Hidden Geometry in Dual Heterotic/F-theory Compactifications

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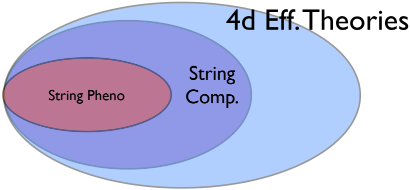
Motivation

Much recent work: Classifying which effective theories arise from string compactifications, scanning for models/patterns

- Goal: Combine two approaches.

  Consider $4D$, $N = 1$, Dual Heterotic-F-theory Vacua

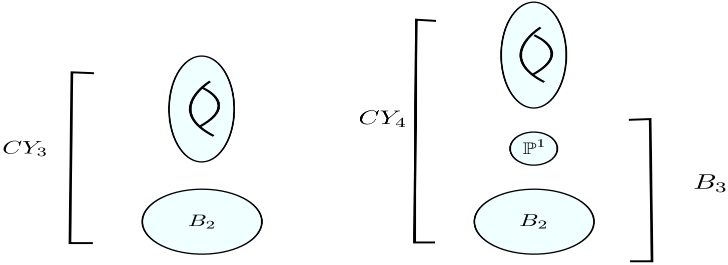
  - Systematically construct and study a large class of vacua
  - Try to understand/classify how topology/geometry constrains effective theories
  - Develop new tools for string pheno?



What possible EFTs?

⇕

Which geometries?



Heterotic **F-theory**

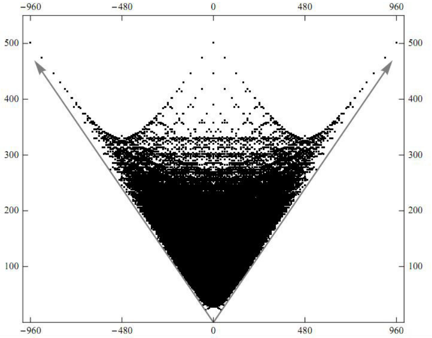
Heterotic on $\pi_h : X_n \xrightarrow{\mathbb{E}} B_{n-1} \;\Leftrightarrow\;$ F-theory on $\pi_f : Y_{n+1} \xrightarrow{K3} B_{n-1}$

(with $\pi : Y_{n+1} \xrightarrow{\mathbb{E}} \mathcal{B}_n$ and $\rho : \mathcal{B}_n \xrightarrow{\mathbb{P}^1} B_{n-1}$)

Descends from 8-dim: Het on $T^2 \leftrightarrow$ F-theory on $\pi : K3 \xrightarrow{\mathbb{E}} \mathbb{P}^1$ (Vafa)

(Rich history: Vafa, Morrison, Friedman, Morgan, Witten, Donagi, Curio, Aspinwall, Katz, Plesser, Andreas, Watari, Hayashi, Toda, Yamazaki, Schafer-Nameki, Saulina, Marsano, Cvetic...)

Where these two theories are dual, there is a finite set of geometries to study



- The number of elliptically fibered CY 3-folds, X_3, is finite (M. Gross)
- $\mathbb{E}$-fibered 3-folds "extremal" in known examples?? (Taylor, Candelas, Ooguri-Keller, etc.)

What about the no. of vector bundles (V_1, V_2) over X_3?

- For *fixed topology* $\mathcal{M}(c(V))$ has only finitely many components
  - $rk(V)$: $H \subset E_8$
  - Spinors: $c_1(V) = 0$
  - Anomaly cancellation: $0 \leq c_2(V_i) \leq c_2(TX)$
  - For fixed $c_2 \Rightarrow$ only finitely many values of c_3 compatible w/ $\mathcal{N} = 1$ SUSY (e.g. Maruyama, Langer (for $H = SU(n)$))
- Bounds on (X_3, V_i) non-constructive

The Plan...

(With W. Taylor)

- Systematically study the general properties/constraints of EFT for this class of string compactifications
- Develop a general formalism: For smooth X_3, possible B_2 classified (generalized del Pezzo). Build an algorithm to construct all $\mathcal{B}_3$ that are non-degenerate $\mathbb{P}^1$ fibrations over any B_2.
- To explore/test general structure: Build dual (X_3, Y_4) pairs using dataset of $61,539$ toric surfaces, B_2 (Morrison + Taylor)
  - Caveats: All fibrations w/ section. $\mathcal{B}_3$ constructed as a $\mathbb{P}^1$-bundle over B_2.
  - Only 16 of these B_2 lead to smooth X_3 $\Rightarrow$ Start with these $\Rightarrow$ 4962 4-folds (Note: Toric manifolds used as examples but constructions/constraints general)

Complex structure of $Y_4 \Leftrightarrow$ bundle moduli space of V

Weierstrass Model for an elliptic fibration:

$$y^2 = x^3 + f(u)x + g(u)$$

w/ $f \in H^0(B_3, K_{\mathcal{B}_3}^{-4})$, $g \in H^0(\mathcal{B}_3, K_{B_3}^{-6})$

E.g. $H = SU(2)$, $G = E_7$:

- F-theory w/ E_7 singularity:
$$y^2 = x^3 + (f_3 z^3 + f_4 z^4)x + (g_5 z^5 + g_6 z^6) + \dots$$
- In the neighborhood of the 7-brane (z=0):

  $y^2 = x^3 + z^3(g_5 z^2 + f_3 x) + \dots$

  Heterotic: $SU(2)$ Spectral Cover, C (w/ $c_2(V) = \eta \wedge \omega_0 + \pi^*(\zeta)$) :
$$a_0 \hat{Z}^2 + a_2 \hat{X} = 0$$
  with $a_0 \in H^0(B_2, \mathcal{O}(\eta))$ and $a_2 \in H^0(B_2, \mathcal{O}(\eta) \otimes K_{B_2}^{\otimes 2})$

  $\{f_3 = a_2, g_5 = a_0\}$, $\{f_4, g_6\} \leftrightarrow X_3$ Weierstrass

η: Building bundles and $\mathcal{B}_3$

- Idea: Choose topology of bundles $(V_1, V_2) \Leftrightarrow$ Build $\rho : B_3 \xrightarrow{\mathbb{P}^1} B_2$

Heterotic:

- Can expand:

  $c_2(V_i) = \eta_i \wedge \omega_0 + \zeta_i$,

  w/ η_i (resp. ζ_i) $\{1,1\}$ (resp. $\{2,2\}$) forms on B_2 and ω_0 dual to the zero section.

- Anomaly Cancellation $\Rightarrow$

$$\eta_{1,2} = 6c_1(B_2) \pm t$$

- Can build $\mathcal{B}_3$ over B_2 by "twisting" the $\mathbb{P}^1$ fibration (analog of $\mathbb{F}_n$ surfaces in $6D$)

  $\mathcal{B}_3 = \mathbb{P}(\mathcal{O} \oplus \mathcal{L})$

- $c_1(\mathcal{B}_3) = c_1(B_2) + 2\Sigma + t$

  where Σ is dual to the zero-section of the $\mathbb{P}^1$-fibration

In Het/F-dual pairs, two t's are the same (FMW), (Grimm + Taylor)

Next: Bounds on twists $\Rightarrow$ finite # $\mathcal{B}_3$ sol'ns/enumeration

$N = 1$ SUSY

- Heterotic: X_3 CY. Bundles, V_i satisfy the Hermitian-YM Eq.s:
  $F_{ab} = F_{\bar{a}\bar{b}} = 0 \qquad g^{a\bar{b}} F_{a\bar{b}} = 0$
- F-theory: Y_4 can be resolved into a smooth Calabi-Yau 4-fold
- Need vanishing degrees of $(f, g, \Delta) \leq (4, 6, 12)$ on every divisor in $\mathcal{B}_3$
- f, g cannot vanish to orders $4, 6$ on any curve.

$\Rightarrow t \Rightarrow \eta$ an effective curve class in B_2.

4D Symmetries

Only certain divisors can carry singular fibers

- η_i base point free $\Rightarrow$ implies that $\nexists$ any eff. curve of negative self-intersection, D such that $\eta_i \cdot D < 0$
  $(-6K_2 \pm t) \cdot D \geq 0$
- $H = SU(n) \Rightarrow \eta$ bpf.
- $D^2 = -2 \Rightarrow$ non-bpf egs: $H = SO(8), G_2, F_4, E_6, E_7, E_8,$ over gen. del Pezzos

Sample Question: How does topology constrain $4D$ Gauge Symmetry?

I.e. given a CY 3-fold, X, does $\exists$ a stable bundle with *given* rank ($rk(V)$), structure group ($H \subset E_8$) and total Chern class ($c(V)$)?

Step 1:

- Study all possible Y_4's with perturbative heterotic duals. Constrain $\mathcal{M}(c(V))$

Step 2:

- Add in G-flux on Y_4 to fully determine $\mathcal{M}(c(V))$

base B_2		$h_{1,1}$	# $\mathcal{B}_3$'s
(1, 1, 1)	($\mathbb{P}^2$)	1	14
(0, 0, 0, 0)	($\mathbb{F}_0$)	2	82
(1, 0, -1, 0)	($\mathbb{F}_1$)	2	109
(2, 0, -2, 0)	($\mathbb{F}_2$)	2	24
(0, 0, -1, -1, -1)	(dP_2)	3	472
(1, -1, -1, -2, 0)		3	173
(-1, -1, -1, -1, -1, -1)	(dP_3)	4	776
(0, -1, -1, -2, -1, -1)		4	729
(0, 0, -2, -1, -2, -1)		4	312
(1, 0, -2, -2, -1, -2)		4	62
(-1, -1, -2, -1, -2, -1, -1)		5	1119
(0, -1, -1, -2, -2, -1, -2)		5	406
(-1, -1, -2, -1, -2, -2, -1, -2)		6	351
(-1, -2, -1, -2, -1, -2, -1, -2)		6	214
(0, -2, -1, -2, -2, -2, -1, -2)		6	83
(-1, -2, -2, -1, -2, -2, -1, -2, -2)		7	36
total			4962

Questions in Deformation theory

Begin with $4d$ symmetry $G \subset E_8$:

Heterotic: Begin with H-bundle V, $rank(V) = n$

- "Higgs" $G \Rightarrow$ Deform $V \oplus \mathcal{O}_{X_3}{}^{\oplus m}$ to V' with $rank(V') = n + m$
- "Enhance" to G': $\Rightarrow$ "Break" $V \to \mathcal{V}_1 \oplus \mathcal{V}_2 \oplus \mathcal{O}_{X_3} \oplus \ldots$ with $rank(\mathcal{V}_i) < rank(V)$

F-theory: Singular Y_4

- "Higgs" $G \Rightarrow$ Deform complex structure of Y_4 to smooth singularities
- "Enhance" to G': $G \subset G' \subset E_8 \Rightarrow$ "Tune" complex structure of Y_4 to produce more singular space

Generic Symmetries

$\times$	$\cdot$	$\mathfrak{su}_2$	$\mathfrak{su}_3$	$\mathfrak{g}_2$	$\mathfrak{so}_8$	$\mathfrak{f}_4$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_8$
$\cdot$	712								
$\mathfrak{su}_2$	499	47							
$\mathfrak{su}_3$	121	11	2						
$\mathfrak{g}_2$	589	62	7	34					
$\mathfrak{so}_8$	276	14	1	12	3				
$\mathfrak{f}_4$	1245	74	6	54	9	32			
$\mathfrak{e}_6$	184	2	0	2	0	2	0		
$\mathfrak{e}_7$	890	24	0	14	2	13	0	4	
$\mathfrak{e}_8$	15	0	0	0	0	0	0	0	0

Bounds on the structure group, H

- "Generic" symmetries on Y_4 provide rank(V)-dependent vanishing criteria for $\mathcal{M}(c(V))$. (First studied by Rajesh and Berglund & Myer)
- Also constraints on which symmetries can be enhanced
- non-Higgsable $SU(2), SU(3) \not\to SU(5)$
- Can be pinned at exactly one symmetry (or a sparse set)
- Intriguing for string pheno...

H	$\eta \geq Nc_1(B_2)$ $N =$
$SU(n)$	$n \quad (n \geq 2)$
$SO(7)$	4
$SO(m)$	$\frac{m}{2} \quad (m \geq 8)$
$Sp(k)$	$2k \quad (k \geq 2)$
F_4	$\frac{13}{3}$
G_2	$\frac{7}{2}$
E_6	$\frac{9}{2}$
E_7	$\frac{14}{3}$
E_8	5

Issues with G-flux

- In the previous discussion we have ignored G-flux
- Does gauge symmetry of the theory match Kodaira/Tate singular fibers of Y_4?
- Up until recently the consensus would have said yes.... (in M-theory limit, Abelian flux cannot break non-Abelian symmetries)
- But in the singular limit, F-theory can be more subtle
- Can never have *more* symmetry than indicated by Kodaira/Weierstrass. Could have less with G-flux in the singular limit...
- D-branes idea (Donagi, Katz, Sharpe) $\Rightarrow$ much recent work in local F-theory ("T-branes" (Cecotti,Cordova, Heckman, Vafa) or "Gluing data", (Donagi,Wijnholt))

An illustrative 6D example (Aspinwall + Donagi)

- Consider the simplest possible heterotic solution. The so-called "Standard Embedding", $V = TK3$, $c_2(V) = 24$.
- Problem: F-theory dual $y^2 = x^3 + g_5 z^5 + \ldots$.
  This is an E_8 singularity not E_7
- Even worse, $\Delta_{Y_4} = z^{10}(g_{24})(\ldots)$ with $g_{24} = \Delta_{K3}$
- To get a smooth CY4, must blow up the base at $g_{24} = \Delta_{K3} \Rightarrow$ This is the dual of Heterotic Small Instantons at 24 I_1 fibers over pts in $\mathbb{P}^1$.
- Question: How can $TK3$ and $\mathcal{I}_{\Delta_{K3}}$ have the same F-theory dual?

T-branes (Local Description)

- Gauge fields on the 7-brane: Hitchin's Equations

$$F - \frac{i}{2}[\Phi, \Phi^\dagger] = 0 \quad , \quad \bar{\partial}_A \Phi = 0 \qquad (\Phi \in H^1(End(V) \times K))$$

- Spectral Equation: $det(\Phi - \lambda \mathcal{I}) = 0$ reproduces local transverse d.o.f. E.g. If $y^2 = x^3 + z^5$ (i.e. E_8 on $z = 0$) can turn on $SU(2)$ gauge flux to break to E_7

$$\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi \end{bmatrix} \quad \Rightarrow \quad y^2 = x^3 + \phi^2 z^3 x + z^5$$

- T-brane:

$$\Phi = \begin{bmatrix} 0 & \phi \\ 0 & 0 \end{bmatrix}$$

- This still breaks $E_8 \to E_7$, but no longer visible in the complex structure.

Global Geometry+ T-branes

(with J. Heckman and S. Katz)

- How to extend local T-brane description to concrete global geometry?
- G-flux defined in Deligne Cohomology:

$$0 \rightarrow J^3(X) \rightarrow \mathcal{D} \rightarrow H^{2,2}(X, \mathbb{Z}) \rightarrow 0$$

- Need an intrinsic notion of these d.o.f in singular limit $(X_t \rightarrow X_0)$
- Key new ingredient: Diaconescu, Donagi, Pantev demonstrated that the moduli space of the Hitchin system over a curve can be identified with the moduli (complex structure and intermediate Jacobian) of a non-compact CY 3-fold....

- We found a partial compactification of the DDP results
- "Emergent" Hitchin System
- Limiting mixed Hodge structure analysis identifies the fibers of the parabolic Hitchin systems with part of limits of intermediate Jacobians $J(X_t)$ of 1-parameter smoothings X_t

$$
\begin{array}{ccc}
 & & M \\
 & \nearrow & \downarrow \\
\pi^* H & \to & \tilde{M}_{\rm cx} \\
\downarrow & & \downarrow \pi \\
H & \to & M_{\rm loc}
\end{array}
$$

A "Transition function" to patch open/closed string descriptions in limit $X_t \to X_0$

M_{loc} =Moduli of 7-brane curve

H = Hitchin Moduli space

$\tilde{M}_{cx}$ = Comp. Struc. of resolved CY

Conclusions and Future Directions

- $N = 1$ Heterotic/F-theory geometries are a fruitful arena for classifying/enumerating (a finite set) of dual geometries/string vacua
- Developed an algorithm to systematically build all 4-folds (w/ $\mathbb{P}^1$ bundle base $\mathcal{B}_3$, over B_2 (gdP))
- Explicitly constructed all Heterotic/F-theory dual pairs over toric bases (such that X_3 smooth).
- Non-trivially matched topological consistency conditions (η eff., bpf, etc) & developed vanishing conditions for $\mathcal{M}(c(V))$ on X_3
- A classification requires understanding G-flux in the singular limit
- $6D$ Global T-branes $\Rightarrow$ limiting mixed Hodge structures and emergent Hitchin systems
- A first step in a systematic study...

Thank you!