# Holographic Phases of Rényi Entropies

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Based on hep-th/1306.2640 with A. Maloney and S. Matsuura

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McGill

Consider a QFT in state $\rho$ with Hilbert space

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

We trace over the DOF in B

$$\rho_A = \mathrm{Tr}_B \rho \qquad\qquad S_A = -\mathrm{Tr} \rho_A \log \rho_A$$

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The Rényi entropies are defined as

$$S_n = \frac{1}{1-n} \log \mathrm{Tr}\left[\rho_A^n\right]$$

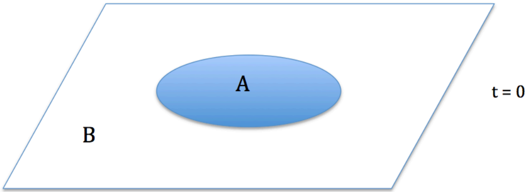
$$\xrightarrow{\text{if analytic}} \qquad S_A = \lim_{n \to 1} S_n$$

But is $S_n$ analytic in n?

# Setup

Large $N$ $CFT_d$ in $|0\rangle$ $\overset{DUAL}{\Longleftrightarrow}$ Einstein-Scalar in $AdS_{d+1}$

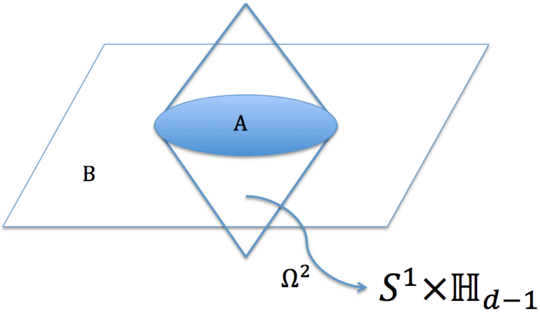
with scalar operator $\mathcal{O}_\Delta$ $\qquad$ $m_\phi^2 \ell^2 = \Delta(\Delta - d)$



$$\rho_A = e^{-2\pi H_E}$$

$\Rightarrow$ Thermal state on $S^1 \times \mathbb{H}_{d-1}$ with $T_0 = 1/2\pi$.

$$S_A = S_{thermal}(\mathbb{H}_{d-1})$$



This can be generalized to Rényi entropies

$$\rho_A^n = e^{-2\pi n H_E} \quad \Rightarrow \quad S_n = \frac{n}{n-1}\frac{1}{T_0}\int_{T_0/n}^{T_0} S(T)dT$$

AdS/CFT $\Rightarrow S_{thermal} = S_{BH}$

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$T \to 0$, near horizon geometry is $AdS_2 \times \mathbb{H}_{d-1}$. If

$$BF_{AdS_{d+1}} < m_{eff}^2 \ell_{AdS_{d+1}}^2 < BF_{AdS_2}$$

Scalar condenses $\rightarrow$ 2nd order phase trans. $\rightarrow$ hairy black hole

$\partial_T S(T)$ is discontinuous $\Rightarrow \partial_n^2 S_n$ is discontinuous

at some $n = n_c$

$\Rightarrow$ $S_n$ is NOT analytic in $n$.

Analytical estimate using near horizon arguments. If

$$\frac{d}{2} - 1 < \Delta < \frac{d + \sqrt{d}}{2}$$

$\rightarrow$ non-analyticity at $n = n_c$.

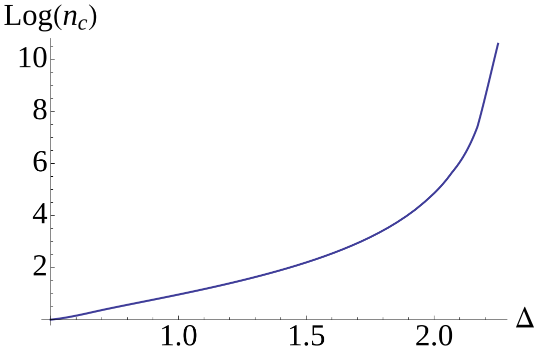


Figure: $d = 3$

# Conclusion and future challenges

- We have found simple examples of non-analyticities in $S_n$ for large $N$ CFTs. This addresses possible subtleties in the use of the replica trick.
- It would be interesting to reproduce similar results from the field theory directly.

Thank you!