Space-Time Action for  $G_2$ Compactifications in Superspace

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Strings 2016, YMSC,Tsinghua University



I. Given a supersymmetric string theory or M-theory compactification in the supergravity approximation, can it be corrected order by order in  $\alpha'$  (or the inverse radius) to give a solution of the corrected equations of motion and supersymmetry transformations?  $\longrightarrow$  Type II string theory

II. What is the manifestly supersymmetric complete space-time action for an arbitrary string theory or M-theory compactification?  $\longrightarrow$  M-theory on  $G_2$  manifolds

I. Type II String Theory on G2 Manifolds



Calabi-Yau*, Spin*(7) manifold

# Tools

### Given a 7d spin manifold  $M$  there is a unit spinor  $\eta$

 $\varphi_{abc} = \eta^T \Gamma_{abc} \eta$ 

and a 4-form...

$$
\boldsymbol{\psi}_{abcd} = \boldsymbol{\eta}^T \boldsymbol{\Gamma}_{abcd} \boldsymbol{\eta}
$$

...related by

$$
\psi = \ast \varphi
$$

$$
g_{ab} = g_{ab} [\varphi] = (\det s)^{-1/9} s_{ab}
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#### If the metric has  $G_2$  holonomy

0  $\overline{0}$  $d\varphi = 0$  $d\psi = 0$  $\varphi = 0$  $\psi = 0$  $= 0$  $= 0$ 

In general, if the manifold is spin (but the spinor might not be covariantly constant) then the space has a  $G_2$  structure and forms If the metric has  $G_2$  holonomy<br>  $d\varphi = 0$ <br>
In general, if the manifold is spin (but the spinor might not b<br>
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ble representations of  $G_2$ <br>  $\tau_1 \wedge \varphi + * \tau_3$ <br>  $\tau_+ \tau_2 \wedge \varphi$ <br>  $\tau_0, \tau_1, \tau_2, \tau_3$  are torsion classes

has 
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\n $d\varphi = 0$   
\n $d\psi = 0$   
\nthe manifold is spin (but the spinor mi  
\nonstant) then the space has a  $G_2$  struct  
\nposed into irreducible representations  
\n $d\varphi = \tau_0 \psi + 3\tau_1 \wedge \varphi + * \tau_3$   
\n $d\psi = 4\tau_1 \wedge \psi + \tau_2 \wedge \varphi$   
\n $\tau_0, \tau_1, \tau_2, \tau_3$  are tors

# Leading Order Correction

Gravitino supersymmetry transformation

*a a a a <sup>b</sup> A iB a a b*

 $\rightarrow$  In 7d dimensions spinors have 8 real components. A basis is  $\{\eta, \Gamma_a \eta\}, a=1,\dots,7.$ 

The coefficients are tensors  $A_a = A_a [\varphi]$   $B_a^b = B_a^b [\varphi]$ 

α' corrections

Gravition supersymmetry transformation  
\n
$$
\delta \psi_a = \nabla_a \eta + A_a \eta + i B_a^b \Gamma_b \eta
$$
\n
$$
\rightarrow \text{In 7d dimensions spinors have 8 real components.}
$$
\n
$$
\rightarrow \text{This conditions has a basis is } \{\eta, \Gamma_a \eta\}, a=1,...,7.
$$
\n
$$
\rightarrow \text{The coefficients are tensors } A_a = A_a [\varphi] \qquad B_a^b = B_a^b [\varphi]
$$
\n
$$
\overbrace{A \sigma(\alpha^{r_3})}^{A \sigma} A_a = 0
$$
\n
$$
B_a^b = \alpha^{r_3} \varphi_{acd} \nabla^c \left( \frac{1}{32g} \varepsilon^{dc_1...c_6} \varepsilon^{bd_1...d_6} R_{c_1c_2d_1d_2} R_{c_3c_4d_3d_4} R_{c_5c_6d_5d_6} \right)
$$

# Supersymmetric Vacuum

To order  $\alpha'^3$ 

$$
\delta \psi_a = \nabla_a^{\dagger} \eta^{\dagger} + \left[ A_a \left[ \varphi \right] \eta + i B_a^{\dagger} \left[ \varphi \right] \Gamma_b \eta \right] = 0
$$

Primed quantities include corrections:  $\eta' = \eta + O(\alpha^{3})$ 

 $\varphi$ ,  $\eta$  of  $G_2$  holonomy manifold

Set up PDE

$$
d\varphi' = \alpha [\varphi]
$$

$$
d\psi' = \beta [\varphi]
$$

$$
\psi' = *' \varphi'
$$

$$
\alpha_{abcd} = 8A_{[a} \varphi_{bcd]} - 8B_{[a}^{\ e} \psi_{bcd]e}
$$

$$
\beta_{abcde}=10A_{[a}\mathstrut \psi_{bcde]}-40B_{[ab}\mathstrut \varphi_{cde]}
$$

A necessary and sufficient condition for this PDE to be solvable is that  $\alpha$  and  $\beta$  should be exact.

To order  $\alpha'$ <sup>3</sup> we can check this explicitly.

The PDE for  $\varphi'$  is solvable!

K. B., D. Robbins, E. Witten, 1404.2460 All Orders in a'

Using induction over the order in  $\alpha'$  it is possible to show that a solution of the supersymmetry conditions exists to any order in  $\alpha'$  provided the corresponding



Exactness of  $\alpha$  and  $\beta$  is not only necessary but also sufficient....

There exists a solution of  $\delta \psi = 0$  to all orders in  $\alpha'$ !

K. B., D. Robbins, E. Witten, 1404.2460

# II. The Space-Time Action of M-theory Compactified to 4d in N=1 Superspace

We wish to describe the fluctuations around the background...





compact  $G_2$ manifold

... these include massless states as well as massive KK modes.

# **Guiding Principles**

 $4d$ supersymmetry

## Assemble fields into 4d superfields

Locality

Keep locality along space-time and M.  $\phi = \phi(x, y)$  $C = \frac{1}{3!}C_{abc}(x, y)dy^a \wedge dy^b \wedge dy^c$ 

11d fields decompose into many 4d fields

 $C_{MNP}, G_{MN} \rightarrow \begin{cases} C_{abc}, C_{ab\mu}, C_{a\mu\nu}, C_{\mu\nu\rho}, C_{\mu\nu\rho}, C_{MN} \end{cases}$ 

The coordinates of flat 4d superspace are  $(x^m, \theta^{\mu}, \overline{\theta}_a)$ upersymmetry $(x^m, \theta^{\mu}, \overline{\theta}_{\mu})$ ates...<br>(computed at established at established at established at each of the set of the ersymmetry $\theta^{\mu}, \bar{\theta}_{\mu})$ <br>... Superfields are functions of these coordinates... Manifest Global 4d Supersyn<br>
oordinates of flat 4d superspace are  $(x^m, \theta^u, \bar{\theta}_a)$ <br>
fields are functions of these coordinates...<br>
<u>Chiral superfields</u><br>  $\overline{\partial}^{\partial}_{\theta} = 0$ <br>  $\Phi(x, \theta) = C(x_+) + \sqrt{2}\theta\psi(x_+) + \theta\theta$ <br>
<br>
(x, y) =  $\hat$ **Manifest Global 4d Supersymmetry**<br> **e** coordinates of flat 4d superspace are  $(x^m, \theta^a, \overline{\theta}_a)$ <br>
perfields are functions of these coordinates...<br>  $-\frac{\partial}{\partial \theta^a} - i\theta^a \overline{\theta^a} \overline{\theta^a}$   $\overline{D}_a \Phi = 0$ <br>  $\Phi(x, \theta) = C(x_+) + \sqrt{2} \$ Lanifiest Global 4d Supersymmetry<br>
dinates of flat 4d superspace are  $(x^m, \theta^u, \overline{\theta}_s)$ <br>
ds are functions of these coordinates...<br>
<u>Chiral superfields</u><br>  $\overline{D}_{\dot{\alpha}}\Phi = 0$ <br>  $\Phi(x, \theta) = C(x_+) + \sqrt{2}\theta\psi(x_+) + \theta\theta F(x_+)$ <br>  $y) = \hat{\phi}_{abc$ **parametry**<br> **parameters**<br> *m*<br> *m<br> <i>x*<br> *x*<br> *x* **Manifest Global**<br>
The coordinates of flat 4d superspack<br>
Superfields are functions of these of<br>  $\bar{D}_a = -\frac{\partial}{\partial \bar{\theta}^a} - i \theta^a \sigma_{aa}^m \partial_m$   $\bar{D}_a$ <br>  $\Phi(x, \theta) = C(x_+) + \Phi(x_+)$ Manifest Global 4d Supersymmetry<br>
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<u>Chiral superfields</u><br>  $\overline{\overline{D}}_{\dot{\alpha}} \Phi = 0$ mifest Global 4d Supersymmetry<br>
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are functions of these coordinates...<br>
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coordinates of flat 4d superspace are  $(x^m, \theta^m, \bar{\theta}_n)$ <br>
erfields are functions of these coordinates...<br>  $=\frac{\partial}{\partial \bar{\theta}^n} - i\theta^n \sigma_m^n \partial_m$ <br>  $\overline{D}_\alpha \Phi = 0$ <br>  $\Phi(x, \theta) = C(x_+) + \sqrt{2}\theta \psi(x_+) + \theta \theta F(x_$ 

Chiral superfields

 $\partial \overline{\theta}^{\dot{\alpha}}$  and  $\alpha \circ m$ 

$$
\mathcal{C}_{abc}(x, y) = \hat{\varphi}_{abc}(x, y) + iC_{abc}(x, y)
$$

 $\alpha$  m  $\sim$ 

 $\hat{\rho}=\varphi$ 

# **Action For Chiral Superfields**

$$
I = \frac{1}{2} \int d^4x \left[ K(\Phi, \Phi^+) \right] \Big|_{D} + \int d^4x \left[ f(\Phi) \right] \Big|_{F} + c.c.
$$

Lagrangian density for bosonic fields

$$
L = -\int_{M \times M} d^7 y d^7 y' \frac{\delta^2 K}{\delta C(y) \delta C(y')} \left( \partial_{\mu} C(y) \partial^{\mu} C(y') - F(y) F(y') \right)
$$

+2 Re 
$$
\int_M d^7 y \frac{\partial J(\nu)}{\partial C(y)} F(y)
$$

Superpotential  
\nA good candidate is 
$$
f(\Phi) = \beta \int_{\text{constant}} \Phi \wedge d\Phi
$$
  
\n $\int_{\text{constant}}^{\text{constant}}$   
\nIn a supersymmetric ground state  
\n $\frac{\delta f}{\delta \Phi} = 0 \Rightarrow d\Phi = 0 \Rightarrow d\hat{\varphi} = 0, G_4 = 0$   
\nComparing with the previous results  $d\varphi = \alpha = d\chi$   
\n $\hat{\varphi} = \varphi' - \chi$   
\nThere is a closed 3-form!

In a supersymmetric ground state

$$
\frac{\delta f}{\delta \Phi} = 0 \Longrightarrow d\Phi = 0 \Longrightarrow d\hat{\varphi} = 0, G_4 = 0
$$

Comparing with the previous results  $d\varphi = \alpha = d\chi$ 

$$
\hat{\varphi}=\varphi^\prime\!-\chi
$$

There is a closed 3-form!

**Example 18 Example 18 Example 19 Example 2011**<br> *abc* =  $\varphi_{abc} + iC_{abc}$  are coordinates of an infinite dimensional<br>
aehler manifold. Eleven-dimensional gauge transformations<br>  $\delta C = d\Lambda$ <br>  $\longleftrightarrow$   $\Lambda \in V$  the space<br>
of 2-forms **EXECUTE:**<br>
FRIM Transform a string the space of 2-forms models of the metric.<br>  $\delta C = d \Lambda$ <br>  $\begin{array}{c}\n\lambda \longleftarrow \lambda \in V \text{ the space of } 2\text{-forms mod} \\
\text{closed } 2\text{-form} \\
\text{of the metric.}\n\end{array}$  $\mathcal{C}_{abc} = \varphi_{abc} + i \mathcal{C}_{abc}$  are coordinates of an infinite dimensional Kaehler manifold. Eleven-dimensional gauge transformations

of 2-forms mod  $\delta C = d\Lambda$   $\longrightarrow$   $\Lambda \in V$  the space

closed 2-form

…gives rise to isometries of the metric.

The Kähler form is invariant and as a result there is a moment map (a concept we borrow from symplectic geometry). As we show in more detail in our paper the vanishing of the moment map implies **Kähler Form**<br>  $\phi_{abc} + iC_{abc}$  are coordinates of an infinite<br>
manifold. Eleven-dimensional gauge trans<br>  $\delta C = d\Lambda \longleftrightarrow_{\text{of the  
left}} \Delta E$ <br>
is rise to isometries of the metric.<br>
hler form is invariant and as a result there<br>
ept we b **Example 18**<br>  $\mathcal{E}_{abc}$  are coordinates of an infinite dimensional<br>
d. Eleven-dimensional gauge transformations<br>  $\delta C = d\Lambda \longleftrightarrow_{\begin{array}{c} \Lambda \in V \text{ the space} \\ 0 \text{ of } 2\text{-forms mod} \\ \text{closed } 2\text{-form} \end{array}}$ <br>
isometries of the metric.<br>
m is invariant **EVALUATE EXECUTE:**<br>
EVALUATE FORM<br>
For manifold. Eleven-dimensional gauge transformations<br>  $\delta C = d\Lambda$ <br>  $\longleftrightarrow$   $\Lambda \in V$  the space<br>
of 2-forms mod<br>
ves rise to isometries of the metric.<br>
<br>
Closed 2-form<br>
<br>
Closed 2-form<br>
<br>
<br> **Example 18**<br>  $\mathcal{E}_{abc}$  are coordinates of an infinite dimensional<br>
d. Eleven-dimensional gauge transformations<br>  $\delta C = d\Lambda \longleftrightarrow_{\begin{array}{c} \Lambda \in V \text{ the space} \\ 0 \text{ of } 2\text{-forms mod} \\ \text{closed } 2\text{-form} \end{array}}$ <br>
isometries of the metric.<br>
m is invariant

$$
\mu = 0 \Longrightarrow \nabla_a \left( \frac{\delta K}{\delta C_{abc} (y)} \right) = 0
$$
 Closed 4-form!

Needles to say it would be interesting to derive these conditions from a Kaluza-Klein reduction of M-theory. We envision this as a two step process:

1) we rewrite the action of 11d supergravity in a form that displays manifest N=1 supersymmetry in 4d.

2) non-renormalization theorems should then give us information about which results hold to all orders in perturbation theory.

## Kaluza-Klein Reduction of M-Theory

Fields are decomposed into a 4+7 split:

**BSosonic Fields**  
\n<sup>3</sup>ields are decomposed into a 4+7 split:  
\n
$$
C_{MNP} \rightarrow C_{abc}, C_{ab\mu}, C_{a\mu\nu}, C_{\mu\nu\rho}
$$
\n
$$
G_{MN} = \begin{pmatrix} h_{\mu\nu} + g_{cd} A^c_{\mu} A^d_{\nu} & g_{bc} A^c_{\mu} \\ g_{ac} A^c_{\nu} & g_{ab} \end{pmatrix}
$$
\n
$$
M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 3} \qquad A
$$
\n
$$
M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 3} \qquad A
$$
\n
$$
M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 4} \qquad B
$$
\n
$$
M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 5} \qquad A
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\n
$$
M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 6} \qquad B
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M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 7} \qquad B
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M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 8} \qquad B
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M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 8} \qquad B
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M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 9} \qquad B
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M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 8} \qquad B
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M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 8} \qquad B
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M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 9} \qquad B
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M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 8} \qquad B
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$$
M, N = 0, ..., 10 \qquad \text{p.v=0, ..., 3} \qquad B
$$
\n
$$
M
$$

Symmetries:

$$
\begin{cases} C \to C + d\Lambda \\ x^M \to x^M - \xi^M \end{cases}
$$

4d system is very complicated but known in detail...

#### **Summary**  $\overline{\mathbf{4}}$

4 Summary  
\nAs a summary we present a concrete example. The space-time effective action for  
\neleven-dimensional supergravity compactified to four dimensions is  
\n
$$
S = -\frac{1}{8\kappa^2} \int dv h^{\alpha} \left( \frac{1}{2} g^{ab} g^{cd} + g^{ac} g^{bd} \right) \mathcal{D}_{\alpha} g_{ab} \mathcal{D}_{\beta} g_{cd} \qquad \alpha, \beta, ...
$$
\nare space-time indices  
\n
$$
+ \frac{1}{2\kappa^2} \int dv \left( h^{\beta \mu} h^{\gamma[\rho} h^{\alpha]\nu} - \frac{1}{2} h^{\alpha \mu} h^{\beta[\nu} h^{\gamma]\rho} \right) \mathcal{D}_{\alpha} h_{\beta} \gamma \mathcal{D}_{\mu} h_{\nu \rho} \qquad \phi
$$
\n
$$
+ \frac{1}{4\kappa^2} \int dv f \left[ g^{ab} h^{\alpha[\beta} h^{\mu]\nu} \hat{\nabla}_{a} h_{\alpha \beta} \hat{\nabla}_{b} h_{\mu \nu} - h^{\alpha \beta} \left( \frac{1}{2} g^{ab} g^{cd} + g^{ac} g^{bd} \right) \hat{\nabla}_{a} h_{\alpha \beta} \hat{\nabla}_{b} g_{cd} \right.
$$
\n
$$
+ \left( g^{\mu t} g^{\alpha u} g^{rs} - \frac{1}{2} g^{\rho s} g^{at} g^{ru} + g^{\nu r} g^{\alpha u} g^{st} \right) \hat{\nabla}_{r} g_{pq} \hat{\nabla}_{u} g_{st} \right] + \frac{1}{8\kappa^2} \int dv f^{-1} \left( \mathcal{F}_{\mu \nu}^a \right)^2
$$
\n
$$
- \frac{1}{24\kappa^2} \int dv \left[ \left( \mathcal{D}_{\mu} \mathcal{C}_{abc} - 3 \partial_{[\alpha} \mathcal{C}_{bc] \mu} \right)^2 + 4 f \left[ \left( \mathcal{F}_{[\alpha} \mathcal{C}_{bcd]} \right)^2 \right]
$$
\n
$$
- \frac{1}{16\kappa^2} \int dv f^{-1} \left( \mathcal{F}_{\mu \nu ab} + \mathcal{F}_{\mu \nu}^c \mathcal{C}_{abc} \right)^2 - \frac{1}{24\kappa^2} \int dv \left[ f^{-2} \left( \mathcal{F}_{\mu \nu \
$$

K. B, M. Becker, D. Robbins, 1412.8198

# Goal: Write this Action in Superspace

## **Kinetic Terms**

#### Use the Kaehler potential

M

$$
K=-\frac{3}{\kappa^2}\int_M d^7y\sqrt{g(F)}
$$

$$
g_{ab} = g_{ab} [\varphi] = (\det s)^{-1/9} s_{ab}
$$
  
etric  

$$
s_{ab} = -\frac{1}{144} \varphi_{amn} \varphi_{bpq} \varphi_{rst} \varepsilon^{mnpqrst}
$$

F is a real superfield whose bottom components is  $\varphi$ 

$$
F_{abc} = \frac{1}{2i} \left( \Phi_{abc} - \overline{\Phi}_{abc} \right) - 3 \partial_{[a} V_{bc]}
$$

Real superfield for  $C_{ab\mu}$ 

#### The kinetic terms obtained from M-theory compactification are

The kinetic terms obtained from M-theory compactification are  
\n
$$
S_{kin} = \frac{1}{24\kappa^2} \int \sqrt{g} \left[ \frac{4}{3} (\pi_i \partial_\mu \varphi)^2 + (\pi_{2i} \partial_\mu \varphi)^2 \right] +
$$
\n
$$
\frac{1}{24\kappa^2} \int \sqrt{g} \left\{ \left[ \left[ \pi_i (\partial_\mu C - 3\partial C_\mu) \right]^2 - \left[ \pi_i (\partial_\mu C - 3\partial C_\mu) \right]^2 \right] - \left[ \pi_{2i} (\partial_\mu C - 3\partial C_\mu) \right]^2 \right\}
$$
\nExpanding in components the kinetic terms obtained from  
\nsuperspace are  
\n
$$
S_{kin} = \frac{1}{24\kappa^2} \int \sqrt{g} \left[ \frac{4}{3} (\pi_i \partial_\mu \varphi)^2 + (\pi_i \partial_\mu \varphi)^2 \right] - (\pi_{2i} \partial_\mu \varphi)^2 \right] +
$$
\n
$$
\frac{1}{24\kappa^2} \int \sqrt{g} \left[ -\left[ \pi_i (\partial_\mu C - 3\partial C_\mu) \right]^2 + \left[ \pi_i (\partial_\mu C - 3\partial C_\mu) \right]^2 \right] - \left[ \pi_{2i} (\partial_\mu C - 3\partial C_\mu) \right]^2
$$
\nThis coefficient only agrees after integrating  
\nout auxiliary fields in the gravity multiple.

Expanding in components the kinetic terms obtained from superspace are

The kinetic terms obtained from M-theory compactification are  
\n
$$
S_{kin} = \frac{1}{24\kappa^2} \int \sqrt{g} \left[ \frac{4}{3} (\pi_0 \partial_\mu \varphi)^2 + (\pi_{22} \partial_\mu \varphi)^2 + \frac{1}{24\kappa^2} \int \sqrt{g} \left\{ \frac{[(\pi_1 (\partial_\mu C - 3\partial C_\mu)]^2 - [\pi_2 (\partial_\mu C - 3\partial C_\mu)]^2 - [\pi_{22} (\partial_\mu C - 3\partial C_\mu)]^2]}{24\kappa^2} \right\}
$$
\nExpanding in components the kinetic terms obtained from  
\nsuperspace are  
\n
$$
S_{kin} = \frac{1}{24\kappa^2} \int \sqrt{g} \left[ \frac{4}{3} (\pi_1 \partial_\mu \varphi)^2 + (\pi_2 \partial_\mu \varphi)^2 - (\pi_{22} \partial_\mu \varphi)^2 \right] + \frac{1}{24\kappa^2} \int \sqrt{g} \left[ -[\pi_1 (\partial_\mu C - 3\partial C_\mu)]^2 + [\pi_2 (\partial_\mu C - 3\partial C_\mu)]^2 \right]
$$
\nThis coefficient only agrees after integrating  
\nout auxiliary fields in the gravity multiplet.

This coefficient only agrees after integrating out auxiliary fields in the gravity multiplet.

## Potential

The potential for the scalar from the metric can be nicely expressed in terms of torsion classes

Scalar curvature of a  $G_2$ structure manifold (Bryant)

Potential	Potential
ne potential for the scalar from the metric can be	
key expressed in terms of torsion classes	Scalar curvature of a $G_2$ structure manifold (Bryant)
$S_{pot} = \frac{1}{2\kappa^2} \int d^7 y \sqrt{g} \left( \frac{21}{8}  \tau_0 ^2 + 30  \tau_1 ^2 - \frac{1}{2}  \tau_3 ^2 - \frac{1}{2}  \tau_2 ^2 \right)$ \n	
Get contributions from the superpotential	Get contributions from the superpotential and from integrating out $D_{ab}$ which is the auxiliary field in the real superfield for $C_{ab\mu}$
his result agrees precisely with the superspace result!	

Get contributions from the superpotential

$$
W=\pm\frac{1}{8\kappa^2}\int\varphi d\varphi
$$

Get contributions from the superpotential and from integrating out  $D_{ab}$  which is the auxiliary field in the real superfield for  $C_{ab\mu}$ 

#### This result agrees precisely with the superspace result!

# Tensor Hierarchy and Chern-Simons Actions in Superspace Iierarchy and Chern-Simons<br>in Superspace<br>ser, W. D. Linch and D. Robbins, 1601.03066,<br>paper we embedded the tensor hierarchy consisting<br>scending from the M-theory three-form<br> $C_{MNP} \rightarrow C_{abc}$ ,  $C_{ab\mu}$ ,  $C_{a\mu\nu}$ ,  $C_{\mu\nu\rho}$ **EXECUTE:** EXECUTE: The Superspace<br> **CERNATION SET SUPER SU**

References: K. B., M. Becker, W. D. Linch and D. Robbins, 1601.03066, 1603.07362

1) In the first paper we embedded the tensor hierarchy consisting of all fields descending from the M-theory three-form

$$
C_{\text{MNP}} \rightarrow C_{\text{abc}}, C_{\text{ab}\mu}, C_{\text{a}\mu\nu}, C_{\mu\nu\rho}
$$

…and the corresponding abelian gauge transformations

$$
\mathcal \delta C = d\Lambda
$$

We explicitly constructed the supersymmetrized Chern-Simons action…

We explicitly constructed the supersymmetrized Chern-Simons  
action...  

$$
S = -\frac{1}{12\kappa^2} \text{Re}[i\int d^4x d^2\theta \left[2\Phi EG + \Phi W^\alpha W_\alpha\right] + 2\Sigma^\alpha EW_\alpha) -
$$

$$
\frac{1}{12\kappa^2} \int d^4x d^4\theta [-2\hat{\Phi}UH + V\hat{E}H + (VD^\alpha U - D^\alpha VU)W_\alpha +
$$

$$
(V\overline{D}_a U - \overline{D}_a VU)\overline{W}^\alpha - \Sigma^\alpha U D_\alpha U - \overline{\Sigma}_a U \overline{D}^\alpha U - X\hat{E}U]
$$
  
2) In the second paper we coupled this system to the non-abelian  
gauge field arising from the metric.

 $\overline{(VD_{\alpha}U - D_{\alpha}VU)W^{\alpha}} - \Sigma^{\alpha}UD_{\alpha}U - \Sigma_{\alpha}UD^{\alpha}U - XEU$ 

2) In the second paper we coupled this system to the non-abelian gauge field arising from the metric.

# Stay Tuned! More To Come...