$\Rightarrow$ Universal properties

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$\Rightarrow$ Universal properties

However, it is not clear

1. how to define the quantum AdS$_3$ gravity?
2. what is the dual CFT?

Semiclassical AdS$_3$ gravity

Let us focus on the semiclassical gravity, which corresponds to the CFT at the large central charge limit

$$c = \frac{3l}{2G}$$

- ▶ The partition function gets contributions from the saddle points

- ▶ For each classical solution, its regularized on-shell action $\propto 1/G \sim c$
- ▶ The 1-loop correction comes from the fluctuations around the solution $\propto O(1)$
- ▶ Possibly there are higher loop correction $\propto O(1/c^{l-1})$

Semi-classical AdS$_3$ Gravity

Semiclassical solutions

$$R_{\mu\nu} = -\frac{2}{l^2} g_{\mu\nu},$$

All solutions are locally AdS$_3$

More precisely, all classical solutions could be obtained as the quotients of global AdS$_3$ by the Kleinian group, a discrete subgroup of $PSL(2, C)$

$$M = AdS_3/\Gamma$$

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For the handlebody solutions, the subgroup Γ is a Schottky group, a finitely generated free group, such that all nontrivial elements are loxodromic

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \sim \begin{pmatrix} p^{1/2} & 0 \\ 0 & p^{-1/2} \end{pmatrix}, \quad 0 < |p| < 1$$

It is often convenient to work in Euclidean version. Then on the boundary, there is a compact Riemann surface.

Schottky uniformization

Every compact Riemann surface could be obtained by the Schottky uniformization "Retrosection theorem" by Koebe (1914)

The Schottky uniformization is determined by a differential equation

$$\psi''(z) + \frac{1}{2} T_{zz}\psi(z) = 0, \qquad (1.1)$$

Two independent solutions: ψ_1 and ψ_2, whose ratio $w = \frac{\psi_1}{\psi_2}$ gives the quotient map

More importantly, T_{zz} is the stress tensor of Liouville CFT. Its explicit form depends on $(3g - 3)$ complex accessory parameters with respect to the holomorphic quadratic differentials on the Riemann surface.

On-shell regularized action

- The essential point is that the on-shell regularized bulk action of gravitational configuration in pure AdS$_3$ gravity is a Liouville type action defined on the fundamental region K. Krasnov (2000),Zograf and Takhtadzhyan (1988)

- More importantly, the dependence of this so-called Zograf-Takhtadzhyan action on the accessory parameters is determined by the differential equation Zograf and Takhtadzhyan (1988)

$$\frac{\partial S_n}{\partial z_i} = -\frac{cn}{6(n-1)}\gamma_i. \qquad (1.2)$$

γ_i are the accessory parameters, being fixed by the monodromy problem of the ordinary differential equation (1.1)

- For a general Riemann surface of high genus, it is a difficult problem to determine this regularized action, even perturbatively

- Nevertheless, for the Riemann surface in computing the Rényi entropy, the problem is simplified due to the replica symmetry
    1. Two-interval case: one cross ratio
    2. Single interval in a torus (finite temperature, finite size)

Rényi entropy in 2D CFT

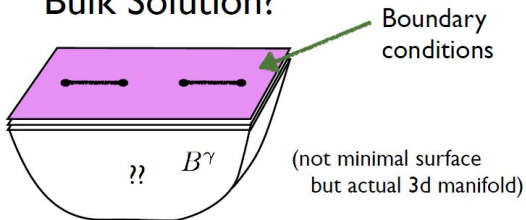$$S_A^{(n)} = -\frac{\ln \operatorname{tr}_A \rho_A^n}{n-1}$$

The partition function on a n-sheeted Riemann surface

1. Double-interval case: $\Rightarrow$ genus $(n-1)$ RS
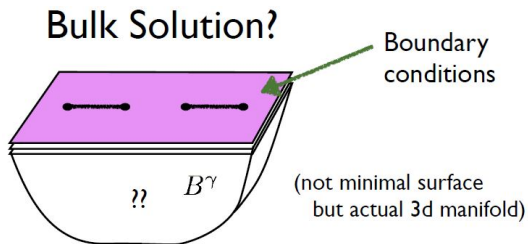2. Single-interval on torus: $\Rightarrow$ genus n RS

Partition function on a higher genus RS: usually hard to compute

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2. Single-interval on torus: $\Rightarrow$ genus n RS

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Let's try holographic computation...

HRE in AdS$_3$: A sketch T. Faulkner 1303.7221

Bulk Solution?

Boundary conditions

B^γ ??

(not minimal surface but actual 3d manifold)

- Find the bulk gravity solutions B^γ such that $\partial B^\gamma = \Sigma_n$
- Assumption: Consider only the handlebody solution, Γ_γ is the Schottky group

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- Assumption: Consider only the handlebody solution, Γ_γ is the Schottky group
- For a fixed Riemann surface Σ_n, find its Schottky uniformization
- Extend the uniformization to the bulk to find the gravitational solution
- From AdS$_3$/CFT$_2$, the classical regularized bulk action should reproduces the leading order partition function on Σ_n.
- Strategy: imposing appropriate monodromy condition to determine the accessory parameters, and integrating the differential equation to read the action
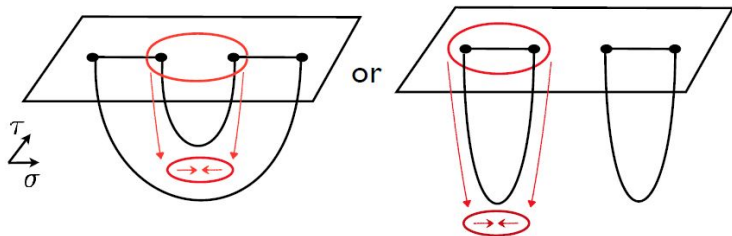
Two-interval case

In this case

$$T_{zz} = \sum_i \frac{\Delta}{(z - z_i)^2} + \frac{\gamma_i}{z - z_i},$$

where

$$\Delta = \frac{1}{2}(1 - \frac{1}{n^2}),$$

The accessory parameters are determined by requiring trivial monodromy at infinity and on one of two cycles (red one)

Single interval on a torus

$$T_{zz} = \sum_i (\Delta \wp(z - z_i) + \gamma_i \zeta(z - z_i)) + \delta,$$

where $\wp, \zeta$ are the doubly periodic Weierstrass elliptic function and zeta function respectively. Barrella et.al. 1306.4682,BC and J.-q. Wu1604.03644

- Torus: $z \sim z + mL + in\beta \Rightarrow$ thermal circle and spacial circle
- We can set trivial monodromy along one circle and the cycle enclosing two branch points, so that the identification of the other circle gives the generator of Schottky group
  1. $T > T_{HP}$, the thermal circle is of trivial monodromy.
  2. $T < T_{HP}$, the spacial circle is of trivial monodromy.

It turns out that the regularized action depends on the moduli of the torus as well.

For the torus at high temperature, the regularized action depends not only on the accessory parameter,

$$\frac{\partial S_n}{\partial z_i} \;=\; -\frac{cn}{6(n-1)}\gamma_i,$$

but also on the size of the torus

$$\frac{\partial S_n}{\partial L} = \frac{c}{12\pi}\frac{n}{n-1}\beta(\tilde{\delta} - \tilde{\delta}_{n=1}). \qquad (1.3)$$

where $\tilde{\delta}$ includes all the constant contribution in $T(z)$.

Beyond classical action

- Simply speaking, the holographic Renyi entropy(HRE) is given by the classical action of the corresponding gravitational configurations

- The $n \to 1$ limit reproduces the RT formula T. Hartman 1303.6955, T. Faulkner 1303.7221

- It captures only the leading order Rényi entropy in CFT

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- The subleading corrections in 2D CFT being independent of c should correspond to the 1-loop partition function around the configurations
- There are good reasons to consider the quantum correction: mutual information, thermal correction, ...
  e.g. the mutual information satisfies M. Wolf et.al. 0704.3906

$$I(A,B) \geq \frac{|<\mathcal{O}_A \cdot \mathcal{O}_B> - <\mathcal{O}_A><\mathcal{O}_B>|^2}{2|\mathcal{O}_A|^2|\mathcal{O}_B|^2}$$

1-loop correction

For a fixed handle-body solution obtained from the Schottky group, its 1-loop partition function

$$Z^{1-loop} = \prod_{\gamma \in \mathcal{P}} \prod_{s} \prod_{m=s}^{\infty} \frac{1}{|1 - q_\gamma^m|}. \tag{1.4}$$

Here the product over s is with respect to the spins of massless fluctuations and $\mathcal{P}$ is a set of representatives of primitive conjugacy classes of the Schottky group Γ. q_γ is defined by writing the two eigenvalues of $\gamma \in \Gamma$ as $q_\gamma^{\pm 1/2}$ with $|q_\gamma| < 1$.

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- Find the Schottky group Γ corresponding to $\mathcal{M}_n$
- Generate $\mathcal{P} = \{\text{non-repeated words up to conjugation}\}$, e.g.

$$\mathcal{P} = \{L_1, L_2, L_1^{-1}, L_2^{-1}, L_1 L_2 \sim L_2 L_1, ...\}$$

- Compute eigenvalues of these words and sum over their contributions
- Difficulty: infinite number of words
- For two intervals with small cross ratio x, only finitely many words contribute to each order in x[Barrella et.al. 1306.4682, BC et.al. 1312.5510]
- For the single interval on a torus, similar thing happens[Barrella et.al. 1306.4682, BC et.al. 1405.6254]

CFT computation

Large c CFT

The semiclassical AdS$_3$ gravity is dual to a large c CFT. One attractive feature of the large c CFT is that the vacuum module plays a fundamental role

1. $g_{\mu\nu} \leftrightarrow T_{\mu\nu}$
2. The study on the conformal block T. Hartman 1303.6955,...

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We only focus on the Virasoro vacuum module in our CFT study

The replica trick requires us to study a orbifold CFT: $(CFT)_n/Z_n$. When the intervals are short, we have the OPE of the twist operators

$$\sigma(z, \bar{z})\tilde{\sigma}(0,0) = c_n \sum_K d_K \sum_{m,r \geq 0} \frac{a_K^m}{m!} \frac{\bar{a}_K^r}{r!} \frac{1}{z^{2h-h_K-m}\bar{z}^{2\bar{h}-\bar{h}_K-r}} \partial^m \bar{\partial}^r \Phi_K(0,0),$$

with the summation K being over all the independent quasiprimary operators of CFT_n.

We are interested in the two-interval case, then

$$
\begin{aligned}
\mathrm{Tr}\rho_A^n &= \langle \sigma(1+y, 1+y)\tilde{\sigma}(1,1)\sigma(y,y)\tilde{\sigma}(0,0)\rangle_C \\
&= c_n^2 x^{-\frac{c}{6}\left(n-\frac{1}{n}\right)} \left(\sum_K \alpha_K d_K^2 x^{h_K} F(h_K, h_K; 2h_K; x) \right)^2
\end{aligned}
$$

where x is the cross ratio.

In the small x limit, to each order only finite number of the quasi-primary operators contribute

Rényi mutual information: leading order

The leading part, being proportional to the central charge c,

$$I_n^{LO} = \frac{c(n-1)(n+1)^2 x^2}{144n^3} + \frac{c(n-1)(n+1)^2 x^3}{144n^3}$$
$$+ \frac{c(n-1)(n+1)^2\left(1309n^4 - 2n^2 - 11\right)x^4}{207360n^7}$$
$$+ \frac{c(n-1)(n+1)^2\left(589n^4 - 2n^2 - 11\right)x^5}{103680n^7}$$
$$+ \frac{c(n-1)(n+1)^2\left(805139n^8 - 4244n^6 - 23397n^4 - 86n^2 + 188\right)x^6}{156764160n^{11}}$$
$$+ \text{(the terms proportional to } x^7 \text{ and } x^8\text{)} + \mathcal{O}\left(x^9\right)$$

It matches exactly with the holographic result up to order x^8.M. Headrick

1006.0047, T. Hartman 1303.6955, T. Faulkner 1303.7221

The classical mutual information ($n = 1$) is vanishing when the two intervals are far apart.

Mutual information: next-to-leading order

The NLO part from the vacuum module, being proportional to c^0, is

$$I_n^{NLO} = \frac{(n+1)\left(n^2+11\right)\left(3n^4+10n^2+227\right)x^4}{3628800n^7}$$
$$+ \frac{(n+1)\left(109n^8+1495n^6+11307n^4+81905n^2-8416\right)x^5}{59875200n^9}$$
$$+ \frac{(n+1)\left(1444050n^{10}+19112974n^8+140565305n^6+1000527837n^4-167731255n^2-14142911\right)x^6}{523069747200n^{11}}$$
$$+ \text{(the terms proportional to } x^7 \text{ and } x^8) + \mathcal{O}\left(x^9\right).$$

It matches exactly the holographic 1-loop result up to order x^8. _{T. Barrella et.al.}

1306.4682,B.C. and J.-J Zhang 1309.5453

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<small>1306.4682,B.C. and J.-J Zhang 1309.5453</small>

The mutual information is not really vanishing due to the quantum correction

Mutual information: NNLO

Remarkably there is also the NNLO contribution, being proportional to $1/c$,

$$
\begin{aligned}
I_n^{NNLO} = & \frac{(n+1)\left(n^2-4\right)\left(19n^8+875n^6+22317n^4+505625n^2+5691964\right)x^6}{70053984000 n^{11} c} \\
& \frac{(n+1)\left(n^2-4\right)\left(276n^{10}+12571n^8+317643n^6+7151253n^4+79361381n^2-9428724\right)x^7}{326918592000 n^{13} c} \\
& +(\text{the terms proportional to } x^8) + \mathcal{O}\left(x^9\right),
\end{aligned}
$$

This is novel, expected to be confirmed by 2-loop computation in gravity

- When $n = 2$, the two-loop correction is vanishing, as $S^{(2)}$ being genus 1 partition function is 1-loop exact A. Maloney and E. Witten 0712.0155
- When $n > 2$, there are nonvanishing 2-loop corrections Xi Yin, 0710.2129
- Actually there is nonvanishing quantum 3-loop contribution, being proportional to $1/c^2$, for $S^{(n)}$, $n > 3$.

Single interval on a torus

- When the interval is not very large, the Rényi entropy could be computed perturbatively for both high and low temperatures
- At a low temperature T in units of $1/L$, the thermal density matrix could be expanded level by level

$$\rho = \frac{e^{-\beta H}}{\mathrm{Tr} e^{-\beta H}} = \frac{1}{\mathrm{Tr} e^{-\beta H}} \sum \mid \phi \rangle \langle \phi \mid e^{-\beta E_\phi}$$

- The expansion is respect to $e^{-2\pi \Delta / TL}$, Δ being the dimension of the excitation
- The expansion could be understood in the following way: cut open the torus and insert the complete basis at the cut
- At the low levels, the computations change to multi-point function on a cylinder, via state-operator correspondence
- At the leading-order and next-leading-order, we found perfect agreements with holographic results up to $e^{-8\pi / TL}$

Entanglement entropy

The entanglement entropy could be read easily

$$
\begin{aligned}
S_{EE} &= c\left(\frac{1}{6}\log\sin^2\frac{\pi l}{L} + const\right) \\
&\quad + 8(1 - \frac{\pi l}{L}\cot(\frac{\pi l}{L}))e^{-\frac{4\pi}{TL}} + 12(1 - \frac{\pi l}{L}\cot(\frac{\pi l}{L}))e^{-\frac{6\pi}{TL}} + O(e^{-\frac{8\pi}{TL}})
\end{aligned}
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- Due to the thermal correction, the symmetry $l \to L - l$ is broken

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S_{EE}(l) \neq S_{EE}(L - l)
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▶ The thermal corrections to EE are independent of c
▶ Correspondingly, they are captured by the quantum corrections in the bulk
▶ This is not true for RE, in which the thermal corrections appear even in the leading order

High temperature case

- In the high temperature case, one may "quantize" the theory along the spacial direction rather than the thermal direction

- In other words, the spacial direction and the thermal direction exchange the role and there is a modular transformation

$$L \to i\beta, \qquad \beta \to iL,$$

relating the two cases

- As a result, we have the density matrix

$$\rho \propto e^{-LH} = e^{-2\pi(L/\beta)(L_0 + \tilde{L}_0 - \frac{c}{12})}$$

- This is in accord with the holographic computation

Large interval limit

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& + 8(1 - \frac{\pi l}{L}\cot(\frac{\pi l}{L}))e^{-\frac{4\pi}{TL}} + 12(1 - \frac{\pi l}{L}\cot(\frac{\pi l}{L}))e^{-\frac{6\pi}{TL}} + O(e^{-\frac{8\pi}{TL}})
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The large interval limit $l \to L$ is singular

Such singular behavior exists for other CFT. For example, the thermal correction of a primary operator to EE takes a universal form J. Cardy and C.P. Herzog 1403.0578

$$
\begin{aligned}
\delta S_n \;=\; & \frac{g}{1-n}\left(\frac{1}{n^{2\Delta-1}}\frac{\sin^{2\Delta}\left(\frac{\pi l}{L}\right)}{\sin^{2\Delta}\left(\frac{\pi l}{nL}\right)} - n\right)e^{-2\pi\Delta/TL} + o(e^{-2\pi\Delta/TL}) \\
\delta S_{EE} \;=\; & 2g\Delta\left(1 - \frac{\pi l}{L}\cot\left(\frac{\pi l}{L}\right)\right)e^{-2\pi\Delta/TL} + o(e^{-2\pi\Delta/TL}), \quad (2.1)
\end{aligned}
$$

It is singular in the limit $l \to L$.

Large interval limit

$$S_{EE} = c\left(\frac{1}{6}\log\sin^2\frac{\pi l}{L} + const\right)$$
$$+ 8(1 - \frac{\pi l}{L}\cot(\frac{\pi l}{L}))e^{-\frac{4\pi}{TL}} + 12(1 - \frac{\pi l}{L}\cot(\frac{\pi l}{L}))e^{-\frac{6\pi}{TL}} + O(e^{-\frac{8\pi}{TL}})$$

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$$\delta S_{EE} = 2g\Delta\left(1 - \frac{\pi l}{L}\cot\left(\frac{\pi l}{L}\right)\right)e^{-2\pi\Delta/TL} + o(e^{-2\pi\Delta/TL}), \quad (2.1)$$

It is singular in the limit $l \to L$. We needs a different way to compute the Rényi entropy in the large interval limit.

Large interval limit

$$S_{EE} = c\left(\frac{1}{6}\log\sin^2\frac{\pi l}{L} + const\right)$$
$$+8(1-\frac{\pi l}{L}\cot(\frac{\pi l}{L}))e^{-\frac{4\pi}{TL}} + 12(1-\frac{\pi l}{L}\cot(\frac{\pi l}{L}))e^{-\frac{6\pi}{TL}} + O(e^{-\frac{8\pi}{TL}})$$

The large interval limit $l \to L$ is singular

Such singular behavior exists for other CFT. For example, the thermal correction of a primary operator to EE takes a universal form J. Cardy and C.P. Herzog 1403.0578

$$\delta S_n = \frac{g}{1-n}\left(\frac{1}{n^{2\Delta-1}}\frac{\sin^{2\Delta}\left(\frac{\pi l}{L}\right)}{\sin^{2\Delta}\left(\frac{\pi l}{nL}\right)} - n\right)e^{-2\pi\Delta/TL} + o(e^{-2\pi\Delta/TL})$$

$$\delta S_{EE} = 2g\Delta\left(1-\frac{\pi l}{L}\cot\left(\frac{\pi l}{L}\right)\right)e^{-2\pi\Delta/TL} + o(e^{-2\pi\Delta/TL}), \quad (2.1)$$

It is singular in the limit $l \to L$. We needs a different way to compute the

Rényi entropy in the large interval limit.

We got inspiration from the holographic computation

Large interval: holographic result

- The entanglement entropy of single interval at high temperature is

$$S_{EE} = \frac{c}{3} \log \sinh(\pi T l) \qquad (2.2)$$

- From holographic point of view, it is given by the geodesic in the BTZ background ending on the interval

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- From holographic point of view, it is given by the geodesic in the BTZ background ending on the interval
- However, it is only true when the interval is not very large
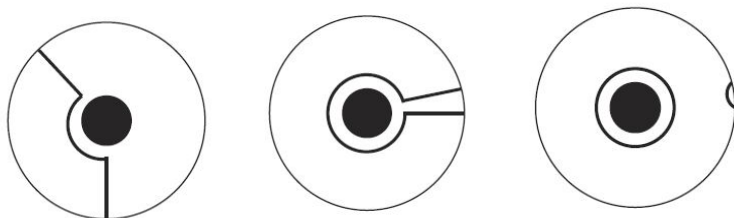
- When the interval is very large, the disconnected curve gives smaller length T. Azeyanagi et.al. 0710.2956
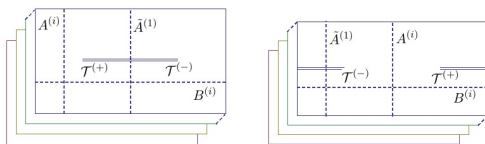
Our proposal in CFT BC and J.-q. Wu 1412.0763
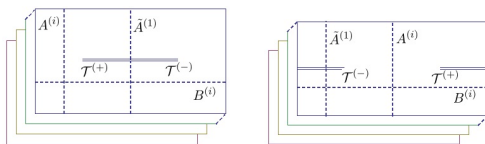
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- Insert a complete basis at $\tilde{A}^{(1)}$ cycle
- This requires us to study the twist sector of the CFT carefully
- This proposal can be applied to any CFT

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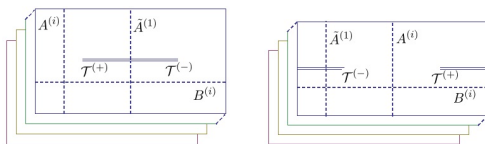
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- ▶ We tested our proposal in the case of free boson, after correcting some errors in the literature_{BC and J.-q. Wu, 1412.0763,1501.00373}
- ▶ Next, we studied the large interval Rényi entropy at high temperature in the context of AdS_3/CFT_2 correspondence

HRE: large interval limit_{BC and J.-q. Wu, 1506.03206}

- ▶ Different gravitational configurations
- ▶ Different set of monodromy conditions
- ▶ Among n cycles of trivial monodromy
  1. One cycle which goes across the branch cut for n times
  2. The other $n-1$ independent cycles enclosing the complementary interval
- ▶ Both the classical and 1-loop contributions are in good agreements with CFT results
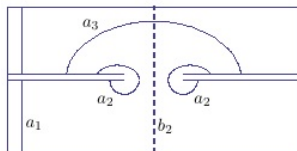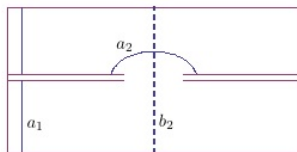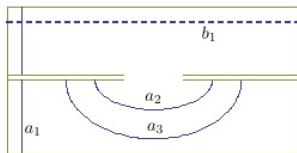
Semi-classical picture

- Rényi entropy opens a new window to study the AdS_3/CFT_2 correspondence
- AdS_3/CFT_2 correspondence at semi-classical level: the leading order partition function on a general (higher genus) RS is captured by the on-shell regularized gravity action, which reduces to the ZT action
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- Moreover, the next-to-leading order partition function is given by the 1-loop partition function in the bulk, this can be proved[BC and J.-q. Wu 1509.02062](See Jie-qiang's poster)

Large c CFT

- For pure AdS_3 quantum gravity, it is the vacuum conformal module in the dual CFT which dominate the contribution
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- What's the CFT dual of quantum AdS_3 gravity?E. Witten 1988, S. Carlip 050302, A. Maloney and E. Witten 0712.0155,H. Verlinde et.al. 1412.5205,...

- ...

Thanks for your attention!