An exact AdS_3/CFT_2 duality

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Based on:
LE, M.R. Gaberdiel and R. Gopakumar: 1812.01007 & *to appear*,
LE, M.R. Gaberdiel: 1903.00421 & 1904.01585,
A. Dei, LE, M.R. Gaberdiel: *to appear*

Motivation

- The AdS/CFT correspondence has been tremendously successful in the last 22 years and many instances have been checked.

- So far, we are missing a solvable stringy AdS/CFT correspondence, in which the underlying physics can be explored in detail.

- AdS_3/CFT_2 is a candidate for an exact stringy correspondence, since both string theory on AdS_3 and CFT_2 are under much better control than in other dimensions.

An exact AdS/CFT duality

Conjecture

Strings on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ with one unit of NS-NS flux

$$=$$

$$\mathrm{Sym}^N(\mathbb{T}^4) \ .$$

Checked on the level of

- (Nonprotected!) spectrum to exact in α'!
  [LE, Gaberdiel, Gopakumar '18, *to appear*]
- Symmetry algebra [LE, Gaberdiel '19]
- Some correlation functions and indications of the correct structure for all higher genus corrections
  [Dei, LE, Gaberdiel, *to appear*; LE, Gaberdiel, Gopakumar, *to appear*]

$\mathrm{AdS}_3/\mathrm{CFT}_2$ holography

- The string background

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

is believed to be on the same moduli space of CFTs that contains the symmetric product orbifold

$$\mathrm{Sym}^N(\mathbb{T}^4) \equiv \left(\mathbb{T}^4\right)^N / S_N \ .$$

[Maldacena '97; . . . ; e.g. David, Mandal, Wadia '02]

- However it is *not clear* what precise string background is being described by the symmetric orbifold theory itself.
see however [Larsen, Martinec '99]

Pure NS-NS flux

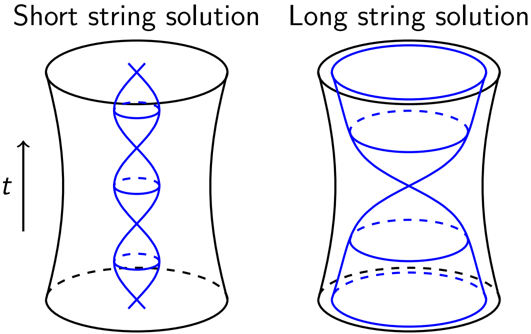
- There is an explicitly solvable worldsheet theory for strings on this background in terms of an $\mathrm{SL}(2,\mathbb{R})$ WZW model (for pure NS-NS flux). [Maldacena, (Son), Ooguri '00 & '01]

- However it is *not known* what precise dual CFT (on the above moduli space) this corresponds to.

Pure NS-NS flux

- A naive argument seems to imply that the pure NS-NS background *cannot* be dual to the symmetric product orbifold CFT.
- The basic reason for this is that the WZW model describes the pure NS-NS background which is known to have long string solutions. [Seiberg, Witten '99]

Long and short strings



- These long strings live close to the boundary and give rise to a continuum of excitations that are not present in the symmetric orbifold theory.

Tensionless strings

- In the tensionless limit

$$\ell_{\text{AdS}} \sim \ell_{\text{string}} ,$$

  the dual CFT becomes (almost) free. Conserved currents correspond to massless higher spin fields in the bulk.
  [Fradkin & Vasiliev, '87; Sundborg, '01; Klebanov & Polyakov '02; ...]

- The symmetric product orbifold contains a *free* subsector and hence seems to describe tensionless strings in AdS_3.
  [Gaberdiel & Gopakumar '14]

- This suggests that the symmetric product orbifold is located at a (near) tensionless point in the string moduli space.

- At pure NS-NS flux, minimal tension is achieved when the background has exactly one unit of NS-NS flux.

What about the continuum?

- I have argued that pure NS-NS AdS_3 string theory has a continuum.
- The worldsheet analysis will show that this continuum vanishes for $k = 1$.
- This aligns with the fact that a single NS5-brane does not produce a throat and hence there is no continuum.
  [Callan, Harvey, Strominger '91; Seiberg, Witten '99]

The ingredients

The RNS formalism

- Strings on pure NS-NS backgrounds can be described by a WZW model on the worldsheet:

$$\mathfrak{sl}(2,\mathbb{R})_k^{(1)} \oplus \mathfrak{su}(2)_k^{(1)} \oplus \mathbb{T}^4 \oplus \text{ghosts} \ .$$

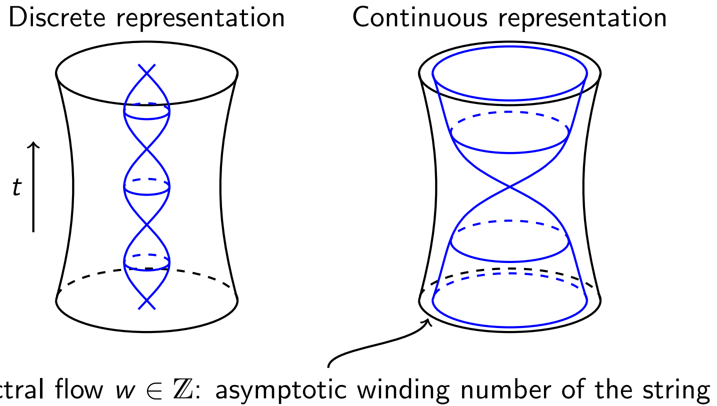
$$\mathfrak{su}(2)_{k-2} \oplus 3 \text{ free fermions}$$

- k: amount of NS-NS flux.

- For $k=1$, the $\mathfrak{su}(2)_{k-2}$ factor has negative level.

Representations of $\mathfrak{sl}(2,\mathbb{R})_k$

- The worldsheet theory contains discrete and continuous representations which are interpreted as short and long strings:



An intuition about $k = 1$

- Since $c(\mathfrak{su}(2)_{-1}) = -3$, this factor eats degrees of freedom.
- In a more precise sense, it eats 4 fermionic degrees of freedom and one bosonic degree of freedom.
- Instead of the usual 8 transverse bosonic and fermionic oscillators, the $k = 1$ background has only 4 transverse bosonic and fermionic oscillators.
- This matches the degrees of freedom of $\mathbb{T}^4$.
  [Gaberdiel, Gopakumar '18]

The hybrid formalism

- One way to make this intuition precise is by considering an alternative description of string theory on this background: [Berkovits, Vafa, Witten '99]

$$\mathsf{PSU}(1,1|2)_k \oplus \mathbb{T}^4 \oplus \text{ghosts} \ .$$

- Worldsheet supersymmetry is traded for spacetime supersymmetry:

$$\mathsf{PSU}(1,1|2)_{\text{bosonic}} = \mathsf{SL}(2,\mathbb{R}) \times \mathsf{SU}(2) \ .$$

- In this formalism, the $k = 1$ theory is defined very naturally. For it, we have to understand the $\mathsf{PSU}(1,1|2)_1$ WZW-model.

The $\mathrm{PSU}(1,1|2)_1$ WZW model

- $\mathrm{PSU}(1,1|2)_1$ has a free field realisation in terms of free fermions and symplectic bosons!

- Similarly to the $\mathrm{SL}(2,\mathbb{R})_k$ WZW model, representations are built on discrete and continuous representations of $\mathrm{PSU}(1,1|2)$ and their spectrally flowed images.

- Since we are considering a supergroup WZW model, the ground state representation can be either short (atypical) of long (typical).

The $PSU(1,1|2)_1$ WZW model

- $\mathrm{PSU}(1,1|2)_1$ contains only short (atypical) representations and in particular, the theory contains no long string continuum! [LE, Gaberdiel, Gopakumar '18]
- This is forced by the representation theory of $\mathfrak{psu}(1,1|2)_1$ (somewhat similar to $\mathfrak{su}(2)_1$).
- The only surviving classical string state is the long string which has just enough energy to escape to the boundary of AdS_3.
- The free field realisation allows us to prove consistency of the model. See also [Gotz, Quella, Schomerus '06; Ridout '10]

The PSU(1, 1|2)$_1$ partition function

- The PSU(1, 1|2)$_1$ partition function takes the form

dual CFT modular parameter

worldsheet modular parameter

$$Z_{\mathrm{PSU}(1,1|2)_1}(\tau,\bar{\tau}) = \int_0^1 \mathrm{d}\lambda \sum_{w=1}^{\infty} \delta_{\mathbb{Z}}^{(2)}(t-\tau w) \times |q|^{w^2-2\lambda w}|x|^{2\lambda}\left|\frac{\vartheta_2(\frac{t}{2};\tau)^2}{\eta(\tau)^4}\right|^2 .$$

integral over continuous representations

only 2 bosonic and 4 fermionic oscillators, instead of 6 bosonic and 8 fermionic

- $q = \mathrm{e}^{2\pi i\tau}$, $x = \mathrm{e}^{2\pi i t}$.

Similar to SU(2)$_1$!

Localisation

- The partition function localises onto holomorphic maps from the worldsheet torus to the boundary torus.
- We have found signs of similar localisation properties in the moduli space of Riemann surfaces of other correlation functions. [LE, Gaberdiel, Gopakumar, *to appear*]
- This is reminiscent of a topological string theory.

The string partition function

- Imposing physical state conditions gives the string partition function of the $k=1$ background [LE, Gaberdiel, Gopakumar '18]

ground state energy of spectrally flowed sectors

$$Z_{\text{string}}(t,\bar{t}) = \sum_{w=1}^{\infty} |x|^{\frac{w}{2}} Z_{\mathbb{T}^4}^{\text{NS/R}'}\left(\tfrac{t}{w}, \tfrac{\bar{t}}{w}\right) \ .$$

modular parameter of dual CFT, $x = e^{2\pi i t}$

Fractionally moded $\mathbb{T}^4$ partition function

Matching with the symmetric product

- This matches the large N limit of the symmetric product orbifold!
- Spectral flow is mapped to the length of the twist in the symmetric orbifold.

What about $k > 1$?

The correspondence for $k > 1$

- For generic $k > 1$, we can use the standard RNS formalism.

- The long string spectrum matches precisely the symmetric orbifold [LE, Gaberdiel '19]

$$\mathrm{Sym}^N\big(\big(\mathcal{N}=4 \text{ Liouville theory with } c=6(k-1)\big) \times \mathbb{T}^4\big) \ .$$

- This contains the $k = 1$ case as a limiting case and hence motivates it also in the RNS formalism.

- The presence of the long string continuum and its disappearance mirrors precisely the string worldsheet.

Further checks

The symmetry algebra

- We also showed that the symmetry algebra of the symmetric product orbifold is reproduced from string theory. [LE, Gaberdiel '19]

- To do so, we considered a set of DDF [Del Giudice, Di Vecchia, Fubini '72] operators which act on the *continuous part of the* physical Hilbert space of string theory on $\text{AdS}_3 \times \text{S}^3 \times \mathbb{T}^4$. [Giveon, Kutasov, Seiberg '98; de Boer, Ooguri, Robins, Tannenhauser '98]

The symmetry algebra

- In the w-th spectrally flowed sector, they satisfy the fractionally moded algebra (where modes take values in $\frac{1}{w}\mathbb{Z}$)

$$\mathcal{N} = 4 \text{ superconformal algebra with } c = 6w(k-1)$$
$$\oplus\, 4 \text{ free bosons \& 4 fermions} .$$

- For $k = 1$, this collapses to the algebra of four fractionally moded bosons and fermions.

- This coincides precisely with the symmetry algebra acting on the single-particle Hilbert space of the w twisted sector of the symmetric product orbifold $\text{Sym}^N((\mathcal{N} = 4 \text{ Liouville}) \times \mathbb{T}^4)$.

Null vectors

- Correlation functions in 2d CFTs are constrained by null-vectors.
- We have shown that null vectors in the CFT are mapped to BRST exact states on the worldsheet which decouple in string theory. [Dei, LE, Gaberdiel, *to appear*]
- As a consequence, the string correlation functions satisfy the same differential equations as the CFT correlation functions.
- This is enough to prove that the seed theory of the symmetric product is described by a Liouville theory. [Teschner '95]

Conclusions

- We have provided evidence that the symmetric orbifold is exactly dual to string theory with one unit of NS-NS flux

$$\mathrm{Sym}^N(\mathbb{T}^4) = \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4 \text{ with } k = 1 \ .$$

- We have similarly motivated that the long-string sector of the pure NS-NS flux background $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ is dual to

$$\mathrm{Sym}^N\big(\mathcal{N} = 4 \text{ Liouville theory with } c = 6(k-1) \ \oplus \mathbb{T}^4\big) \ .$$

- We have checked this at the level of the spectrum, the symmetry algebra and partially for correlation functions.

- This is an AdS/CFT correspondence, where both sides are completely accessible!