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Amplitudes and GR

Review Talk

Thanks to Y.F. Bautista, C. Kavanagh, A. Ochirov, , A. Laddha, J. Parra-Martinez, A. Strominger, J. Vines, D. O'Connell,... for many discussions!

Alfredo Guevara
Harvard University
Society of Fellows
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General Relativity is one of the prime examples of effective field theory describing the low energy regime of quantum gravity.

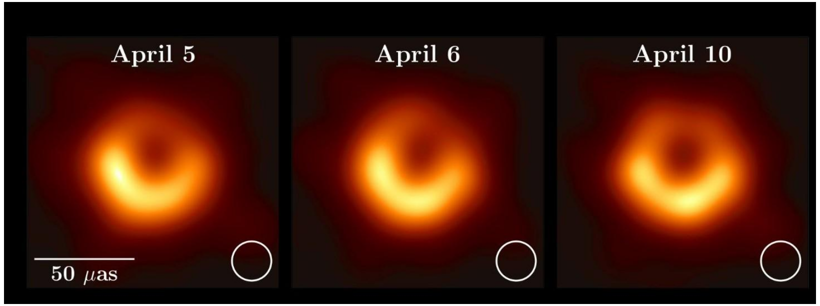
As such it has beautifully succeeded in making precise predictions at scales we can measure with our technology.

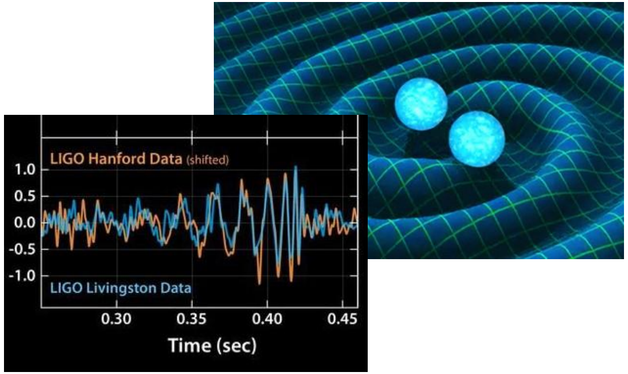
As technology progresses, so does the precision of our detectors. While quantum measurements are still out of reach, we can nowadays test GR to an unprecedented precision, expected to increase over the coming years!

This brings up to major problems, patent over the last 100 years.

1) How do we solve Einstein’s equations for realistic dynamical processes?

2) What are the observables of GR (as an EFT) we should be discussing?





P1) In order to devise exact solutions in GR a high amount of symmetry is implemented (stationary or static/cylindrical or spherical/various degrees of SUSY, etc….)

TABLE 2–3.1. Degenerate Static Vacuum Fields

1	2	3	4	5
Class	First Fundamental Form G	Coordinate Ranges	α	ν_S
A1	$r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) + \dfrac{dr^2}{1 - b/r} - \left(1 - \dfrac{b}{r}\right) dt^2$	$0 \leqslant \vartheta \leqslant \pi$, φ mod 2π $0 < b < r < \infty$ or $b < 0 < r < \infty$	$-\dfrac{b}{r^3}$	
A2	$z^2\,(dr^2 + \sinh^2 r\, d\varphi^2) + \dfrac{dz^2}{b/z - 1} - \left(\dfrac{b}{z} - 1\right) dt^2$	$0 \leqslant r < \infty$ φ mod 2π $0 < z < b$	$\dfrac{b}{z^3}$	0
A3	$z^2(dr^2 + r^2\, d\varphi^2) + z\, dz^2 - \dfrac{dt^2}{z}$	$0 \leqslant r < \infty$ φ mod 2π $0 < z < \infty$	$\dfrac{1}{z^3}$	
B1	$\dfrac{dt^2}{1 - b/r} + \left(1 - \dfrac{b}{r}\right) d\varphi^2 + r^2(d\vartheta^2 - \sin^2\vartheta\, dt^2)$	$0 < b < r < \infty$ or $b < 0 < r < \infty$ $0 < \vartheta < \pi$	$-\dfrac{b}{r^3}$	$-\left(\dfrac{\alpha}{b}\right.$
B2	$\dfrac{dz^2}{b/z - 1} + \left(\dfrac{b}{z} - 1\right) d\varphi^2 + z^2(dr^2 - \sinh^2 r\, dt^2)$	$0 < z < b$ $0 < r < \infty$	$\dfrac{b}{z^3}$	$-\left(\dfrac{\alpha}{b}\right.$
B3	$z\, dz^2 + \dfrac{d\varphi^2}{z} + z^2(dr^2 - r^2\, dt^2)$	$0 < z < \infty$ $0 < r < \infty$	$\dfrac{1}{z^3}$	0
C	$\dfrac{1}{(x + y)^2}\left(\dfrac{dx^2}{f(x)} + f(x)\, d\varphi^2 + \dfrac{dy^2}{\lvert f(-y)\rvert}\lvert f(-y)\rvert\, dt^2\right)$ $f(u) \equiv \pm(u^3 + au + b)$	$0 < x + y$ $f(-y) < 0$ $0 < f(x)$	$\pm(x + y)^3$	$f(x) -$

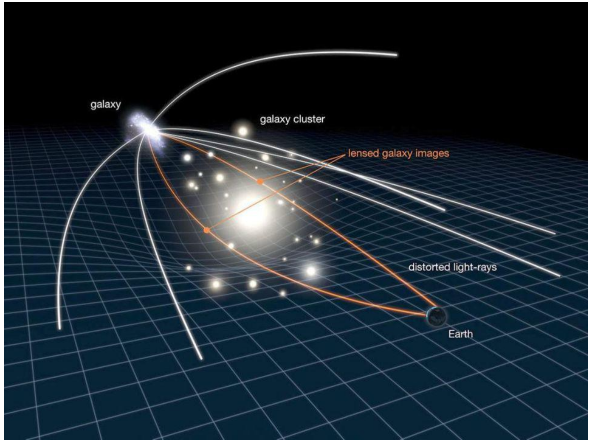
from Ehlers and Kundt, "Exact solutions of the gravitational field equations" (1962)

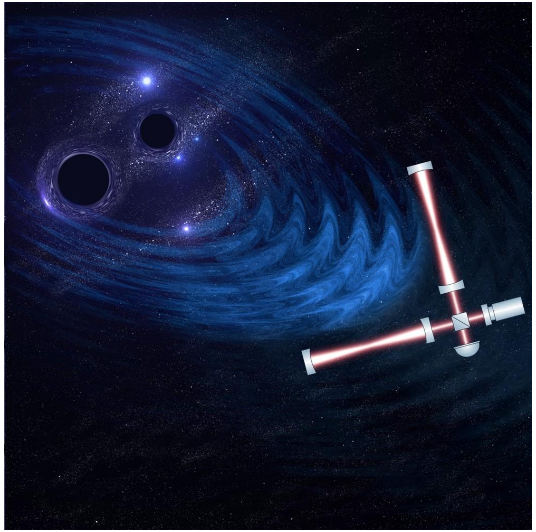
These solutions, albeit remarkable, fall short in describing more realistic time dependent scenarios, such as the perturbation of a BH background by a non-spherical wave, or the merger of two BHs!

P2) The main observables of QFT are scattering amplitudes. They have so far driven the success of predictions at large particle accelerators such as LHC.

Such a framework is infamously not applicable to quantum gravity due to UV divergences which make scattering amplitudes incapable of generating predictions.

Instead, in GR one considers a seemingly unrelated set of observables for different processes: light-bending angles, polarizations, radiated gravitational fluxes, time delays, etc





But most of these observables are extremely difficult to compute in realistic situations. For instance, in the case of GW mergers we need to rely on a subtle combination of analytic methods (the so-called Post-Newtonian or PN theory) and numerical relativity. The PN theory in turn relies on a perturbative approach to solve Einstein's equations. But these equations are highly non-linear and PN estimates become increasingly cumbersome as more precision is required.

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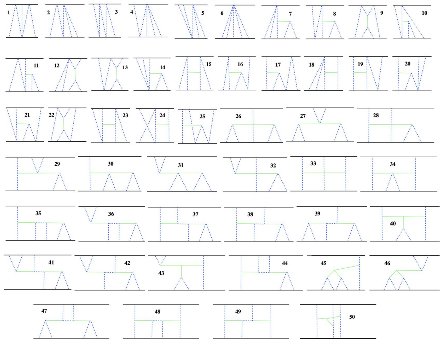
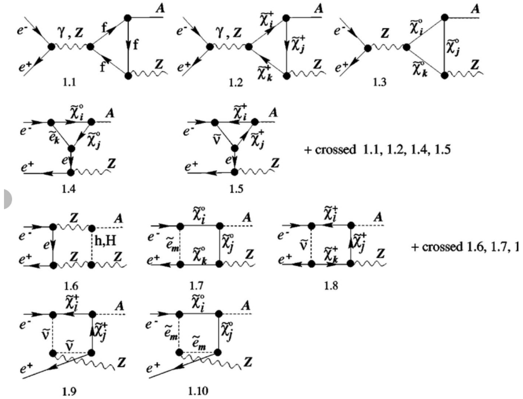




Note that the situation is reminiscent of (and indeed closely related to) the evaluation of a high number of Feynman diagrams when dealing with precision scattering of fundamental particles!

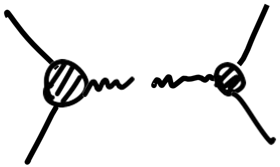
Goal of the talk

To give you a flavor of how **scattering amplitudes in QFT** can be used to define and compute (classical) **observables of GR**. Moreover, such observables characterize dynamical escenarios such as BH perturbations and the two-body problem.

They are free from UV divergences since they are defined from a classical limit of the scattering amplitude; involving low exchange momentum. Classically, we expect to describe dynamics at distances greater than the Schwarzschild radius.

In this regime we can fully trust GR as an EFT, and indeed we can compute observables to high accuracy in G/r with **exact dependence on the relative velocities** of the bodies. We will refer to such a scheme as the Post-Minkowskian (PM) approach.

The relevant scattering amplitudes can be computed using novel on-shell methods. This enforces unitarity in the underlying quantum theory, bypassing the evaluation of (tons of) Feynman diagrams. Such a powerful tool is completely ‘invisible’ if we just attempt to solve Einstein’s equations perturbatively.



This effectively provides a new analytical window into perturbation theory in GR, opening the door for new and exciting predictions!

Amplitudes as Gravitational Observables (I)

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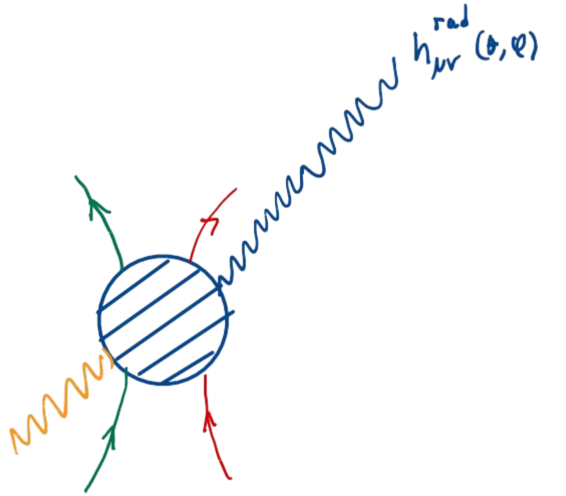
To begin with, we will be interested in observables that can be measured at far distances for asymptotically flat spacetimes.

Consider a perturbative approach

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$\hookrightarrow$ all orders in $\kappa = \sqrt{32\pi G}$



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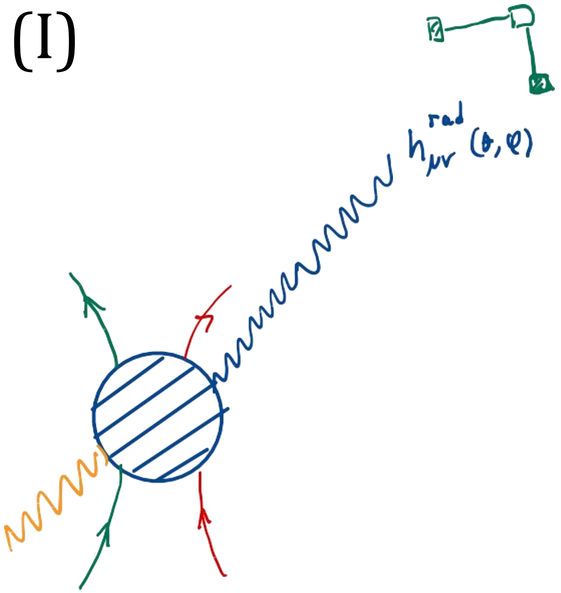

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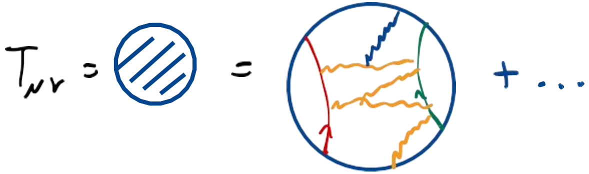
The radiative part of the metric will be measured at null infinity via

$$h^{rad}_{\mu\nu}(x) = \lim_{|x|,t\to\infty} \frac{\kappa}{2}\int d^4k \, \frac{e^{ik\cdot x}}{k^2} T_{\mu\nu}(k)$$

$$\sim \frac{\kappa}{|x|} e^{ik\cdot x} T_{\mu\nu}(k)\Big|_{k^2=0,\ \vec{k}\parallel\vec{x}}$$

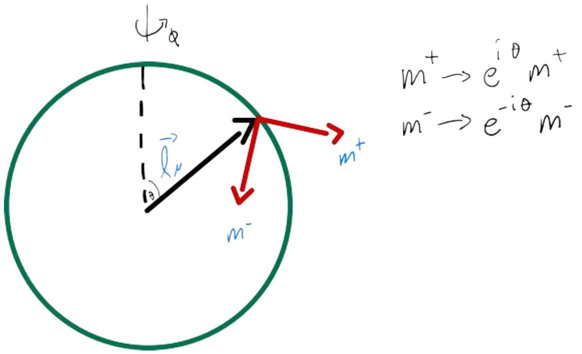


for an effective source including both matter and (non-linear) gravitational contributions.

The metric itself is not an observable. However, we can encode the gauge invariant information from the radiative data into a classical amplitude. Consider the null tetrad

$$\eta_{\mu\nu} = m^-_\mu m^+_\nu + m^+_\mu m^-_\nu + l_\mu n_\nu + n_\mu l_\nu$$

$$m^- \cdot m^+ = l \cdot n = 1$$



Here l_μ parametrizes a direction in the celestial sphere with tangent complex vectors m^+_μ, m^-_μ

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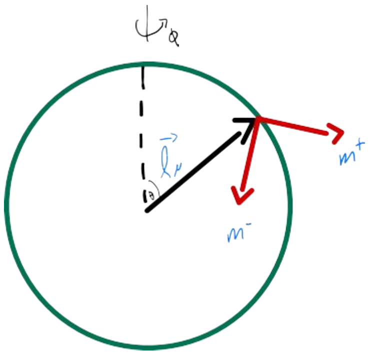

$$m^+ \rightarrow e^{i\theta} m^+$$
$$m^- \rightarrow e^{-i\theta} m^-$$

Here l_μ parametrizes a direction in the celestial sphere with tangent complex vectors m^+_μ, m^-_μ

For algebraically special spacetimes (type D or N) it is enough to consider the Newman-Penrose amplitude of helicity 2, given by

$$\Psi_4(\theta,\phi) = m^+_\mu n_\nu m^+_\rho n_\sigma \, C^{\mu\nu\rho\sigma} ,$$

$$\sim \frac{\kappa}{r} e^{ikx} \underbrace{m^+_\mu m^+_\nu}_{\epsilon^+_{\mu\nu}} T^{\mu\nu}(k) \Big|\Big|_{k^\mu = \omega l^\mu}$$

$$\overset{LSZ}{=} \frac{\kappa}{r} e^{ikx} \langle A_n(k) \rangle$$

$$h^{rad}_{\mu\nu} \sim \frac{\kappa}{r} e^{ik\cdot x} T_{\mu}(k) , \; k^2 = 0$$

The scattering amplitude $\langle A_n(k) \rangle$ is obtained by considering the classical limit of the graviton emission in QFT [Guevara, Bautista, Kavanagh, Vines; O’Connell, Monteiro, O’Connell, Veiga,Sergola].

Interestingly, the same derivation can be done in QED, where it is directly related to electromagnetic radiation. Indeed, for algebraically special spacetimes (D,N,O) both observables are related via the so-called classical double copy [White, Monteiro. O’connell;Luna, Ricardo Monteiro, Isobel Nicholson, Donal O'Connell, Guevara, Bautista, ...]

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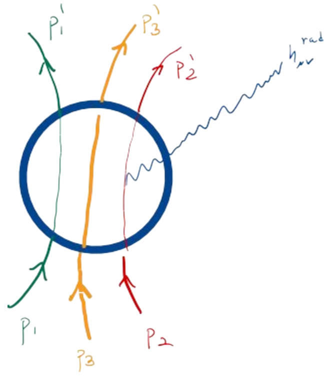
Now, the classical limit formally requires to incorporate an infinite number of gravitons in a coherent state in order to produce a macroscopic observable [Sahoo, Sen, Laddha; Strominger et al; O'Connell, Monteiro, O'Connell, Veiga,Sergola]. We will explain the mechanism in a second. But let us just mention that in practice, coherent means that we can consider only a single graviton as long as we define its classical limit with some care.

Let us see some examples. The simplest case comes from linear gravity: Massive bodies accelerating in flat space are equivalent to the following matter source [Braginski, Thorne]

$$T^{\mu\nu}(k) = \frac{\kappa}{2}\sum_i \frac{p_i^\mu p_i^\nu}{p_i\cdot k + i\epsilon} - \frac{p_i'^\mu p_i'^\nu}{p_i'\cdot k - i\epsilon} + O(k^0)$$



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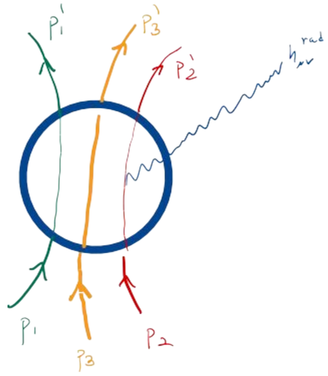
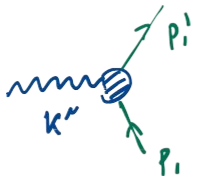


where $O(k^0)$ depends on details of acceleration and finite size effects.

For an isolated particle, $p_1' = p_1 + k$ and

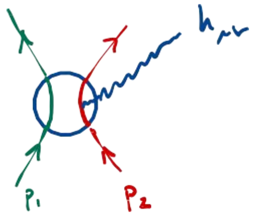
$$\langle A_3(k)\rangle = m^+_\mu m^+_\nu T^{\mu\nu}(k) = \frac{\kappa}{2}\,\delta(p_1\cdot k)\,\epsilon^+_{\mu\nu} p_1^\mu p_1^\nu =$$



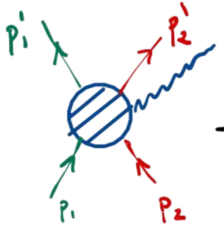
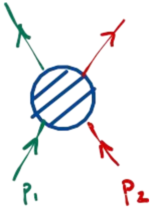
The (unique) 3pt amplitude for a graviton-scalar coupling!

For more particles, the classical current takes precisely the form of the well-known Weinberg soft-factor, e.g.

$$S^0(k) := \langle A_5(k) \rangle = \epsilon^+_{\mu\nu} T^{\mu\nu}(k) = \frac{\kappa}{2} \sum_{i=1,2} \left[\frac{\epsilon^+_{\mu\nu} p_i^\mu p_i^\nu}{p_i \cdot k + i\epsilon} - \frac{\epsilon^+_{\mu\nu} p_i'^\mu p_i'^\nu}{p_i' \cdot k - i\epsilon} \right]$$



This poses a small but interesting puzzle. In QFT we know that in the soft limit

$A_5(k) =$  $\longrightarrow S^0(k) \times$ 

What happens to the hard amplitude when we compute the classical current $\quad A_5(k) \longrightarrow \langle A_5(k) \rangle \quad$?

A way to explain this is that the QFT amplitude and the classical current are solving different scattering problems.

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In the classical scattering problem we fix instead the **initial velocities and positions** of the particles, i.e. in the far past. This is an initial value problem from which the final velocities are determined by integrating the equations of motion.

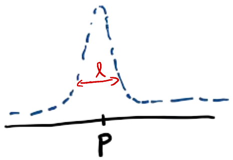
Luckily we can translate one amplitude into the other. For this, consider wavefunctions associated to the asymptotic trajectories (we can ignore Coulomb "drag" at this order)

$$x_1^\nu(\tau) = b_1^\nu + \frac{p_1^\nu}{m}\tau \qquad \Longrightarrow \qquad \psi_1(q) = e^{i b_1\cdot q}\,\Phi_{p_1}(q)$$

$$x_2^\nu(\tau) = b_2^\nu + \frac{p_2^\nu}{m}\tau \qquad \qquad \psi_2(q) = e^{i b_2\cdot q}\,\Phi_{p_2}(q)$$

where $\Phi_p(q)$ projects the amplitude into its classical momenta.

$\Phi_p(q)$:

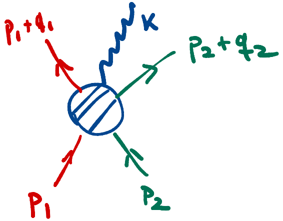


observers distance : $x >> \ell$ (radiation)

momentum transfer: $q << m$

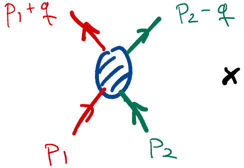
We thus obtain the following relation valid in linearized gravity:

$$\langle A_5(k)\rangle = \int d^4q_1\, d^4q_2\; e^{i q_1\cdot b_1}\, e^{i q_2\cdot b_2}\, \delta(q_1\cdot p_1)\,\delta(q_2\cdot p_2)$$



Taking the soft limit in both sides, i.e. equating the soft factors, we obtain [Guevara, Bautista '19]

$$\Delta p^\mu(b) = \int_{-\infty}^{\infty} \frac{dp^\mu}{d\tau}\, d\tau = \int d^4q\, e^{iq\cdot b}\, \delta(p_1\cdot q)\,\delta(p_2\cdot q)$$



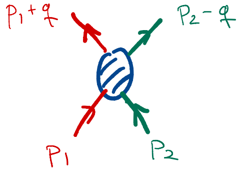
$$\times\ q^\mu$$

which reveals that the role of the 4-pt amplitude is precisely to integrate the equations of motion providing a formula for classical deflection solely as function of the initial data (the impact parameter $b=b_1-b_2$)

Only the singular term momentum transfer will contribute to the Fourier transform. We confirm that the classical limit corresponds to small momentum transfer/large impact parameter as assumed by the previous inequalities.

Eq. (1) is nothing but the well known relation between the S matrix and an interaction potential controlling 2-body dynamics, i.e. the Born approximation. In a certain "non-relativistic" gauge:

$$V(r) = \int d^3q\, e^{iq\cdot r}$$



However, note that eq. (1) is expressed in terms of observables of the classical problem/gauge invariant QFT amplitudes!

The previous formulae for the 5pt and 4pt amplitudes are the simplest (i.e. linear in G) instances of the Kosower-Maybe-O’Connell (KMO) formulae linking QFT amplitudes to classical observables of the scattering problem.

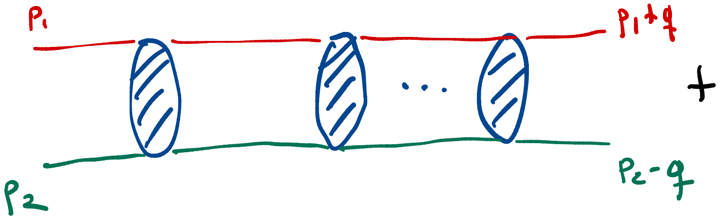
Let us provide a complementary perspective on such formulae, suitable for higher orders in G. Moreover, this will help us translate between the (classical) scattering problem and the (classical) bounded orbit problem.

Amplitudes as Gravitational Observables (II)

We have not yet given a satisfactory explanation on how microscopic effects (scattering of a single quanta) lead to macroscopic observables in GR.

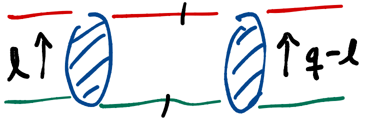
To the best of my knowledge the simplest way to understand this is through the eikonal approximation in QFT amplitudes ['t Hooft 87; Amati, Ciafaloni, Veneziano 88-92; Kabat-Ortiz '93; Verlinde '92...]

Consider the 4-pt S-matrix of two massive particles interacting through gravity. We have seen that the classical limit corresponds to low momentum-transfer, i.e. forward scattering. In this regime the amplitude is dominated by the following ladder diagrams:



$+$ perm , with $|q| \to 0$

The massive propagators can be put on-shell as a consequence of the forward limit. The ladder does not quite factorize since there are loop integrations running in the rings. The integrand is indeed a convolution:



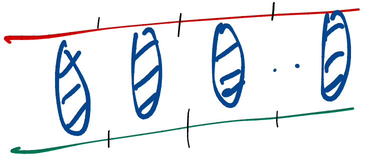
$$= \int d^4 l \, \delta(p_1 \cdot l)\, \delta(p_2 \cdot l)\, M_4(l)\, M_4(q-l)$$

If we define the Fourier transform

$$\boxed{i\chi(b) := \int d^4 q \, e^{i q \cdot b}\, \delta(p_1 \cdot q)\, \delta(p_2 \cdot q)\, M_4(q)}$$

the full ladder indeed factors and we obtain the eikonal exponentiation:

$$\sum_{\text{Loops}}$$



$$= \sum_n \frac{(i\chi(b))^n}{n!} = e^{i\chi(b)} - 1$$

Thus the contribution arising from infinitely many microscopic graviton exchanges indeed generates a macroscopic effect, the eikonal phase. This can be used to construct observables as we will explain in a moment. For instance, note that

$$\frac{\partial \chi(b)}{\partial b^\mu} = \int d^4 q \; e^{iq\cdot b} \, \delta(\;)\,\delta(\;)\, q^\mu \, \mathcal{M}_4(q) = \Delta p^\mu(b)$$

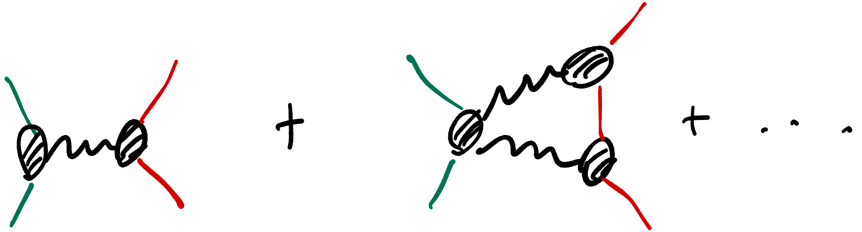
is precisely the KMO formula.

The exponential structure of the S-Matrix reflects that the gravitons are in coherent superposition. In particular, this picture shows that classical information can be contained in loop diagrams!

We have only considered factorization into tree-amplitudes, this is the equivalent of a computation in linearized gravity where we consider a geodesic in a linearized background (no-backreaction). The massless case, where the background is described by a shockwave metric, has been considered in many contexts since the 80's. The eikonal phase in the probe limit can also be computed via similar methods in AdS backgrounds and related to correlation functions of the dual CFT [Camanho, Edelstein, Maldacena, Zhiboedov; Afkhami-Jeddi, Kundu, Tajdinim]

However, in the recent years we have understood how the eikonal exponentiation indeed holds at higher-orders in G, including the full gravitational backreaction! This is, as long as we consider the classical limit via low momentum transfer [Amati,Ciafaloni, Veneziano;Saotome, Akhouri; Guevara, Ochirov, Vines; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove,...]

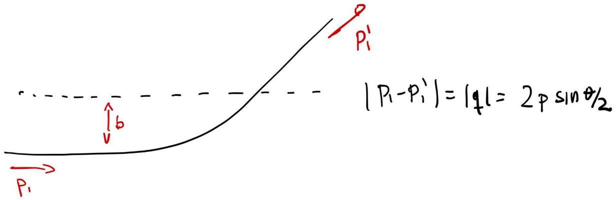
$\chi(b) =$



Observables from the eikonal phase

We can interpret the eikonal integral as a **path integral** with initial and final momenta fixed. The quantum trajectories are integrated as functions of the impact parameter, i.e. the radial distance. The classical trajectory is easily obtained from a saddle point, i.e. WKB approximation.

$$\mathcal{M}(\theta) = \int d^2 b \, e^{i q(\theta)\cdot b} \left(e^{i\chi(b)} - 1\right)$$

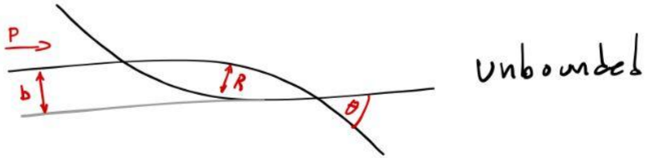


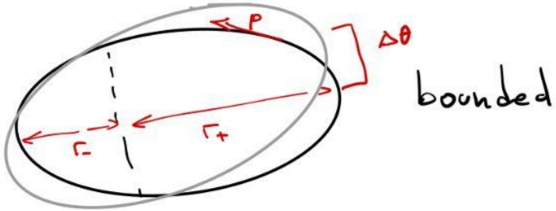
This perspective reveals that the eikonal phase is precisely the radial piece of **single-particle action**(*). The action describes the relative motion of one of the bodies, **scattering off an effective source (in the sense that it includes gravitational backreaction)** [Damour,Schaeffer 02; Porto,Kallin 19]**.**

$$S(E,b) = -Et + J\theta + \underbrace{\int_b^\infty p_r \, dr}_{\chi(b)} \quad , \; J = pb$$

The radial action allows for a direct passage between observables for hyperbolic (unbounded) or elliptic (bounded) motions.

Unbounded	Bounded
$S_r = \int_{R(b)}^{\infty} p_r\, dr = \chi(b)$	$S_r = \int_{r_-}^{r_+} p_r\, dr$
$\theta = \frac{\partial S_r}{\partial J} - \pi$ (scattering angle)	$\Delta\theta = \frac{\partial S_r}{\partial J}$ (periastron precession)
$\Delta t = \frac{\partial S_r}{\partial E}$ (Shappiro time-delay)	$\Delta T = \frac{\partial S_r}{\partial E}$ (period precession)



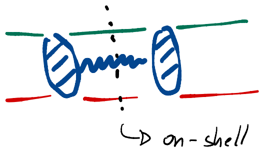


Nicely, we can perform the analytic continuation $p \to ip$ in the scattering momenta in order to obtain the bounded case and relate the observables. This leads to states with $E<0$; in fact the eikonal S-Matrix has poles for such negative energy values corresponding to the bounded spectrum [Kabat,Ortiz 92].

Using either the eikonal approach or a complementary EFT matching approach [Rothstein, Cheung, Solon '18] the classical part of the amplitude has been computed to an unprecedented precision by several groups [Bern, Parra-Martinez, Hermann, Roiban, Ruf, Shen;Bjerrum-Bohr, Damgaard, Planté, Vanhove ;Dlapa, Kalin, Liu, Porto]. This has led to results up to **3-loops, also known as 4PM or simply G^4**.

These results are consistent with state-of-the-art Post-Newtonian (PN) computations. Such computations considered low orbital velocities $v^2 \sim GM/r \ll 1$ adequate for bounded orbits. From the point of view of scattering amplitudes it is natural to obtain the exact velocity dependence: Indeed this has been implemented at the multiloop level via a system of simple differential equations in the velocities (IBP relations) which use PN data as initial conditions [Parra-Martinez, Ruf, Zeng].

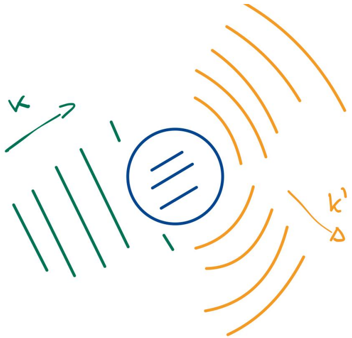
Radiation effects to the observables have also been incorporated. They can be linked to the imaginary part of the scattering phase, which leads to dissipation. This is controlled by on-shell internal gravitons:



[Bautista, Guevara, Kavanagh, Vines TBP;
Cristofoli, Gonzo, Kosower, O'Connell TBP]

Beyond the eikonal: A Wave-Particle duality

A seemingly unrelated problem in BH dynamics is the perturbation of a BH spacetime by a non-spherical wave. This is an important open problem: It arises if we ask about the stability of Schwarzschild or Kerr spacetimes under certain initial perturbations, and it is closely related to the study of their quasinormal modes [Regge,Wheeler 57; Christodoulou, Klainerman ‘90; **Dafermos, Holzegel, Rodnianski, Taylor ‘21**;...].



Again, the absence of symmetry makes the problem extremely hard to track, so we resort to perturbation theory. The first step was made by Regge and Wheeler in the 50’s, who studied linearized perturbations of helicity h=0,½,1,2 in the full (non-linear) Schwarzschild background.

Recall that the gauge invariant information of perturbations can be encoded in the Newman-Penrose amplitudes. Using this language we can decompose NP scalars in partial waves of helicity h:

$$\psi^{(h)}(t,r,\theta,\phi) \propto \frac{1}{r} e^{-i\omega t} \sum_{\ell} R_{\ell\omega}(r) \times {}_{h}Y_{\ell 0}(\theta,\phi)$$

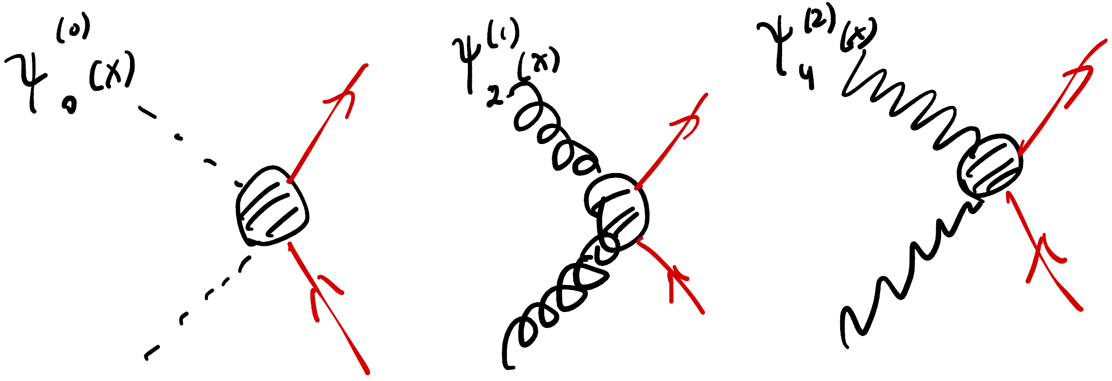
(h=2 for gravitational waves). The RW equation then determines the radial component of the NP scalar.

$$\frac{d^2 R_\ell(r)}{dr_*^2} + \left[E - V_\ell^h(r)\right] R_\ell(r) = 0 \qquad V_\ell^h(r) = \left(1 - 2\frac{GM}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2GM(1-h^2)}{r^3}\right]$$

This provides a direct link to QFT: The RW equation is nothing but a time independent Schrodinger equation, hence scattering can be mapped directly to a gravitational QFT amplitude. In other words, we can scatter the wave with a matter particle that plays the role of the BH effective potential.

We have argued that scattering amplitudes for graviton emission are directly related to classical sources emitting GWs.

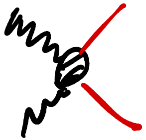
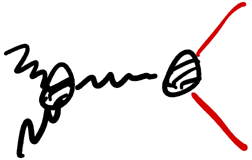
The correspondence holds for all helicities with the adequately defined NP scalars.



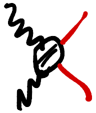
Crucially, none of the above amplitudes requires small momentum transfer to define its classical limit. We only require the wave frequency to be small $GM\omega = \frac{r_s}{\lambda}$ (and ignore corrections in $\frac{\hbar\omega}{M}$) so that the BH appears particle-like. This is the Born approximation.

If we further consider the eikonal limit of these amplitudes, $|q| \to 0$, we can interpret the wave as a classical particle propagating in a Schwarzschild background.

First, universality occurs in the sense that the amplitudes for helicities h=0,½,1,2 agree up to an arbitrary phase at leading order in G.

$\lim_{q \to 0}$  $=$ 

This is nothing but the equivalence principle. Indeed in the eikonal limit we have



$$= \sum_{\ell} \left(e^{i\delta_\ell} - 1\right) {}_hY_{\ell 0}(\theta, \varphi) \underset{\substack{\theta \to 0 \\ \ell = p\frac{b}{\hbar} \to \infty}}{=} \int d^2b \, e^{iq\cdot b} \left(e^{i\delta(b)} - 1\right)$$

[Ford & Wheeler]

Note: In the massless/boosted limit of the source the background is not Schwarzschild but the Aichelburg-Sexl shockwave. The wave-particle duality was considered long ago by `t Hooft.

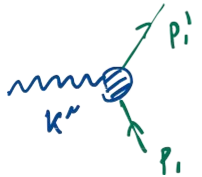
Final thoughts:

Classical Black Holes = (spinning) elementary particles?

Classical Black Holes = elementary particles?

Let us come back to the 3-pt amplitude of a graviton coupled to a scalar source:

$$\langle A_3(k)\rangle = m^+_\mu m^+_\nu T^{\mu\nu}(k) = \frac{\kappa}{2}\,\delta(p_1\cdot k)\,\epsilon^+_{\mu\nu}p_1^\mu p_1^\nu =$$

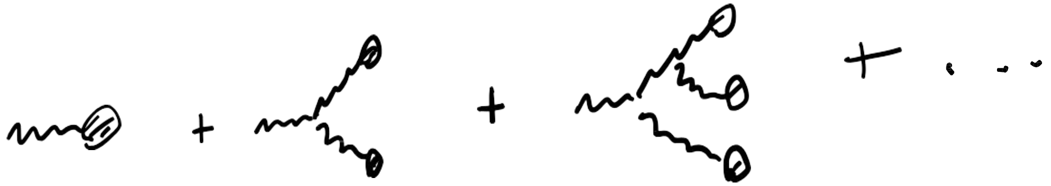


Due to the delta function in front, it does not have support on (3,1) signature!

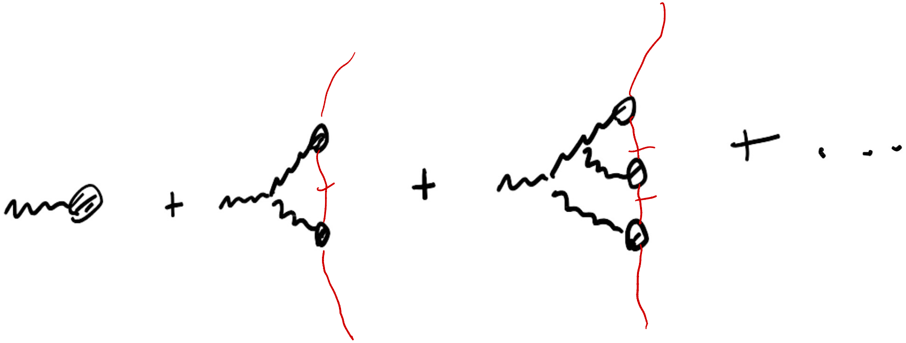
This reflects that a single isolated body yields a stationary metric and cannot emit radiation. Indeed the Schwarzschild metric has only Coulomb-like modes which are off-shell (they don't reach null infinity)

Duff [73] studied this off-shell modes in momentum space and showed that the Schwarzschild metric, in harmonic gauge, can be obtained perturbatively by iterating a classical source

$$\text{for } |x|<2GM: \quad h_{\mu\nu}(x) = \frac{\kappa}{\Box} T^{\text{full}}_{\mu\nu} =$$



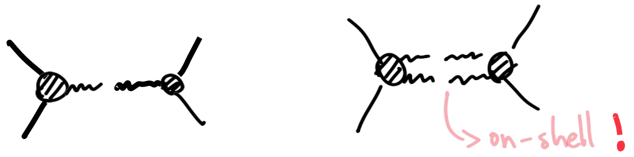
In more modern terms, this is indeed a multiloop computation (with cut propagators), and the classical source can be readily written in terms of the worldline formalism



But such metric is neither an observable nor a scattering amplitude, indeed it was obtained in the harmonic gauge. So how can we recover a BH spacetime solely by scattering massive particles?

For starters, we have seen that 2-body dynamics and BH perturbations are encoded in 4-pt scattering amplitudes, so the background spacetime must be somehow encoded in the corresponding observables. For instance, one can reconstruct the Schwarzschild spacetime by studying the effective potential of a test particle, which in turn is controlled by the scattering angle.

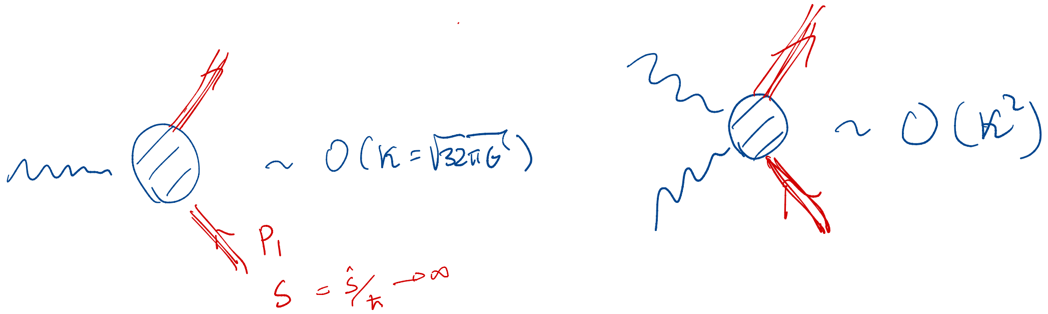
This hints that the on-shell internal gravitons, which indeed contain the classical information of the 4pt amplitudes, are more than enough to recover the background spacetime.



Even though the previous 3pt amplitude vanishes, it still has support on **complexified momenta** and can be used as a **building block** to construct higher-point observables associated to BH spacetimes. This is nothing but the standard lore of the **on-shell program**.

An alternative perspective: We can analytically continue to (2,2) signature where the 3-pt amplitude does not vanish, and indeed matches the (2,2) background metric! This shows that the the amplitude contains the necessary information (after analytic continuation the Coulomb modes indeed reach null infinity) [Guevara, Maybee, Ochirov, O'connell, Vines; Ricardo Monteiro, O'Connell, Peinador Veiga, Sergola]

Moreover, very recently it has been noted that the 3-pt amplitude for massive particles of infinite quantum spin number can be used to construct observables associated to the Kerr black hole [Guevara, Bautista, Ochirov, Vines; Huang, Kim, Chung, Arkani-Hamed;....]



The corresponding 4-pt amplitude can be easily constructed at low orders in spin (aw) via dimensional reduction [Guevara, Bautista], but in order to reconstruct the full spin dependence at 4-pts we need to consider higher-spin amplitudes and match them to the corresponding NP scalars in Kerr backgrounds [Guevara, Bautista, Kavanagh, Vines]. This shows a fascinating link between higher-spin particles and Kerr perturbations!

Related: QNM? Kerr/CFT?

Thanks!

Other discussion points:

- Most of the construction can be extended to include spin.
- Supergravity results, graviton dominance?
- Non-minimal couplings, tidal effects,....
- Methods seem to escalate well---> current 3-loop results
- Is there hope for non-perturbative understanding?