Scattering Amplitudes at Strong Coupling Beyond the Area Paradigm

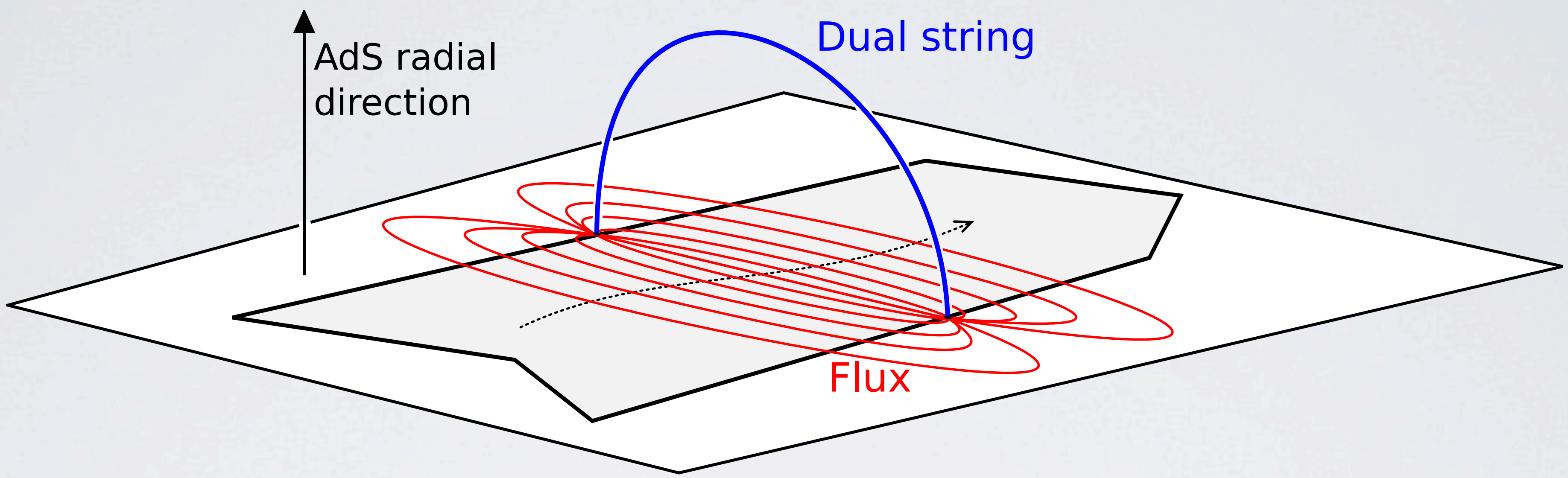
Benjamin Basso
ENS Paris

Strings 14 Princeton

based on work with Amit Sever and Pedro Vieira

Wilson loops at finite coupling in N=4 SYM

[Alday,Gaiotto,Maldacena,Sever,Vieira'10]



1+1d background : *flux tube* sourced by two parallel null lines

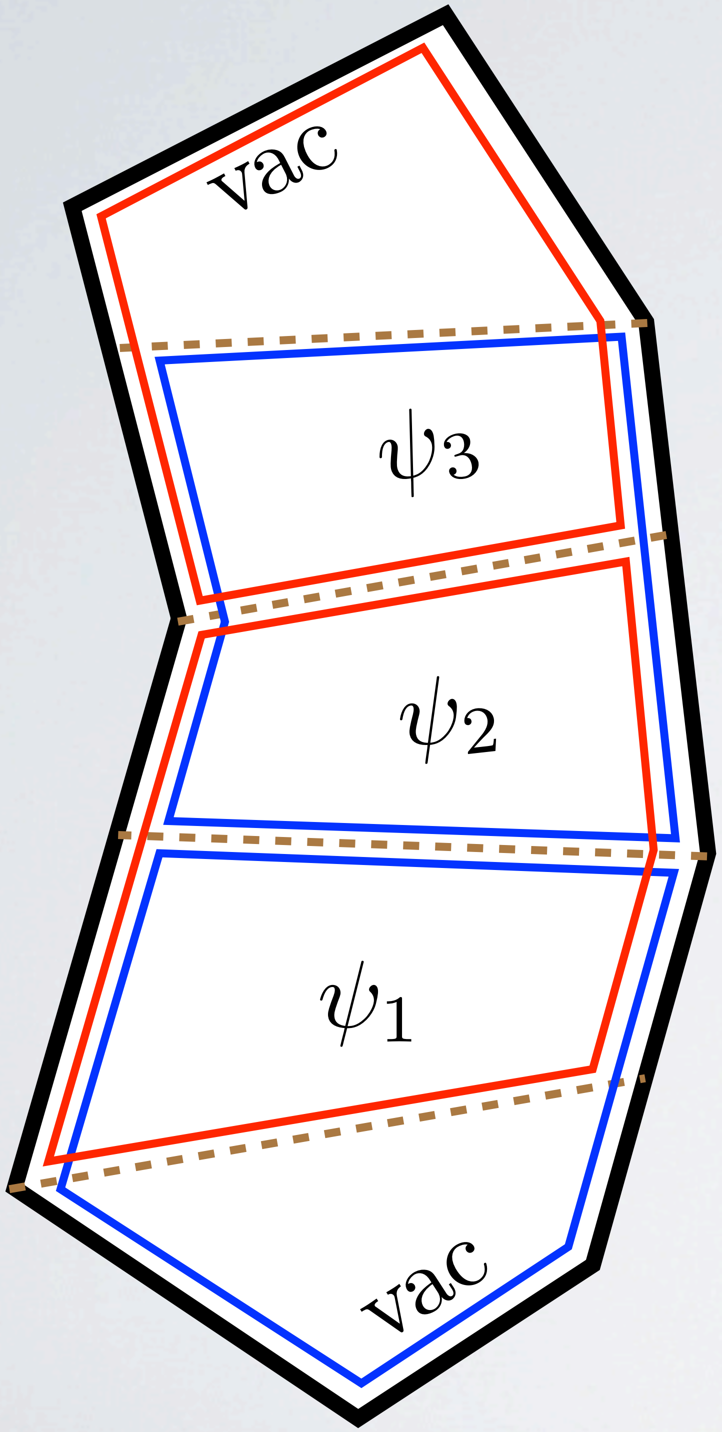
bottom&top cap excite the *flux tube* out of its ground state

→ *Sum over all flux-tube eigenstates*

$$\mathcal{W} = \sum_{\text{states } \psi} C_{\text{bot}}(\psi) \times e^{-E(\psi)\tau + ip(\psi)\sigma + im(\psi)\phi} \times C_{\text{top}}(\psi)$$

Refinement : the pentagon way

[BB,Sever,Vieira'13]



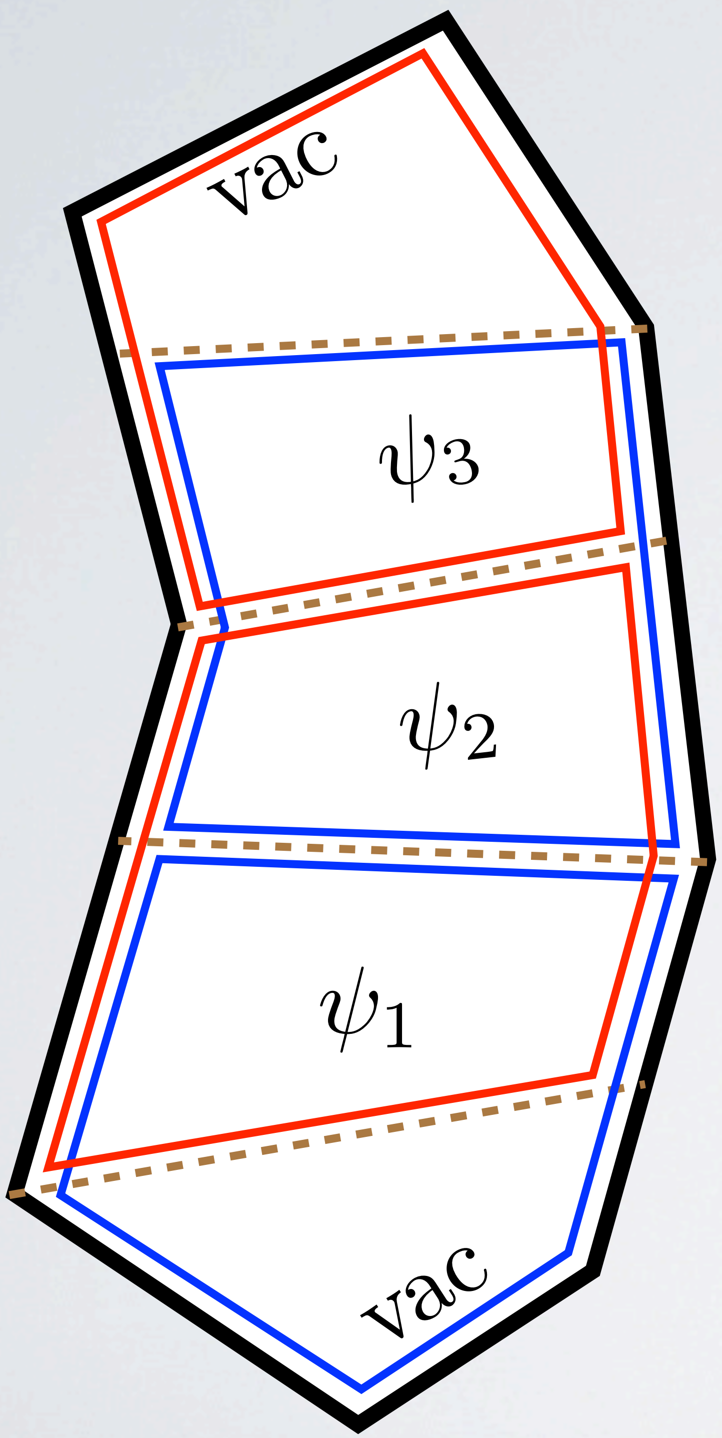
Valid at any coupling

$$= \sum_{\psi_i} \left[\prod_i e^{-E_i \tau_i + i p_i \sigma_i + i m_i \phi_i} \right] \times$$

$$P(0|\psi_1) P(\psi_1|\psi_2) P(\psi_2|\psi_3) P(\psi_3|0)$$

Refinement : the pentagon way

[BB,Sever,Vieira'13]



Valid at any coupling

$$= \sum_{\psi_i} \left[\prod_i e^{-E_i \tau_i + i p_i \sigma_i + i m_i \phi_i} \right] \times$$

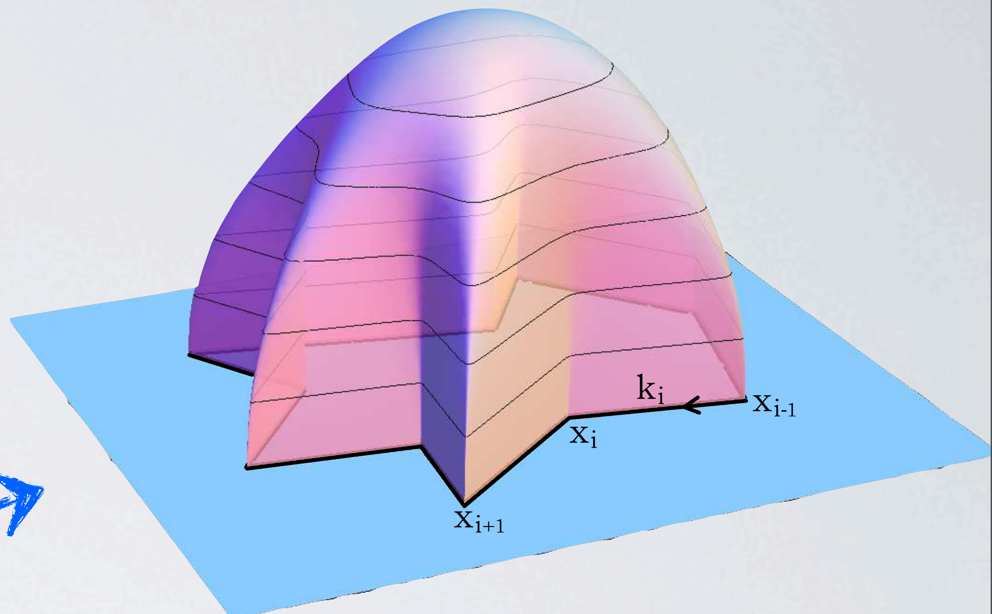
$$P(0|\psi_1) P(\psi_1|\psi_2) P(\psi_2|\psi_3) P(\psi_3|0)$$

To compute amplitudes we need

- The spectrum of flux-tube states ψ
- All the pentagon transitions $P(\psi_1|\psi_2)$

Beyond the area paradigm

Simplest case : hexagon (n = 6) WL



minimal area in AdS_5

[Alday,Gaiotto,Maldacena'09]
[Alday,Maldacena,Sever,Vieira'10]

classical

$$\mathcal{W}_{n=6} = f_6\, \lambda^{-\frac{7}{288}}\, e^{\frac{\sqrt{\lambda}}{144} - \frac{\sqrt{\lambda}}{2\pi} A_{n=6}} (1 + O(1/\sqrt{\lambda}))$$

quantum

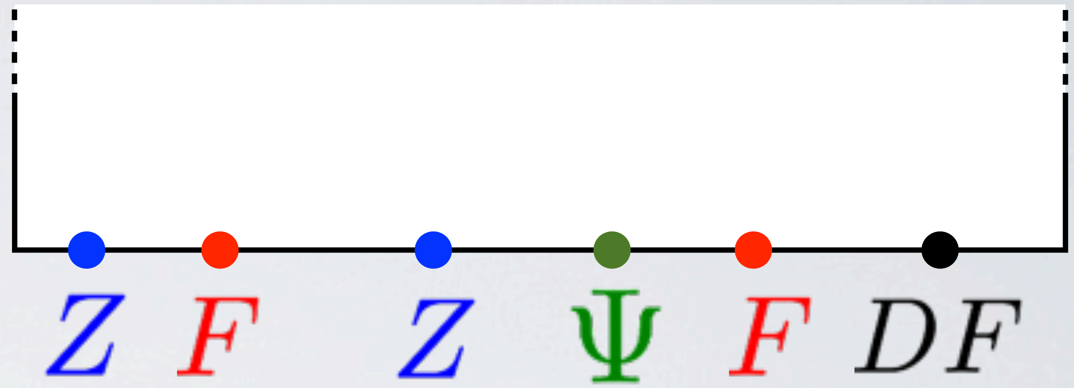
Pre-factor

$$f_6 = \frac{1.04}{(\sigma^2 + \tau^2)^{1/72}} + O(e^{-\sqrt{2}\tau})$$

[BB,Sever,Vieira'14]

The flux-tube eigenstates

$\psi = N$ particles state



(Adjoint) field insertions along a light-ray :
create/annihilate state on the flux tube

Spectral data

$E = E(u_1) + E(u_2) + \ldots + E(u_N)$

rapidity

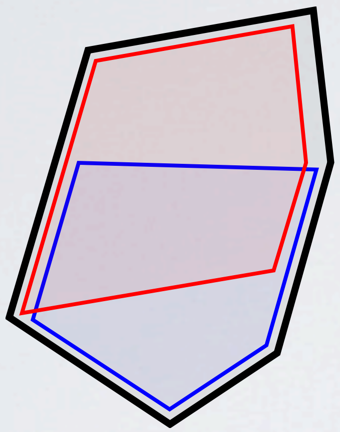
$p = p(u_1) + \cdots + p(u_N)$

$E(u) = \text{twist} + g^2 \ldots$

$p(u) = 2u + g^2 \ldots$

can be found using integrability

Pentagon/OPE series for hexagon

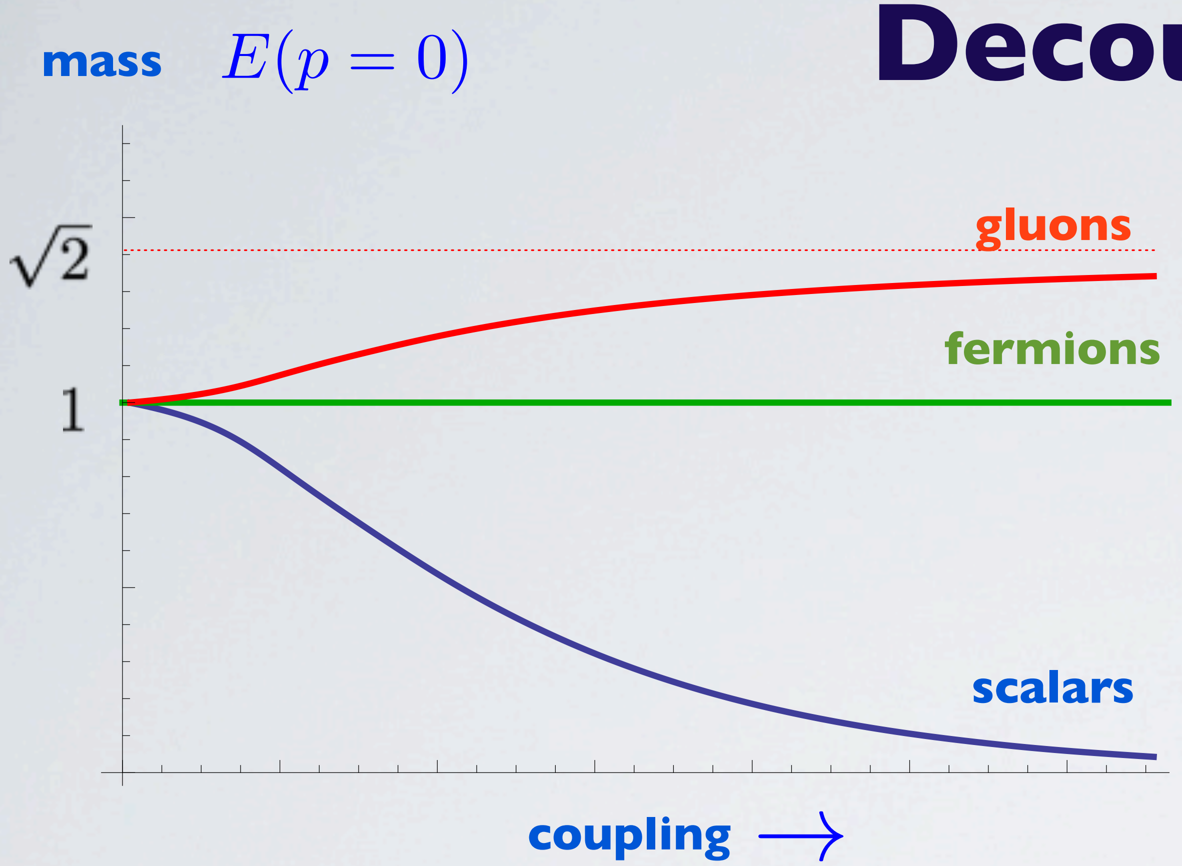
$$\mathcal{W}_{\text{hex}} =$$  $$= \sum_{\mathbf{a}} \int d\mathbf{u}\, P_{\mathbf{a}}(0|\mathbf{u})\, e^{-E(\mathbf{u})\tau + ip(\mathbf{u})\sigma + im\phi} P_{\mathbf{a}}(\bar{\mathbf{u}}|0)$$

i.e. in collinear limit

Lightest states dominate at large τ

What are they?

Decoupling limit



Scalar mass is *exponentially small* at strong coupling

$$m = \frac{2^{1/4}}{\Gamma(5/4)} \lambda^{1/8} e^{-\frac{\sqrt{\lambda}}{4}} (1 + O(1/\sqrt{\lambda})) \ll 1$$

For $\tau \gg 1$ all heavy flux tube excitations decouple

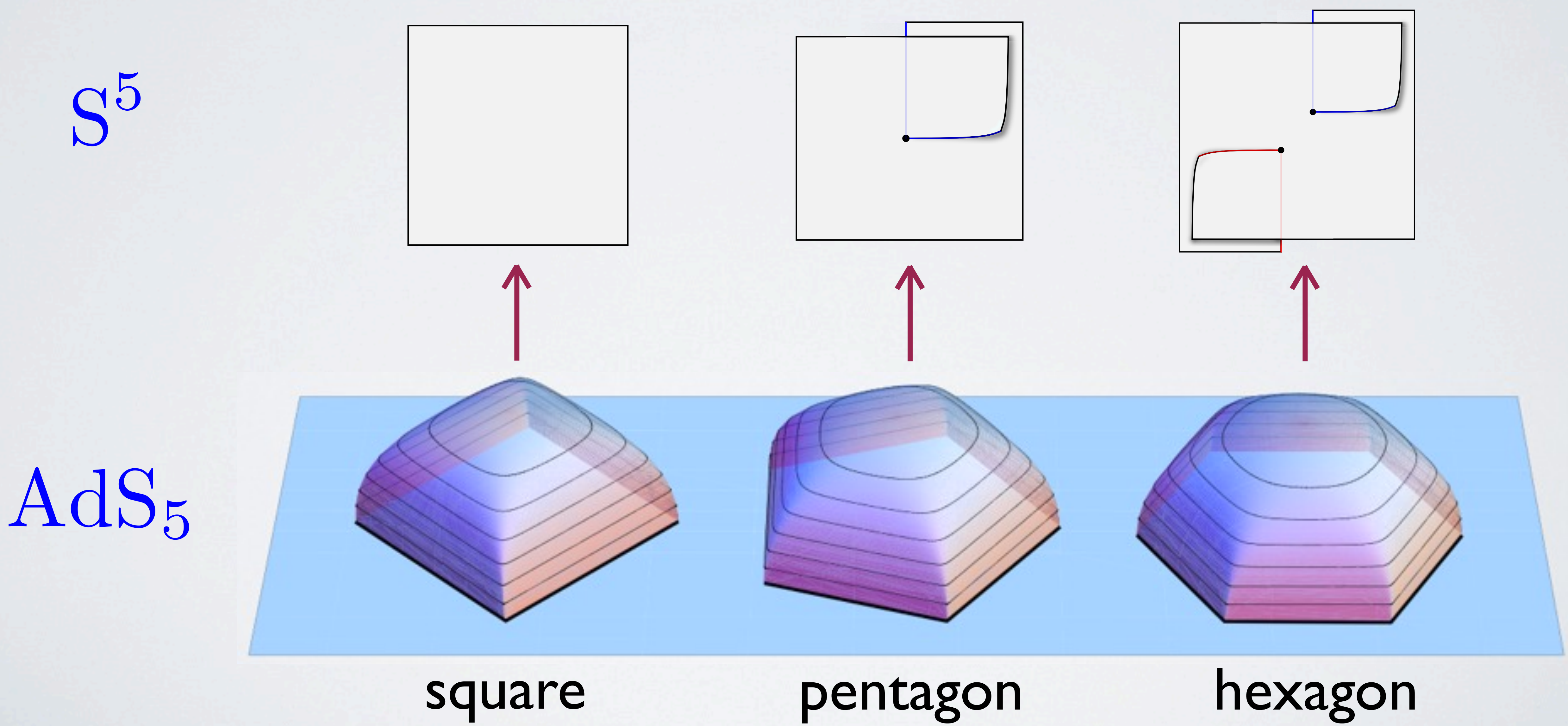
Low energy effective theory :
(relativistic) O(6) sigma model

[Alday,Maldacena'07]

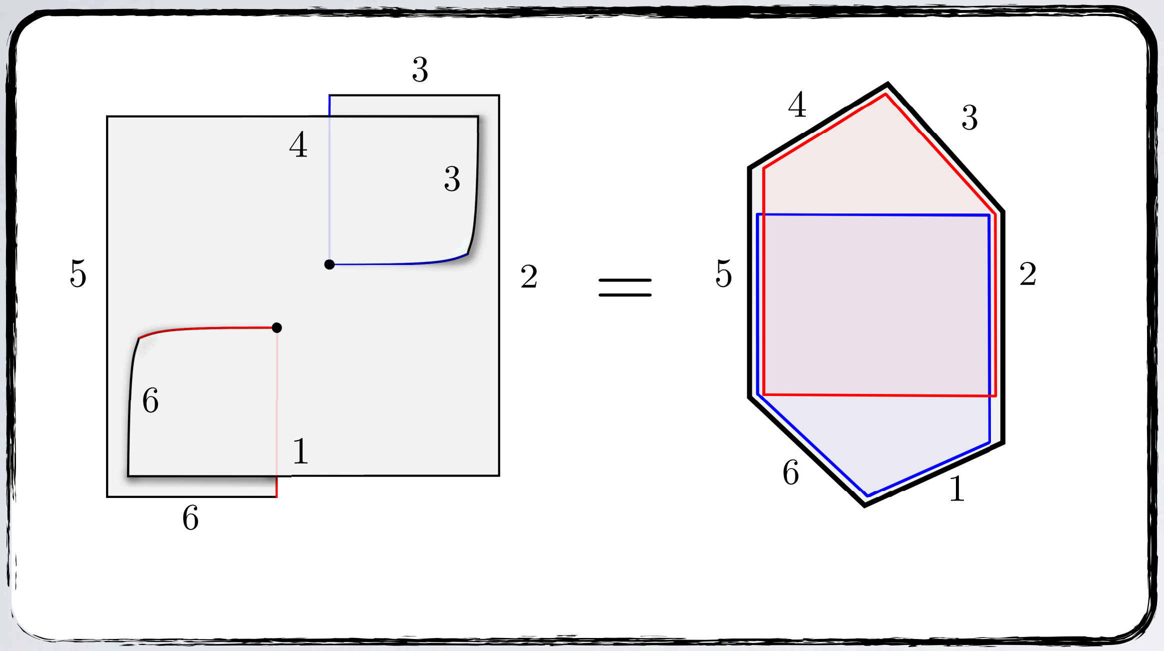
$$\mathcal{L}_{\text{eff}} = \frac{\sqrt{\lambda}}{4\pi} \partial X \cdot \partial X \qquad \text{with} \qquad X^2 = \sum_{i=1}^{6} X_i^2 = 1$$

The pentagon/twist operator

$$S_{\rm NG} = -\frac{1}{2\pi\alpha'}\sqrt{-\det\left(\partial_\alpha x^\mu \partial_\beta x^\nu g^{AdS}_{\mu\nu}\right) - \det\left(\partial_\alpha y^\mu \partial_\beta y^\nu g^{S^5}_{\mu\nu}\right)}$$



Hexagon as a correlator of twist operators



corrections from heavy modes irrelevant in collinear limit

$$\mathcal{W}_6 = \langle 0|\, \phi_{\pentagon}(\tau,\sigma)\phi_{\pentagon}(0,0)\,|0\rangle \; + O(e^{-\sqrt{2}\tau})$$

$$\downarrow$$

Probes the physics of the O(6) sigma model :

$$\mathcal{W}_{O(6)}(z) \qquad\qquad z = m\sqrt{\sigma^2+\tau^2}$$

Large distance $\quad z \gg 1 \quad \mathcal{W}_{O(6)} = 1 + O(e^{-2z})$

Short distance $\quad z \ll 1 \quad \mathcal{W}_{O(6)} = \;\; ?$

OPE as form factor expansion

See [Cardy,Castro-Alvaredo,Doyon'07] for similar considerations for computing entanglement entropy in integrable QFT

Insert complete basis of states

$$\mathcal{W}_{O(6)} = \sum_N \frac{1}{N!} \langle 0| \phi_{\pentagon} |\theta_1, \ldots, \theta_N\rangle \langle \theta_1, \ldots, \theta_N| \phi_{\pentagon} |0\rangle \, e^{-z \sum_i \cosh\theta_i}$$

Pentagon transition = form factor of twist operator

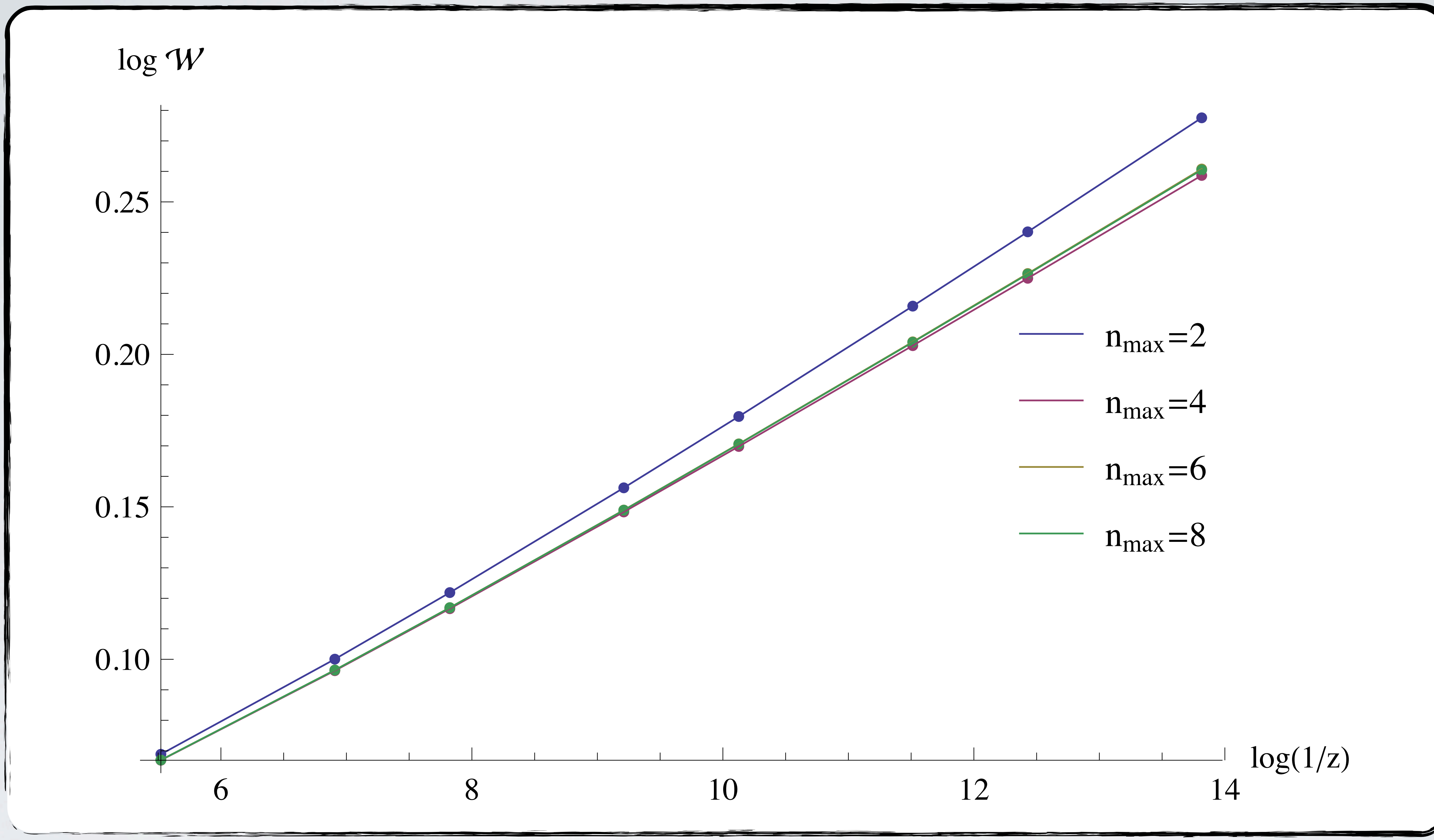
$$P(0|\theta_1, \ldots, \theta_N) = \langle \theta_1, \ldots, \theta_N| \phi_{\pentagon} |0\rangle$$

We found all these transitions so we can plot

Normalization

$\langle 0| \phi_{\pentagon} |0\rangle = 1$ which enforces that $\mathcal{W}_{O(6)} \to 1 \quad z \to \infty$

Numerical analysis



Plot of the truncated OPE/form factor series representation for $\log \mathcal{W}_{O(6)}$

Short distance analysis

Short distance OPE (valid for $z \ll 1$)

$$\phi_{\text{⬠}}(\tau,\sigma)\phi_{\text{⬠}}(0,0) \sim \frac{\log(1/z)^{B}}{z^{A}}\phi_{\text{⬡}}(0,0)$$

3-point function

Critical exponent A

$$A = 2\Delta_{\text{⬠}} - \Delta_{\text{⬡}} = 2\Delta_{5/4} - \Delta_{3/2}$$

with Δ_k the scaling dimension of the twist operator ϕ_k

$$\Delta_k = \frac{c}{12}\left(k - \frac{1}{k}\right)$$

$$\begin{cases} c = & \text{central charge} \\ 2\pi(k-1) = & \text{excess angle for } \phi_k \end{cases}$$

[Knizhnik'87]
[Lunin,Mathur'00]
[Calabrese,Cardy'04]

Short distance analysis

Short distance OPE (valid for $z \ll 1$)

$$\phi_{\pentagon}(\tau,\sigma)\phi_{\pentagon}(0,0) \sim \frac{\log(1/z)^{B}}{z^{A}}\phi_{\hexagon}(0,0)$$

3-point function

Critical exponent A

$$A = \frac{1}{36} \qquad \text{since in our case } c = 5$$

Critical exponent B from one-loop anomalous dimensions

$$B = -\frac{3}{2}A = -\frac{1}{24}$$

Short distance analysis

For $z \ll 1$

include subleading RG logs

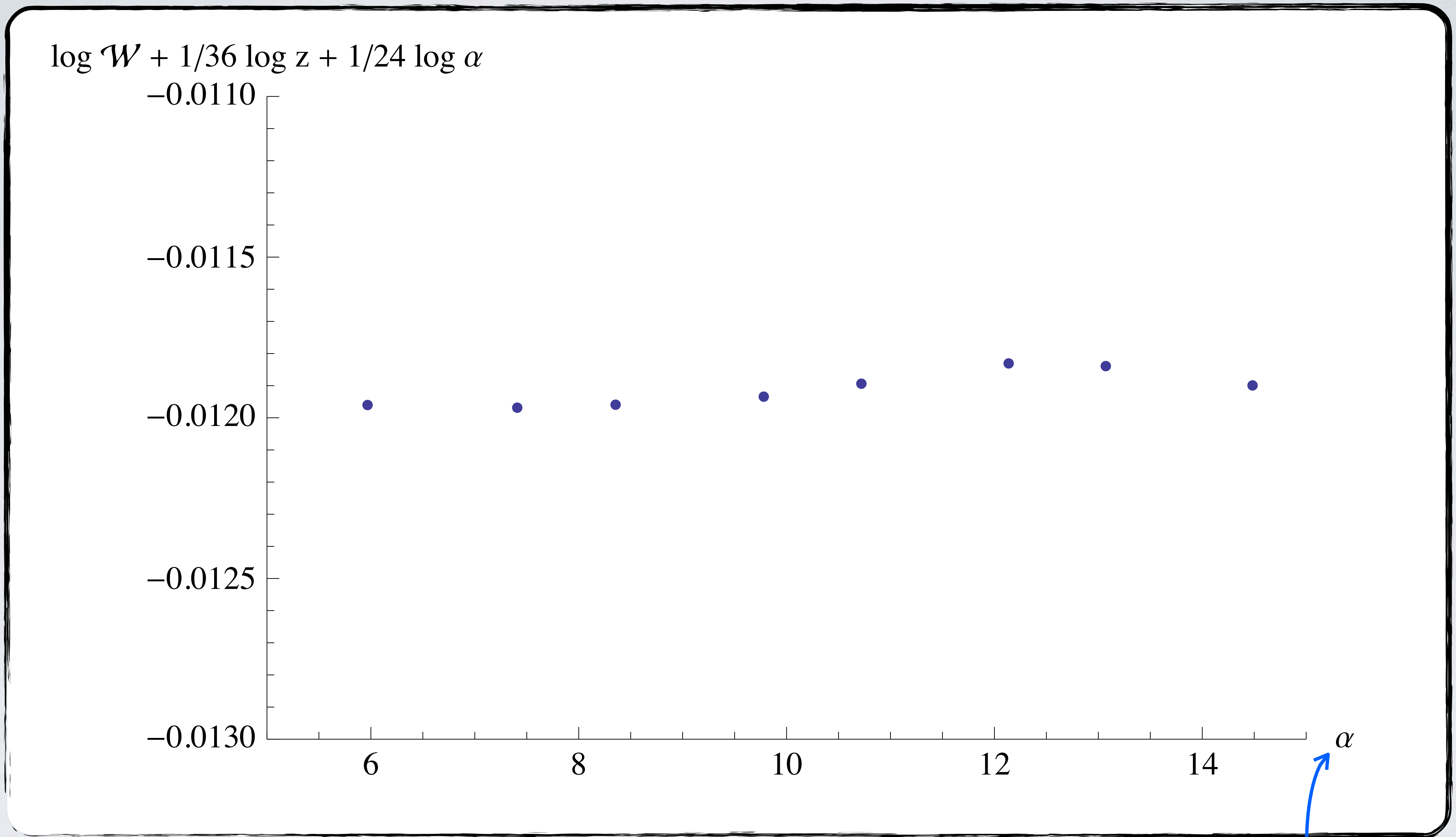
$$\mathcal{W}_{O(6)} = \frac{C}{z^{1/36} \log(1/z)^{1/24}} + \dots$$

Constant C is fixed in the IR by

$$\mathcal{W}_{O(6)} \to 1 \qquad \text{when} \qquad z \to \infty$$

and is thus *non perturbative*

Numerical analysis



$$\log C = -0.01$$

running coupling

$$\alpha = \log(1/z) + \ldots$$

Short distance analysis

For $z \ll 1$ (i.e. $1 \ll \tau \ll e^{\sqrt{\lambda}/4}$)

$$m \simeq \frac{2^{1/4}}{\Gamma(5/4)} \lambda^{1/8} e^{-\frac{\sqrt{\lambda}}{4}}$$

$$\mathcal{W}_{O(6)} = \frac{C}{z^{1/36} \log(1/z)^{1/24}} + \dots$$

$$z = m\sqrt{\sigma^2 + \tau^2}$$

controlled by the gluons

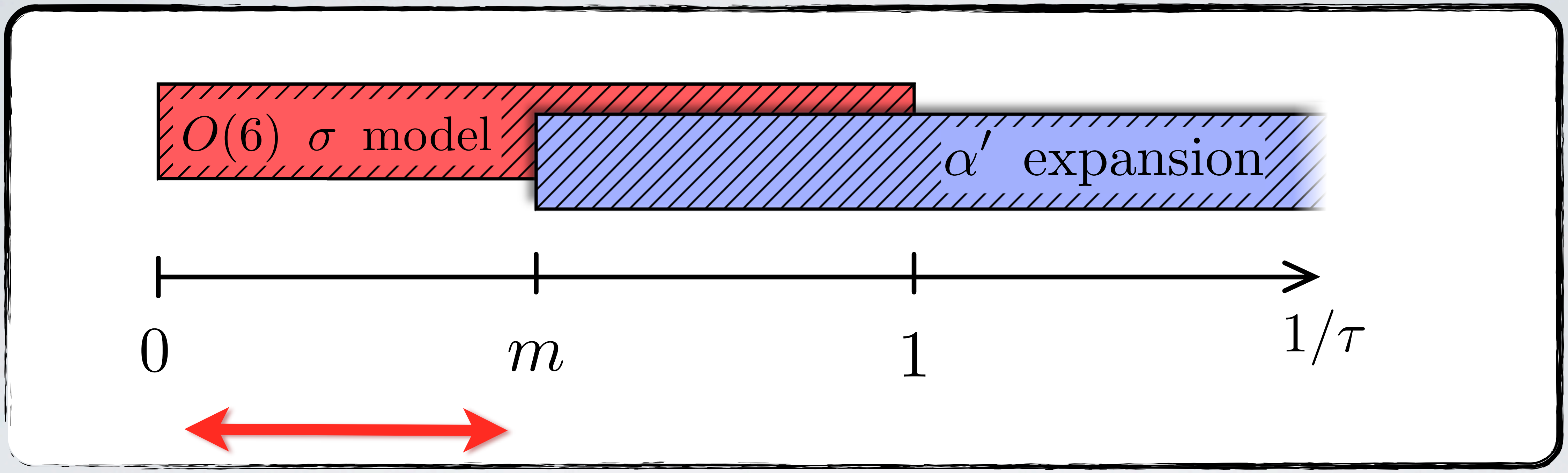
$$A_{n=6} = O(e^{-\sqrt{2}\tau})$$

$$\mathcal{W}_{n=6} = f_6 \, \lambda^{-\frac{7}{288}} e^{\frac{\sqrt{\lambda}}{144} - \frac{\sqrt{\lambda}}{2\pi} A_{n=6}} (1 + O(1/\sqrt{\lambda}))$$

Pre-factor

$$f_6 = \frac{1.04}{(\sigma^2 + \tau^2)^{1/72}} + O(e^{-\sqrt{2}\tau})$$

Infrared/non-perturbative regime

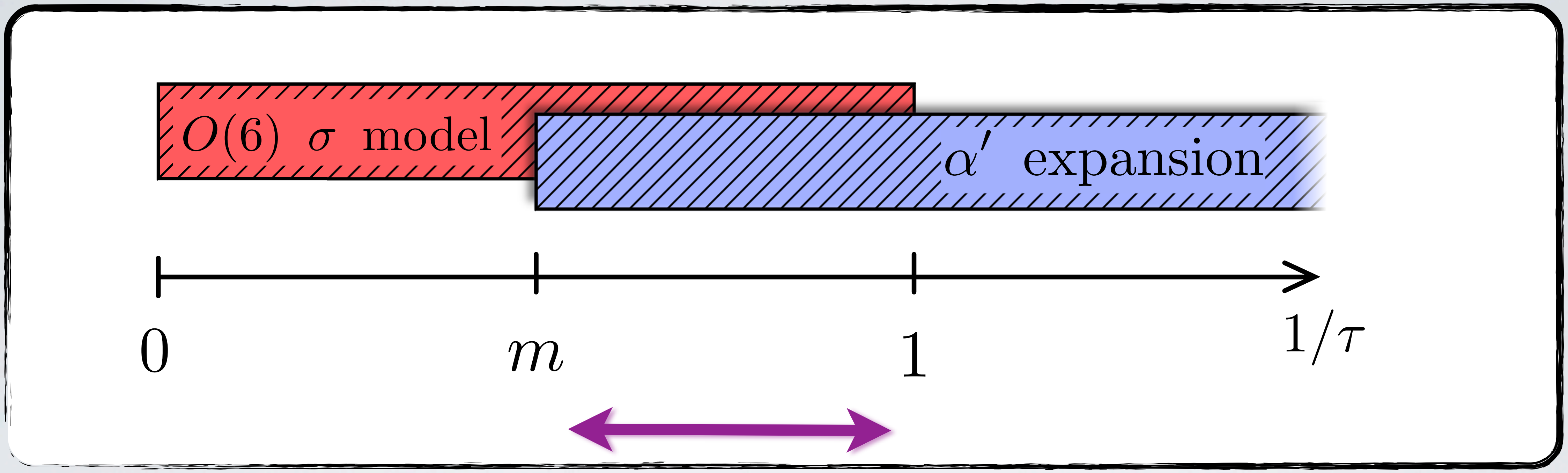


$z \gg 1$ equivalently $\tau \gg e^{\sqrt{\lambda}/4}$

Deep (infrared) collinear limit

Completely non perturbative

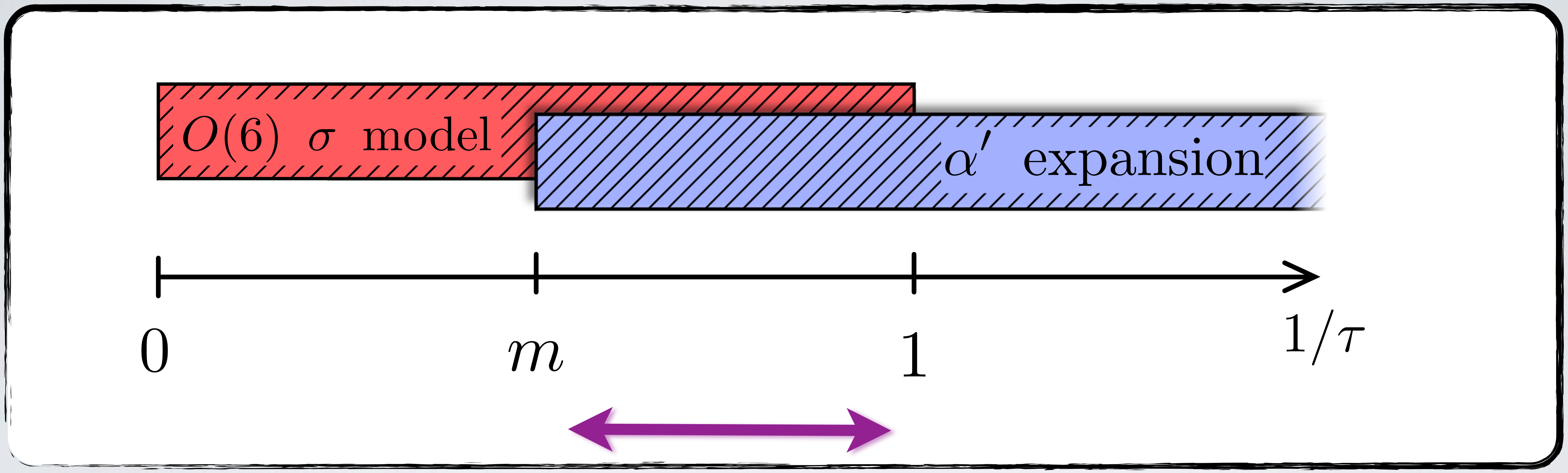
Cross over



$z \ll 1$ equivalently $1 \ll \tau \ll e^{\sqrt{\lambda}/4}$

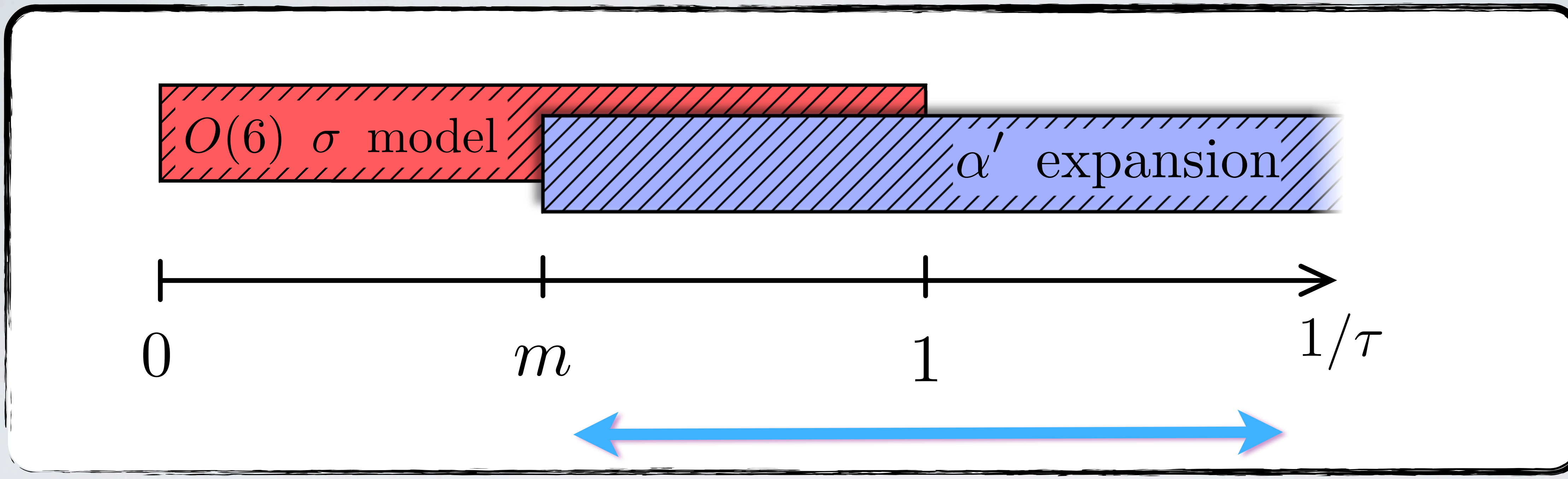
UV regime of O(6) model :
perturbative collinear limit

Cross over



here
we could match O(6) analysis
with
string perturbative expansion

Full stringy pre-factor



full thing :

include all heavy modes

gluons, *fermions*, ...

Next Strings maybe .... :)

$$f_6 = \frac{1.04}{(\sigma^2 + \tau^2)^{1/72}} + O(e^{-\sqrt{2}\tau}) \ \ + O(e^{-2\tau})$$

Conclusions

At strong coupling SA develop a non-perturbative regime in the near collinear limit

The string α' expansion breaks down for extremely large values of $\tau \sim -\log u_2 \sim e^{\sqrt{\lambda}/4}$

That's because flux tube mass gap m becomes extremely small

One should think in terms of correlators of twist operators

This fixes the collinear limit of SA at strong coupling

Outlook

Higher multiplicity (heptagon, ...)?

Next-to-MHV amplitudes?

Full one-loop pre-factor?

One-loop Thermodynamical-Bubble-Ansatz equations?

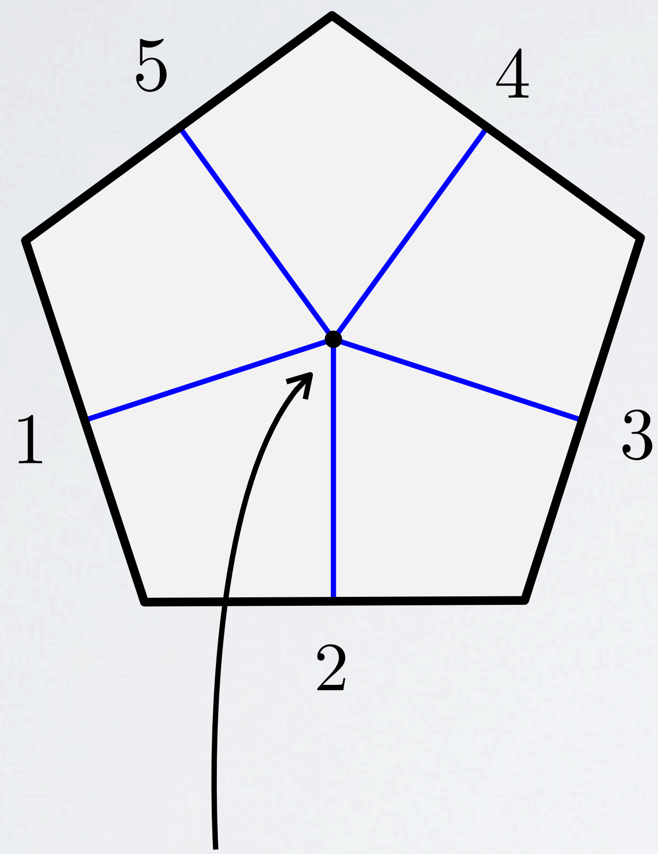
... and many other questions...

THANK YOU!

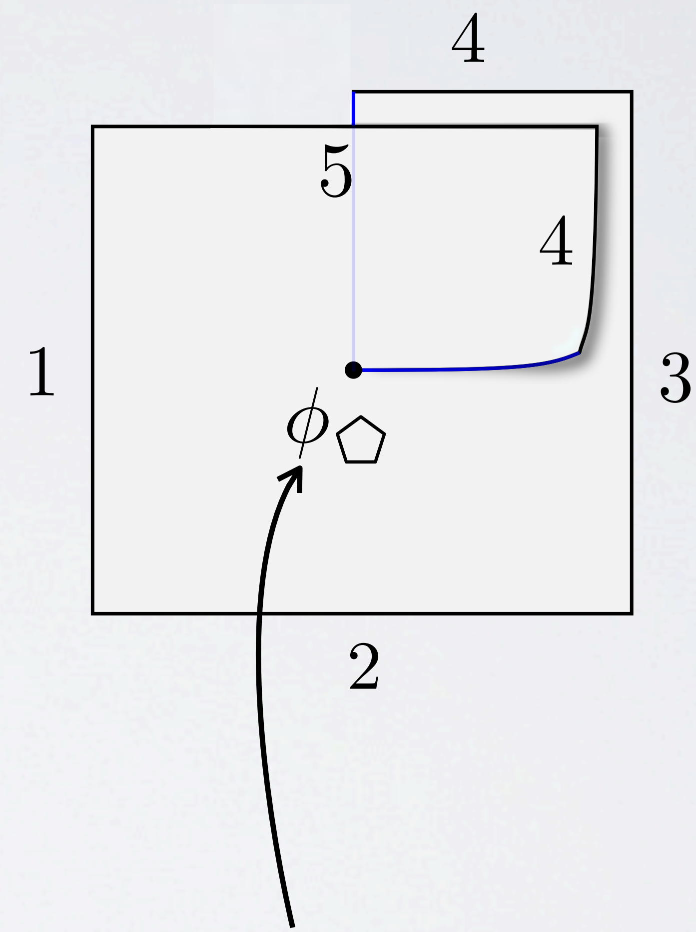
BACK UP

Pentagon as twist operator

Asympotically a pentagon = 5 quadrants glued together



$=$



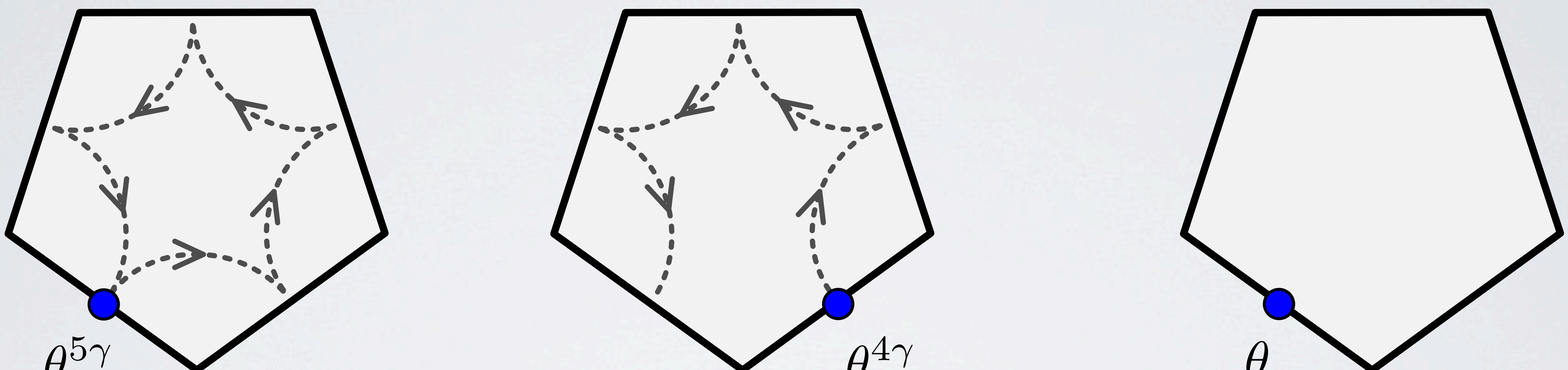
excess angle $= \dfrac{\pi}{2}$

twist operator

$$P(\psi_{\text{edge }2}|\psi'_{\text{edge }5}) = \langle\psi'|\,\phi_{\pentagon}\,|\psi\rangle$$

Monodromy

One can go around the pentagon with 5 mirror rotations

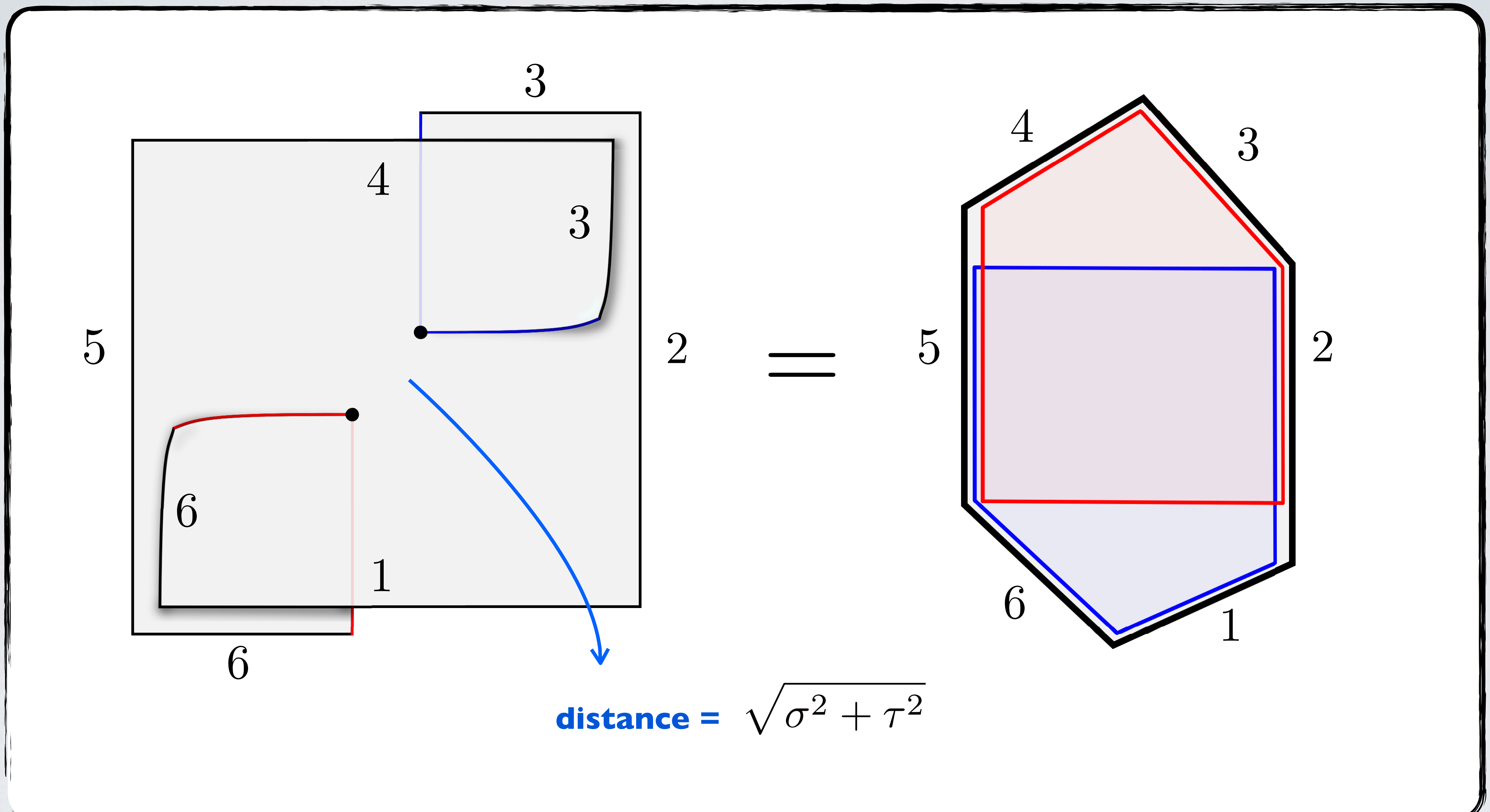


$\theta^{5\gamma} = \theta + 5i\frac{\pi}{2}$ $\qquad$ $\theta^{4\gamma} = \theta + 4i\frac{\pi}{2}$ $\qquad$ θ

This is one more than for a square

$$E \xrightarrow{\gamma} ip \longrightarrow -E \longrightarrow -ip \longrightarrow E$$

Hexagon as a correlator of twist operators



$$\boxed{\mathcal{W}_6 = \langle 0|\, \phi_{\pentagon}(\tau,\sigma)\phi_{\pentagon}(0,0)\, |0\rangle}$$

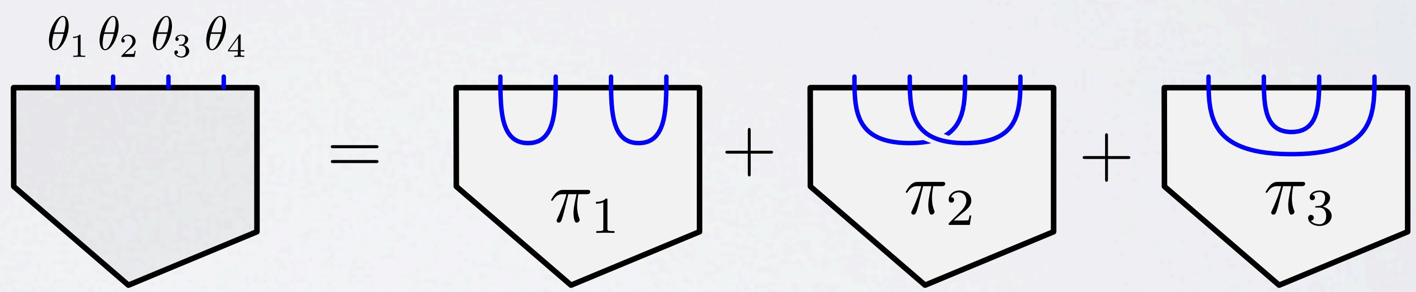
computed in O(6) sigma model

Hexagon beyond 2pt approximation

$$\mathcal{W}_6 = 1 + \frac{1}{2}\int \frac{d\theta_1 d\theta_2}{(2\pi)^2} |P(0|\theta_1,\theta_2)|^2 e^{-m\tau(\cosh\theta_1+\cosh\theta_2)+im\sigma(\sinh\theta_1+\sinh\theta_2)} + \dots$$

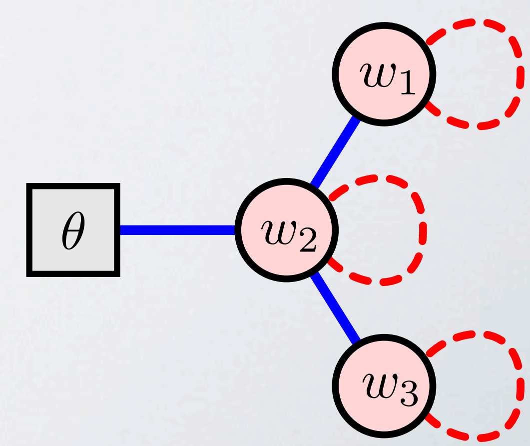
multi-particle states

Multi-particle transitions



Understood!

$$\text{integrand} = \prod_{i<j} \frac{1}{P(\theta_i|\theta_j)P(\theta_j|\theta_i)} \times \text{rational}$$



Higher multiplicity

Higher-point amplitudes correspond to higher-points correlators

$$\mathcal{W}_n = \langle 0| \, \phi_{\pentagon}(\tau_{n-4}, \sigma_{n-4}) \ldots \phi_{\pentagon}(\tau_1, \sigma_1) \, |0\rangle$$

Overall short-distance scaling is controlled by OPE

$$\underbrace{\phi_{\pentagon} \ldots \phi_{\pentagon}}_{n-4} \sim m^{-(n-4)\Delta(\frac{5}{4})+\Delta(\frac{n}{4})} \phi_{\varphi}$$

with final excess angle $\varphi = 2\pi \times \dfrac{n-4}{4}$

This leads to the addition

$$\mathcal{W}_n = e^{-\frac{\sqrt{\lambda}}{2\pi} A_n + \frac{\sqrt{\lambda}(n-4)(n-5)}{48n} + o(\sqrt{\lambda})}$$