

The background of the slide is a reproduction of the painting 'The Starry Night' by Vincent van Gogh. It features a turbulent, swirling blue sky filled with bright, glowing yellow stars and a large, luminous moon. In the foreground, there are dark, jagged cypress trees on the left and a small town with a church spire in the distance.

# **B-modes and the Nature of Inflation**

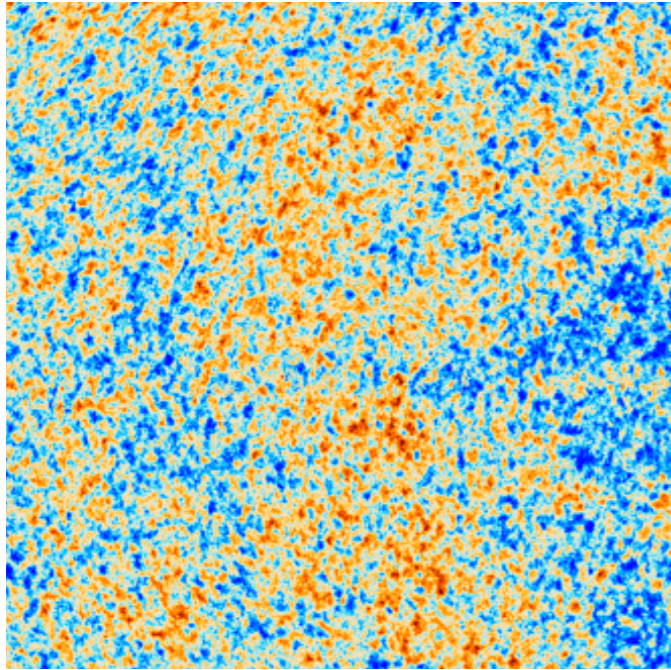
**Daniel Baumann**  
Cambridge University

**with Daniel Green and Rafael Porto**

STRINGS 2014



# Data-Driven Cosmology



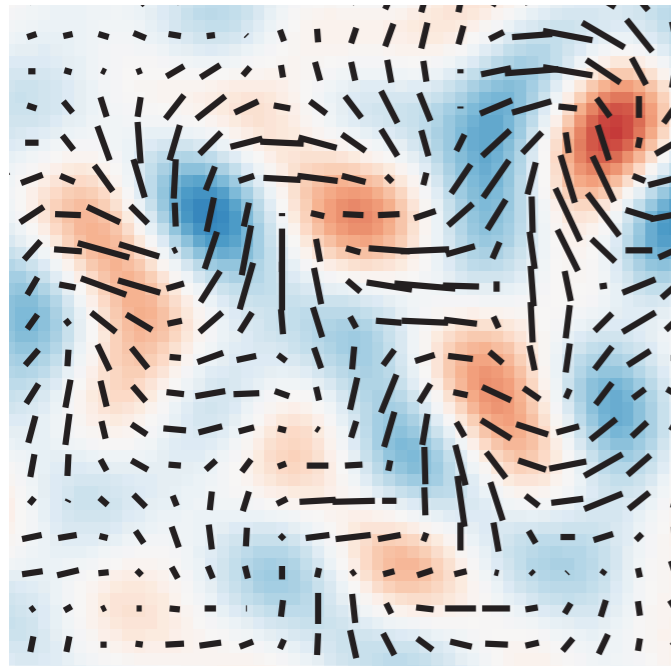
**Planck**

Primordial density perturbations are:

- superhorizon
- scale-invariant
- Gaussian
- adiabatic

$$r \sim 0.1 \quad ?$$

Have primordial gravitational waves been detected?



**BICEP2**

**What does this teach us about  
the UV-completion of inflation?**

In **effective field theory**, we parameterize the effects of the UV-completion by higher-dimension operators.

In this talk, I will consider the leading **higher-derivative corrections** to the slow-roll action:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{(\partial\phi)^4}{\Lambda^4} + \dots$$



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In this talk, I will consider the leading **higher-derivative corrections** to the slow-roll action:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{(\partial\phi)^4}{\Lambda^4} + \dots$$

This induces a non-trivial **speed of sound** for the inflaton fluctuations:

$$\mathcal{L} = -\frac{1}{2} \left[ (\partial_t \delta\phi)^2 - c_s^2 (\partial_i \delta\phi)^2 \right] + \dots$$

I will discuss what the data from Planck and BICEP teaches us about this important class of deformations of slow-roll inflation.



# Higgs vs. Technicolor

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{(\partial\phi)^4}{\Lambda^4} + \dots$$

*perturbative*

$$\Lambda^2 \gg \dot{\phi}$$



$$c_s \sim 1$$

*non-perturbative*

$$\Lambda^2 \sim \dot{\phi}$$



$$c_s \ll 1$$

or

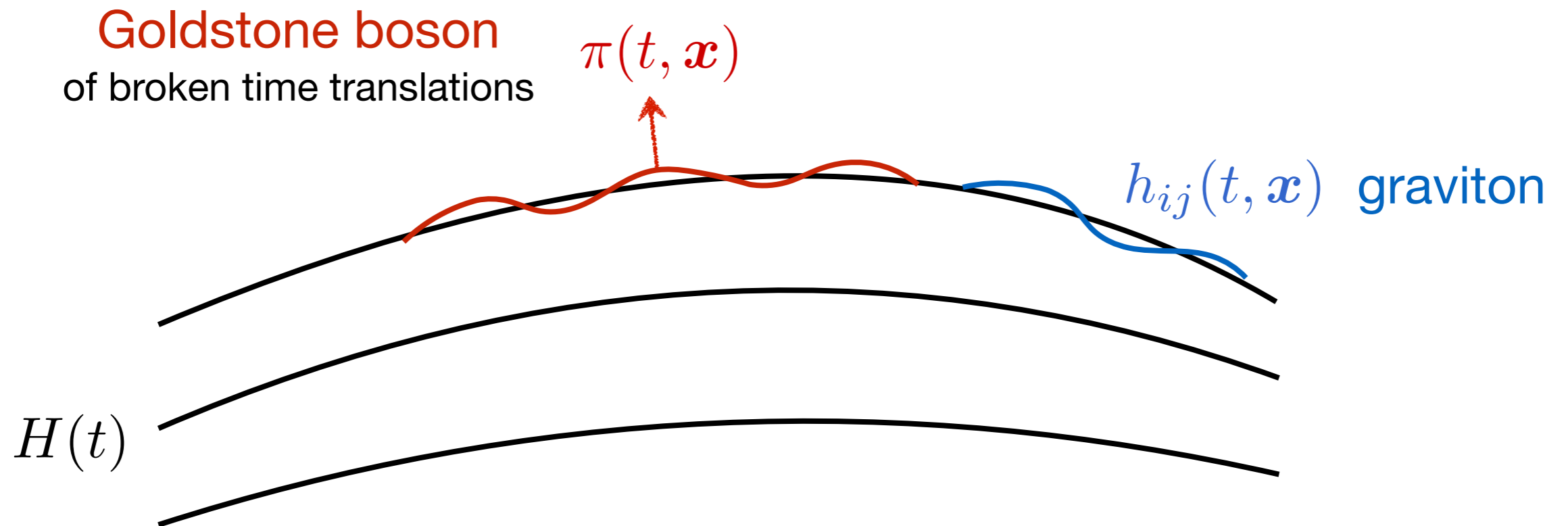
?

*can't be described by small  
corrections to slow-roll inflation*



# Effective Theory of Inflation

Cheung et al.



The Goldstone and the graviton are massless, so their quantum fluctuations are amplified during inflation.



# Effective Theory of Inflation

Cheung et al.

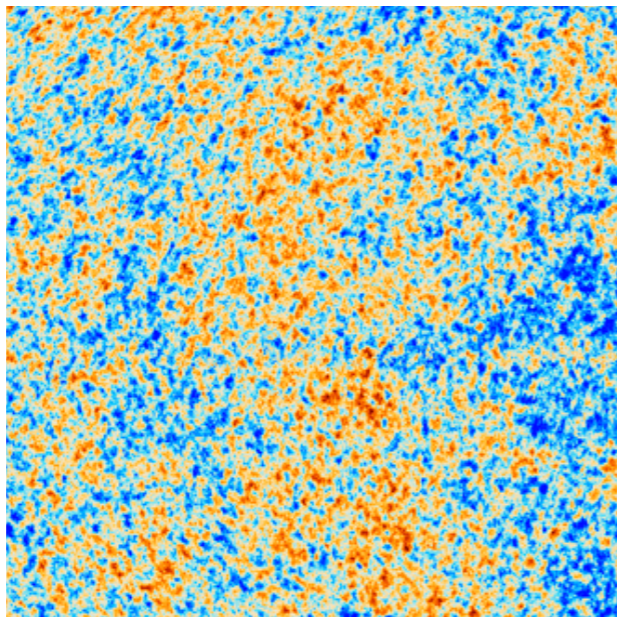
Goldstone boson

$$\pi(t, \boldsymbol{x})$$

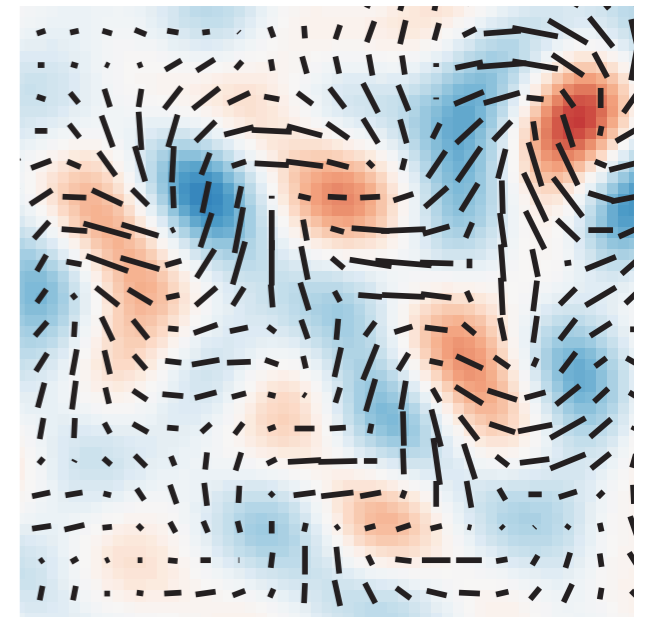


graviton

$$g_{ij} = a^2(t) \left[ (1 + 2\zeta(t, \boldsymbol{x}))\delta_{ij} + 2h_{ij}(t, \boldsymbol{x}) \right]$$



temperature anisotropies



B-mode polarization

# Slow-Roll Inflation

Slow-roll inflation corresponds to nearly free Goldstone bosons with relativistic dispersion relation:


$$\mathcal{L}_\pi = M_{\text{pl}}^2 |\dot{H}| \left[ \dot{\pi}^2 - (\partial_i \pi)^2 \right]$$

$$\mathcal{L}_h = \frac{1}{8} M_{\text{pl}}^2 \left[ (\dot{h}_{ij})^2 - (\partial_k h_{ij})^2 \right]$$




# Slow-Roll Inflation

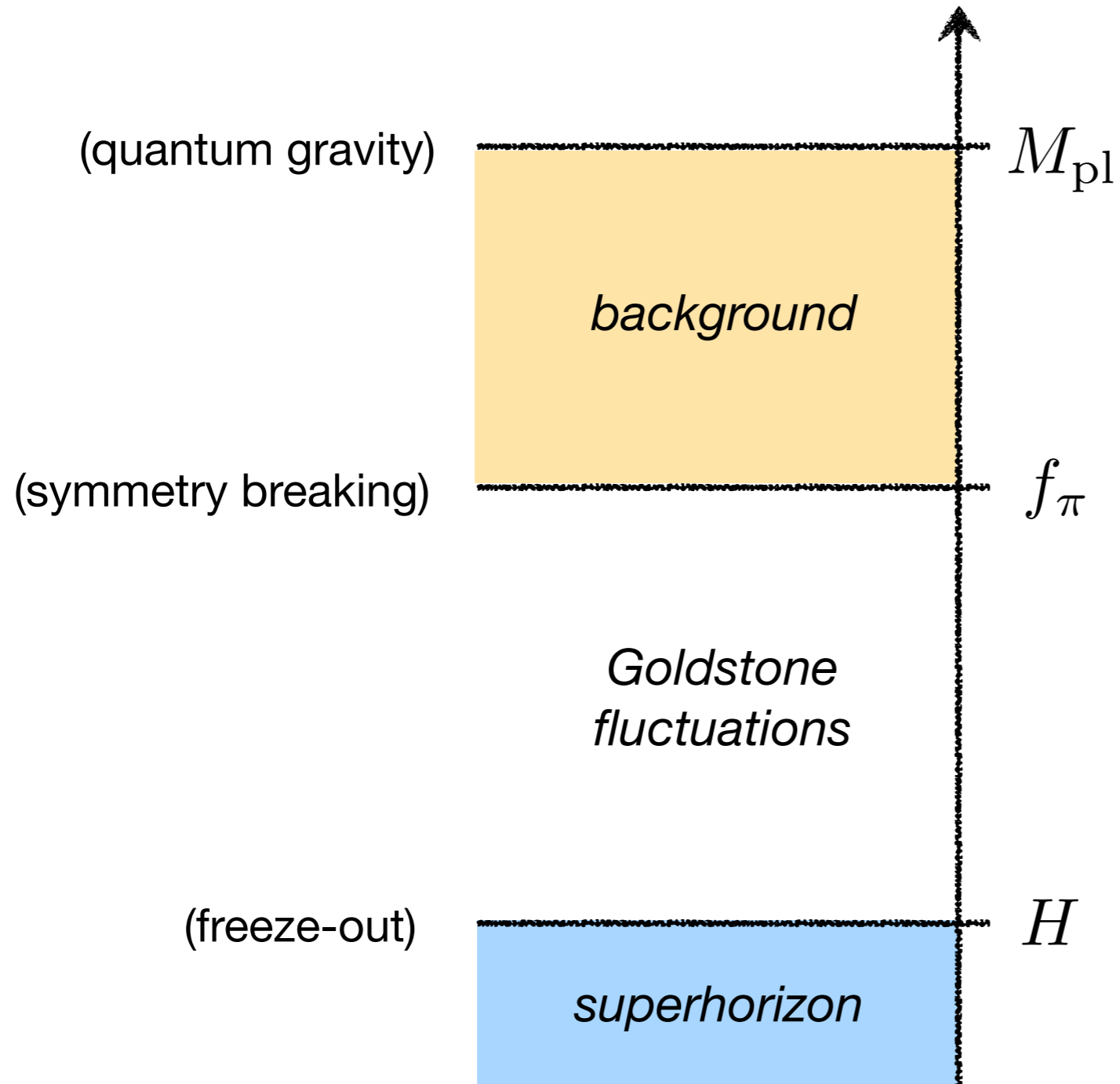
Slow-roll inflation corresponds to nearly free Goldstone bosons with relativistic dispersion relation:

 **symmetry breaking scale**  $f_\pi^4 = \dot{\phi}^2$

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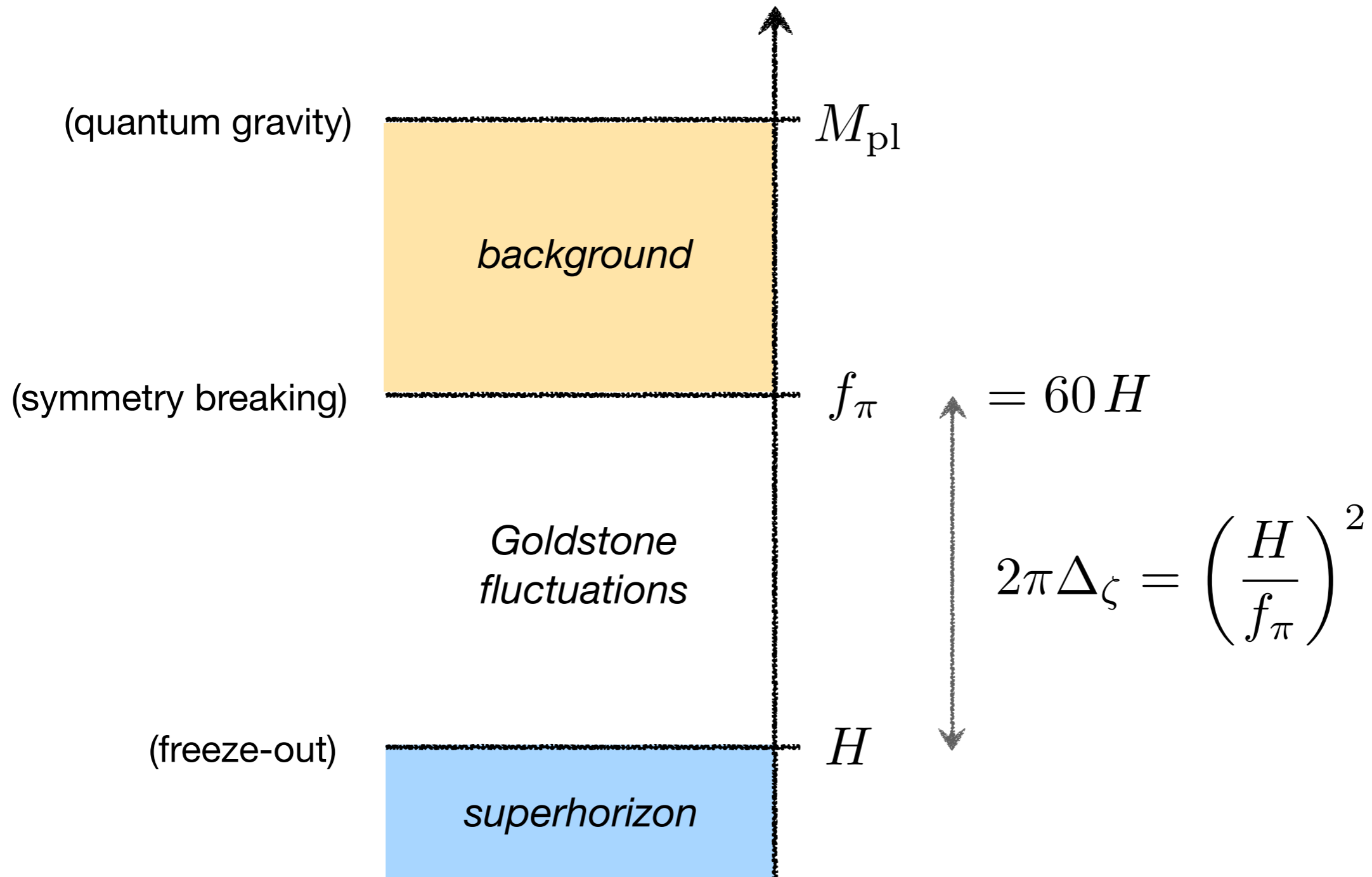
 **quantum gravity scale**

# Slow-Roll Inflation

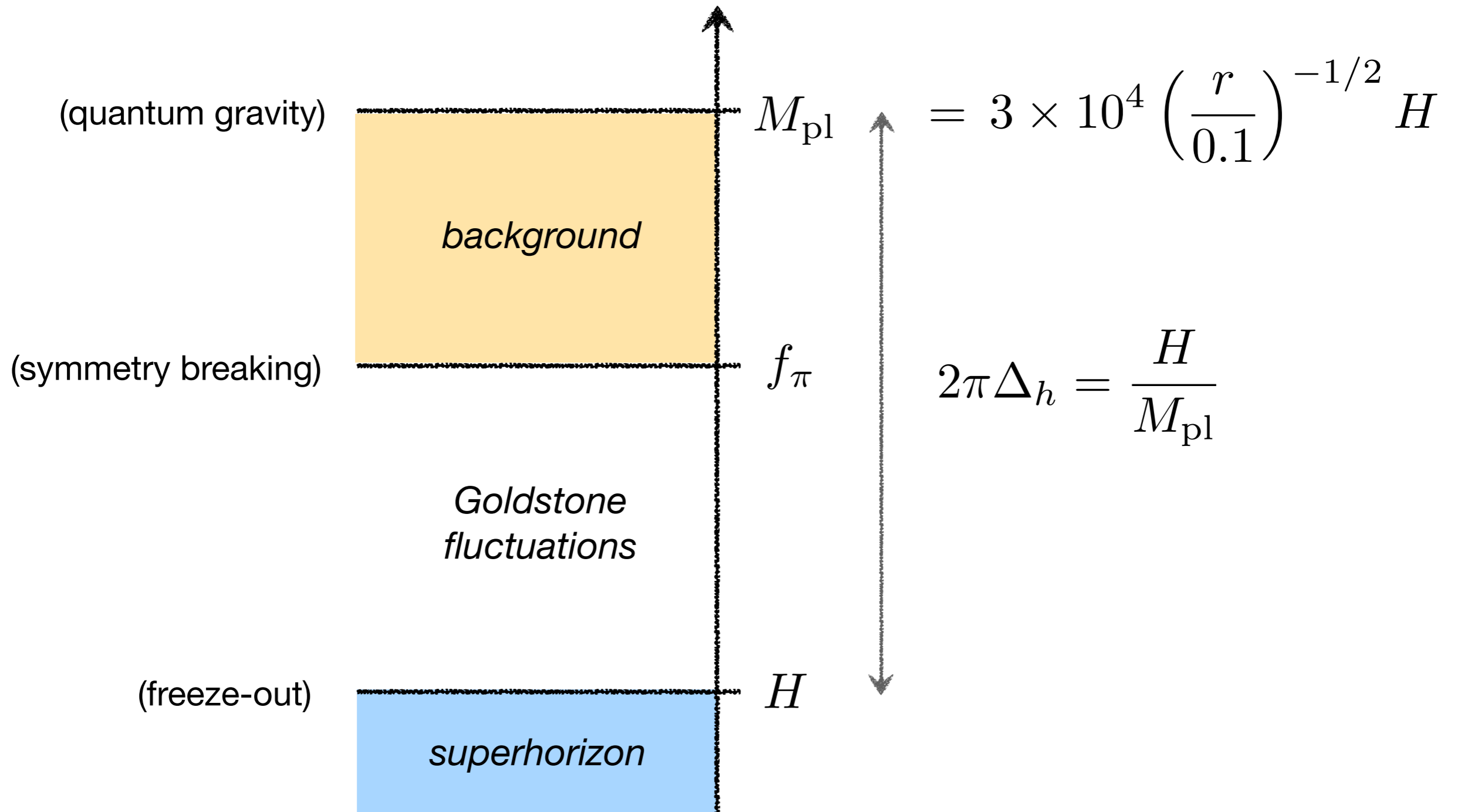




# Slow-Roll Inflation



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# Beyond Slow-Roll

Deviations from slow-roll inflation are parameterized by higher-order self-interactions and/or a non-trivial dispersion relation.

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A well-motivated possibility is a non-trivial **sound speed**:

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**non-linearly realized symmetry**

*allows power spectrum measurements  
to constrain the interacting theory.*

# Beyond Slow-Roll

Writing

$$x \mapsto \tilde{x} \equiv c_s x$$
$$\pi_c \equiv (2M_{\text{pl}}^2 |\dot{H}| c_s)^{1/2} \pi \equiv f_\pi^2 \pi$$

**symmetry breaking scale**

gives

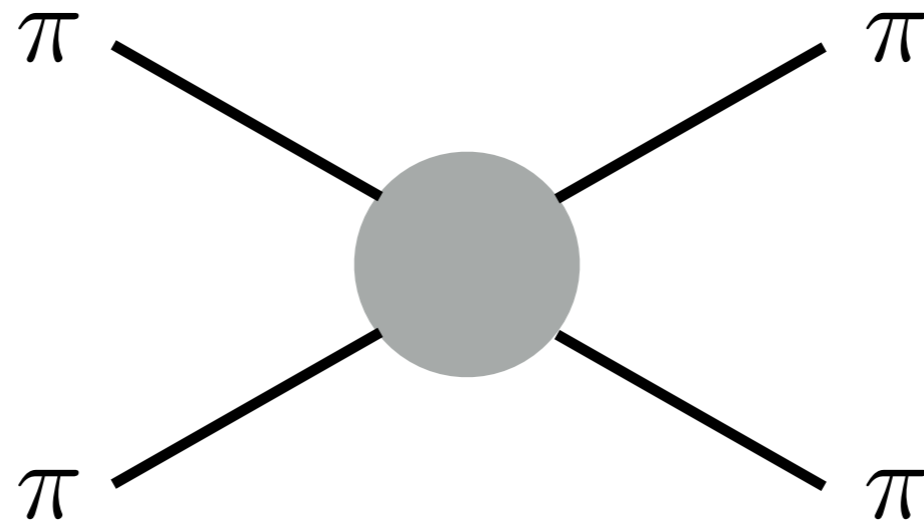
$$\mathcal{L}_\pi = -\frac{1}{2} (\tilde{\partial}_\mu \pi_c)^2 - \frac{\dot{\pi}_c (\tilde{\partial}_i \pi_c)^2}{\Lambda^2} + \dots$$

**strong coupling scale**

$$\Lambda^2 = f_\pi^2 \frac{c_s^2}{1 - c_s^2}$$



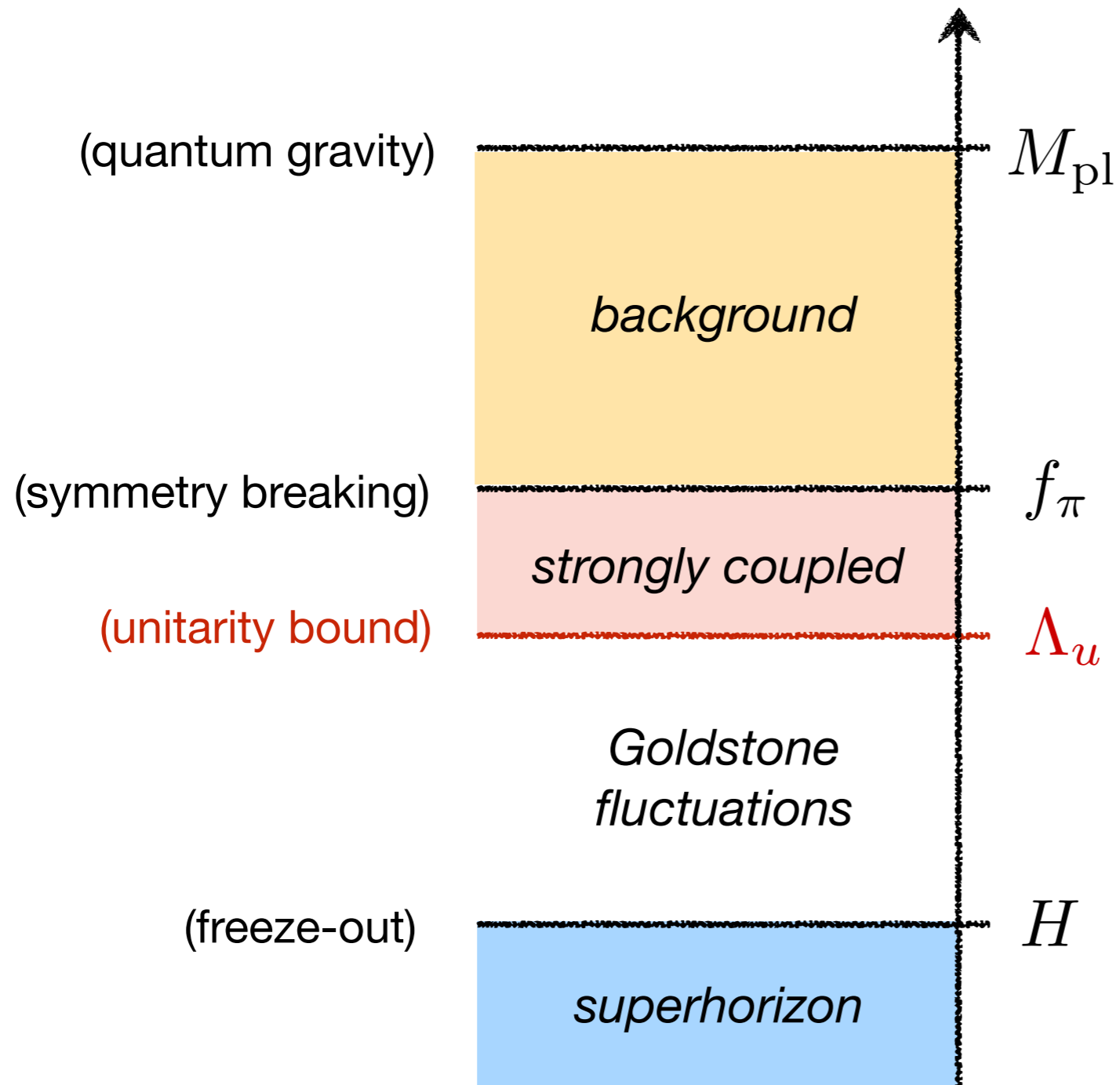
# Unitarity Bound



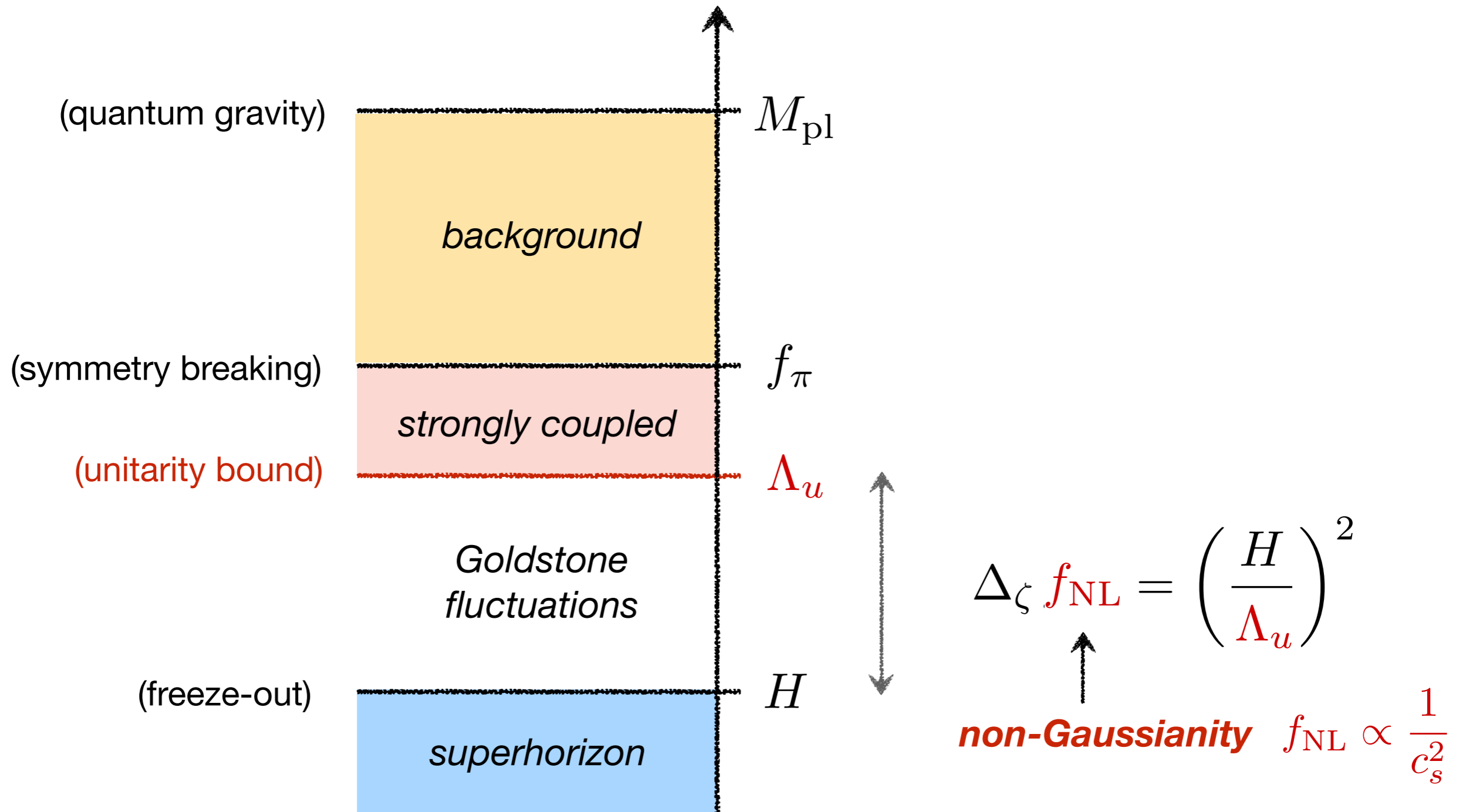
2-to-2 Goldstone scattering violates **unitarity** when

$$E^4 > \frac{24\pi}{5} \Lambda^4 \equiv \Lambda_u^4$$

# Beyond Slow-Roll



# Beyond Slow-Roll



# A Theoretical Threshold

$$\Lambda_u = f_\pi$$



$$\Lambda_u < f_\pi$$

$$\Lambda_u > f_\pi$$

0.47

1



$$\left| f_{\text{NL}}^{\dot{\pi}(\partial_i \pi)^2} \right| = 0.93$$



# A Theoretical Threshold

$$|f_{\text{NL}}^{\dot{\pi}(\partial_i \pi)^2}| < 138$$



0.02

0.47

1

$c_s$

*ruled out by Planck*

*non-perturbative*

*perturbative*

*superluminal*



$$|f_{\text{NL}}^{\dot{\pi}(\partial_i \pi)^2}| = 0.93$$

# A New Bound on the Sound Speed

DB, Daniel Green and Rafael Porto

see also: [Creminelli et al. \[arXiv:0404.1065\]](#)

[D'Amico and Kleban \[arXiv:0404.6478\]](#)

A small sound speed enhances the scalar power spectrum and suppresses the tensor-to-scalar ratio:

$$\left. \begin{aligned} \Delta_{\zeta}^2 &= \frac{1}{8\pi^2} \frac{H^4}{M_{\text{pl}}^2 |\dot{H}| c_s} \\ \Delta_h^2 &= \frac{2}{\pi^2} \frac{H^2}{M_{\text{pl}}^2} \end{aligned} \right\}$$

$$r \equiv \frac{\Delta_h^2}{\Delta_{\zeta}^2} = 16 \varepsilon c_s$$



$$\varepsilon \equiv \frac{|\dot{H}|}{H^2}$$



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BICEP2 then implies a lower bound on the sound speed:

$$c_s = \frac{r}{16\varepsilon} > \frac{0.01}{\varepsilon}$$

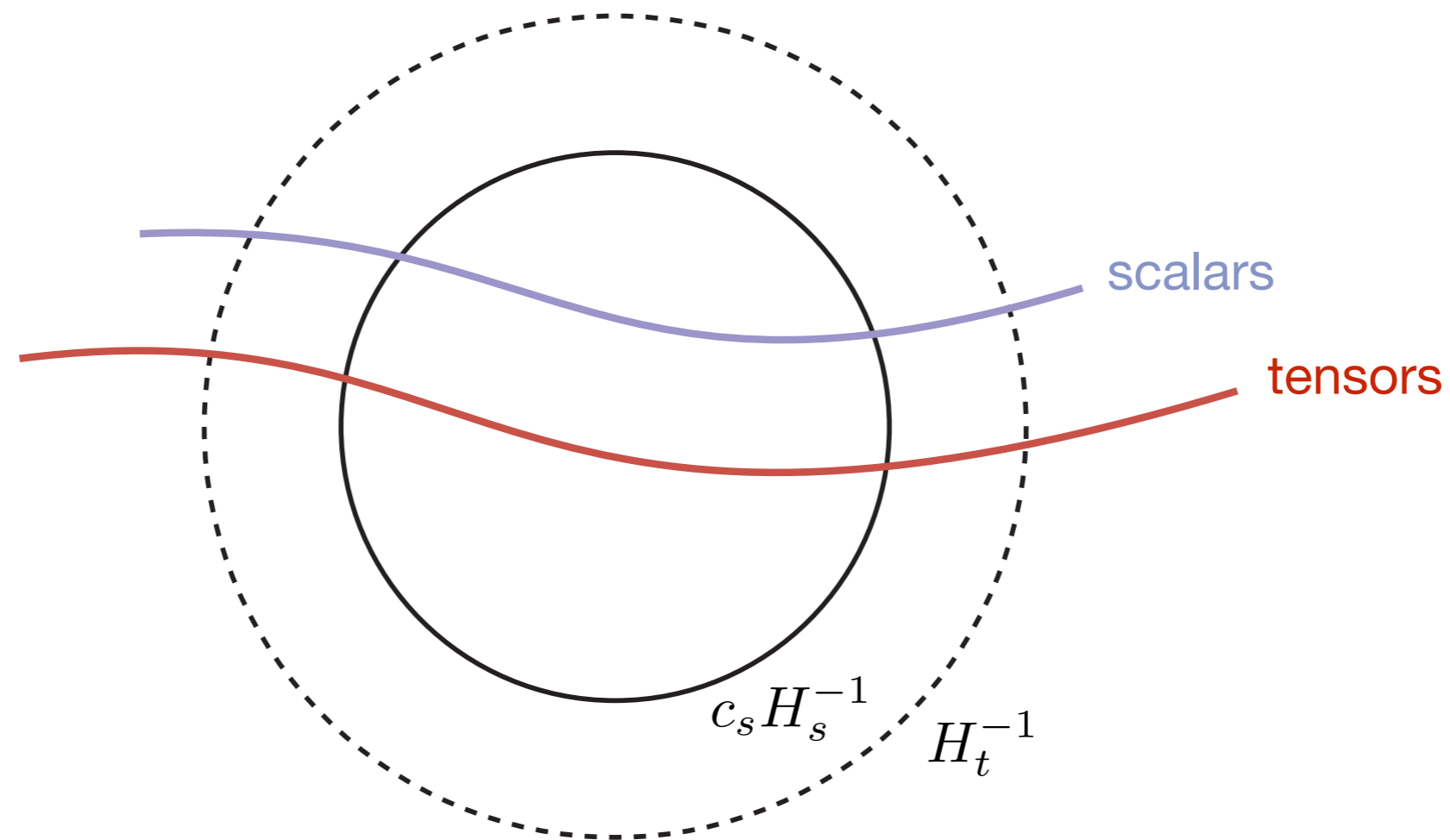
Creminelli et al.  
D'Amico and Kleban

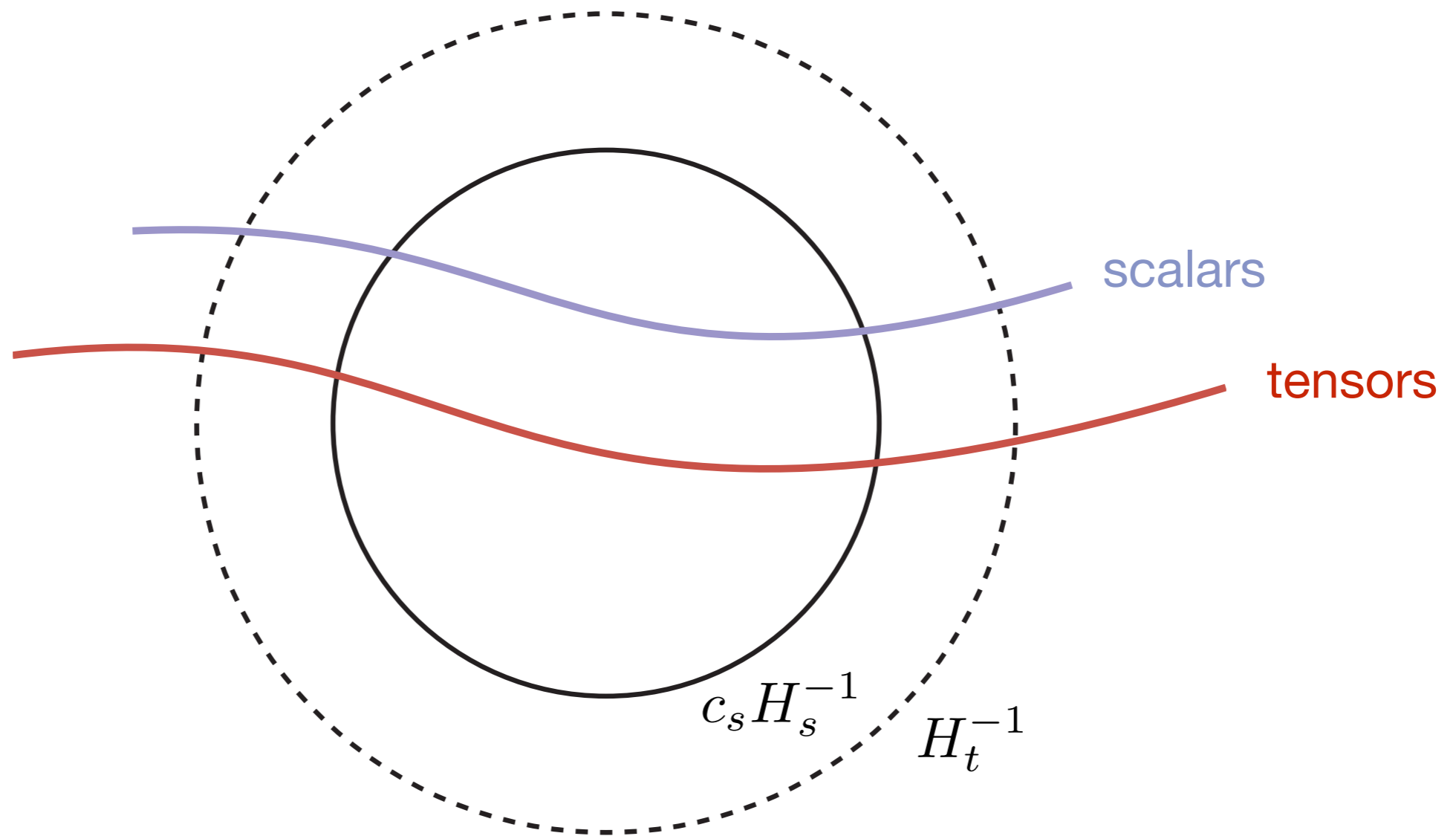


Naively, the bound weakens for large  $\varepsilon$ .

But, for  $\varepsilon > 0.1$  new effects kick in:

1. scale-invariance of the scalars is in danger
2. tensors and scalars freeze at different times





This leads to an extra suppression in the tensor-to-scalar ratio:

$$r = 16\epsilon c_s \left( \frac{H_t}{H_s} \right)^2$$

# Summing Large Logs

At next-to-leading order in slow-roll, one finds:

$$r = 16\varepsilon c_s \left( \frac{H_t}{H_s} \right)^2 = 16\varepsilon c_s \left[ 1 + 2\varepsilon \ln c_s + \dots \right]$$



This is large in the regime of interest.

# Summing Large Logs

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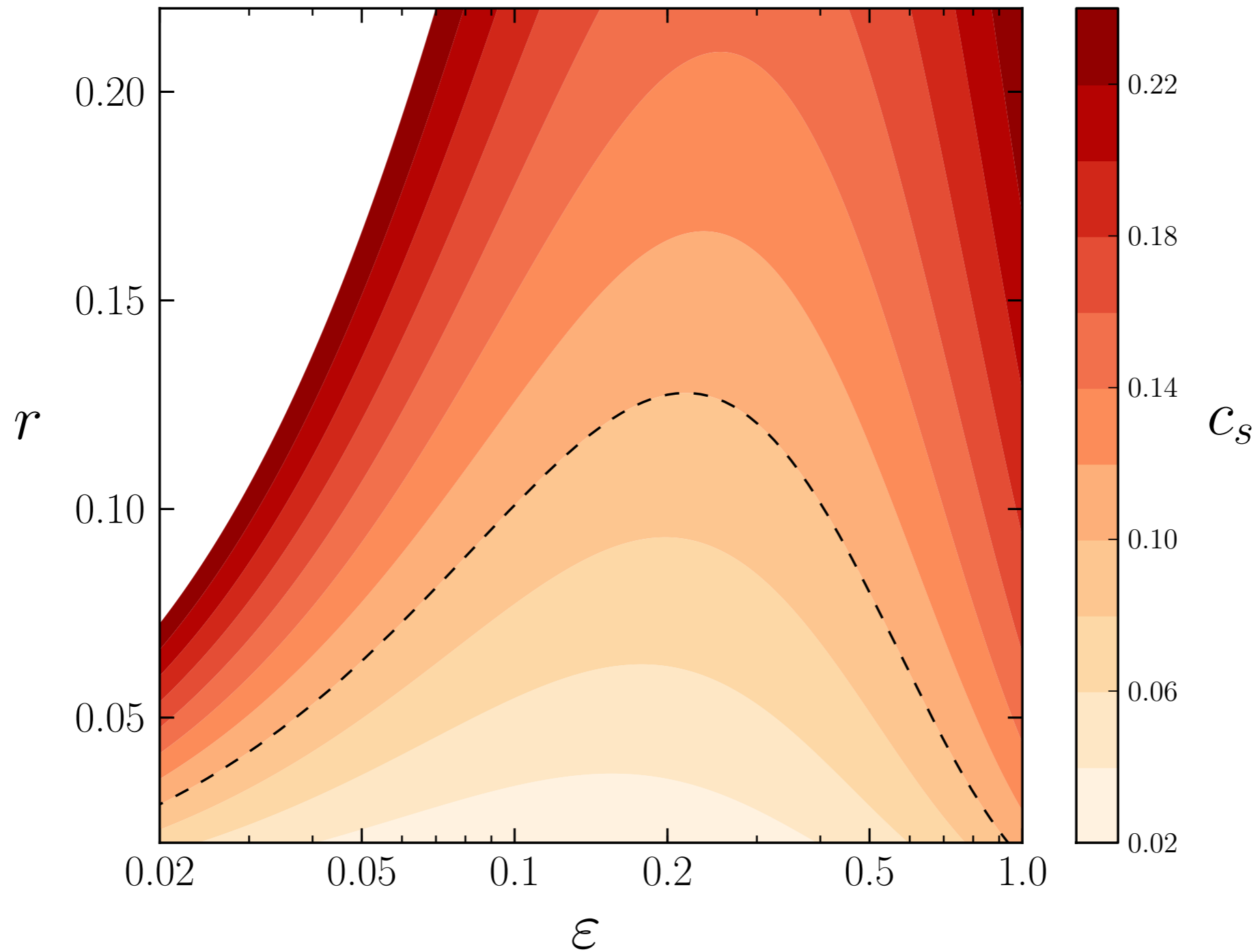
This is large in the regime of interest.

For  $\epsilon \approx \text{const.}$  we can solve the evolution exactly:

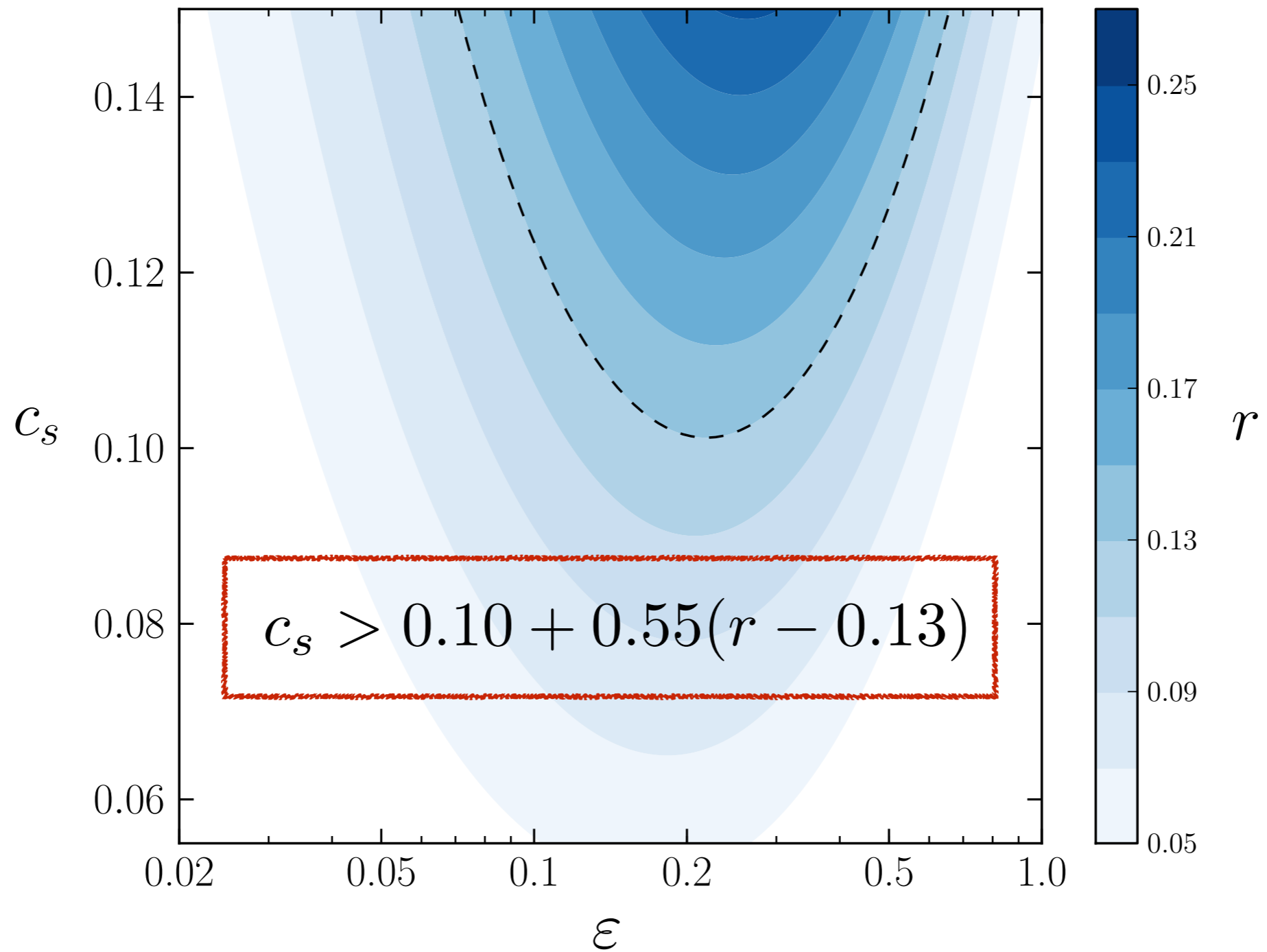
$$r = 16\epsilon c_s^{\frac{1+\epsilon}{1-\epsilon}}$$



# A New Bound on the Sound Speed



# A New Bound on the Sound Speed



# Summing Large Logs

Extending to  $\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \neq 0$  and  $s \equiv \frac{\dot{c}_s}{Hc_s} \neq 0$ , we find:

tensors

$$r = 16\epsilon c_s^{1+2\epsilon \cdot (1-c_s^{-\eta})/(\eta \ln c_s)} \left[ 1 - \mathcal{C}\eta + 2(1 - \mathcal{C})s \right]$$

scalars

$$\Delta_{\zeta}^2(k) \propto \left( \frac{k}{k_{\star}} \right)^{\overbrace{-2\epsilon-\eta-s}^{n_s-1} + \overbrace{(-2\epsilon\eta)}^{\alpha_s} \ln(k/k_{\star})}$$

# Expected Degeneracies

Our bound would weaken if large  $\varepsilon$  is possible.

But this has to be consistent  
with the scalar spectrum:

$$\begin{aligned}n_s - 1 &= -2\varepsilon - \eta - s \\ \alpha_s &= -2\varepsilon\eta\end{aligned}$$

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Taking this into account strengthens the bound:

$$I. \quad \eta \approx -2\varepsilon \xrightarrow{|\alpha_s| \lesssim 0.01} \varepsilon < 0.05 \xrightarrow{r > 0.13} \boxed{c_s > 0.17}$$

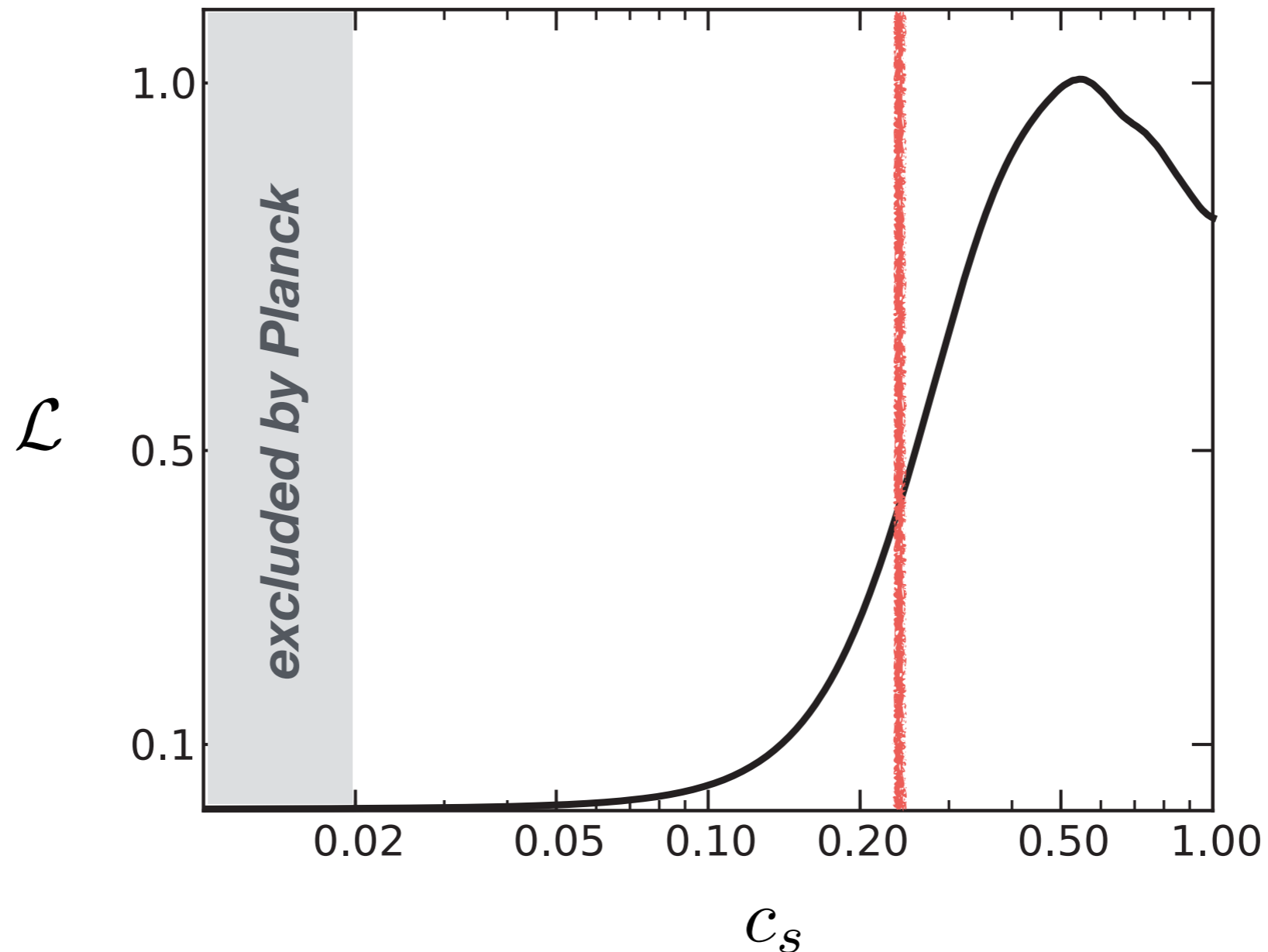
$$II. \quad s \approx -2\varepsilon \longrightarrow r \approx 16\varepsilon c_s^{1+2\varepsilon} \underbrace{\left[ 1 - 2.5\varepsilon \right]}_{\text{strengthens the bound}} \xrightarrow{r > 0.13} \boxed{c_s > 0.15}$$

strengthens the bound

# Data Analysis

A joint likelihood analysis of Planck and BICEP2 \* gives:

$$c_s > 0.25$$



\* warning: no foreground subtraction

# A New Bound on the Sound Speed

$$|f_{\text{NL}}^{\dot{\pi}(\partial_i \pi)^2}| < 138$$

$$|f_{\text{NL}}^{\dot{\pi}(\partial_i \pi)^2}| < 4$$



0.02



0.25

0.47

1

$c_s$

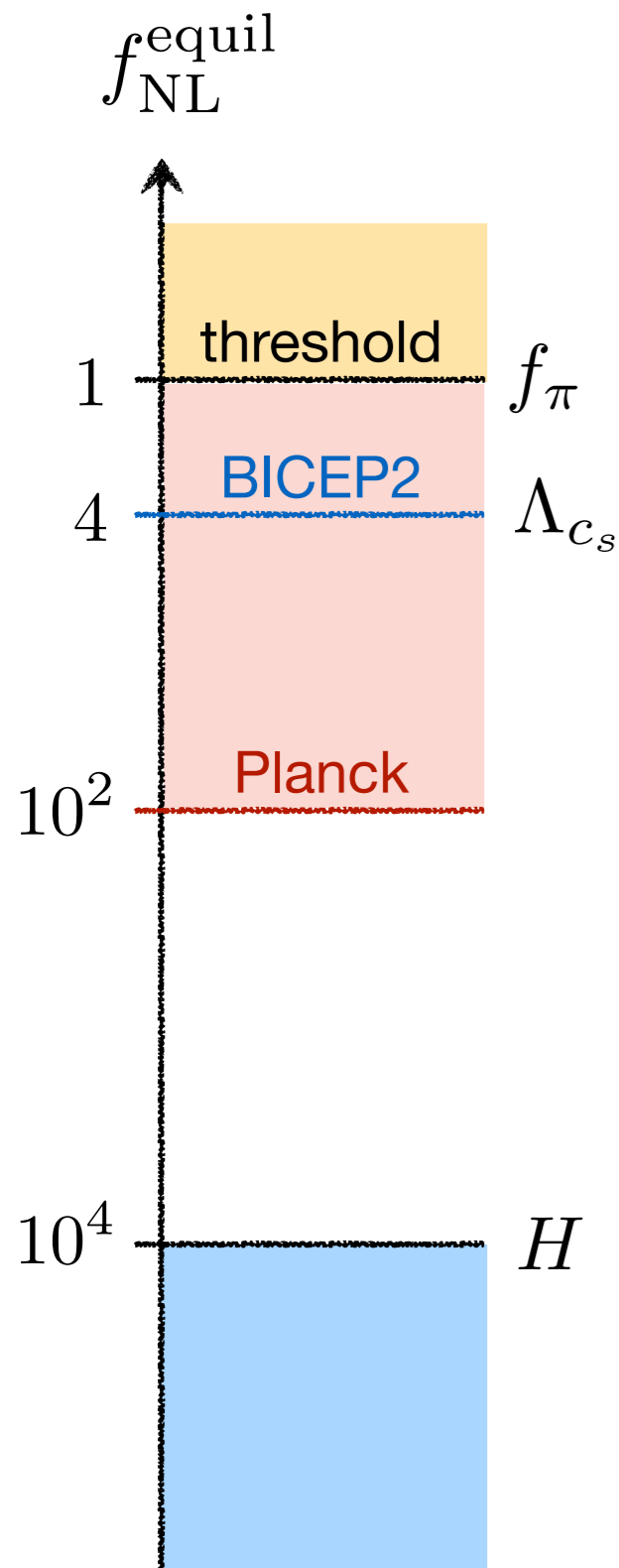
*ruled out by Planck + BICEP2*

*non-  
perturbative*

*perturbative*



# Conclusions



- If the BICEP2 result survives, then  $c_s > 0.25$  almost reaching the unitarity threshold  $(c_s)_* = 0.47$ .
- This corresponds to  $|f_{\text{NL}}^{\dot{\pi}(\partial_i \pi)^2}| < 3.3$ , two orders of magnitude stronger than the Planck-only bound.
- This does not rule out large equilateral non-Gaussianity from other operators in the EFT of inflation:

$$\text{e.g. } \mathcal{L}_\pi^{(3)} = -\frac{\dot{\pi}_c (\tilde{\partial}_i \pi_c)^2}{\Lambda_{c_s}^2} - \frac{\dot{\pi}_c^3}{\Lambda^2} \quad \text{with } \Lambda \ll \Lambda_{c_s}, \text{ is radiatively stable!}$$

- Order-one equilateral non-Gaussianity remains a well-motivated experimental target.


$$f_{NL}^{\text{equil}} = 1$$

*“If you build it they will come.”*

**Thank you for your attention!**



# Robustness of the Bound

